PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 28 – Geometric Optics

Spring 2014 Semester

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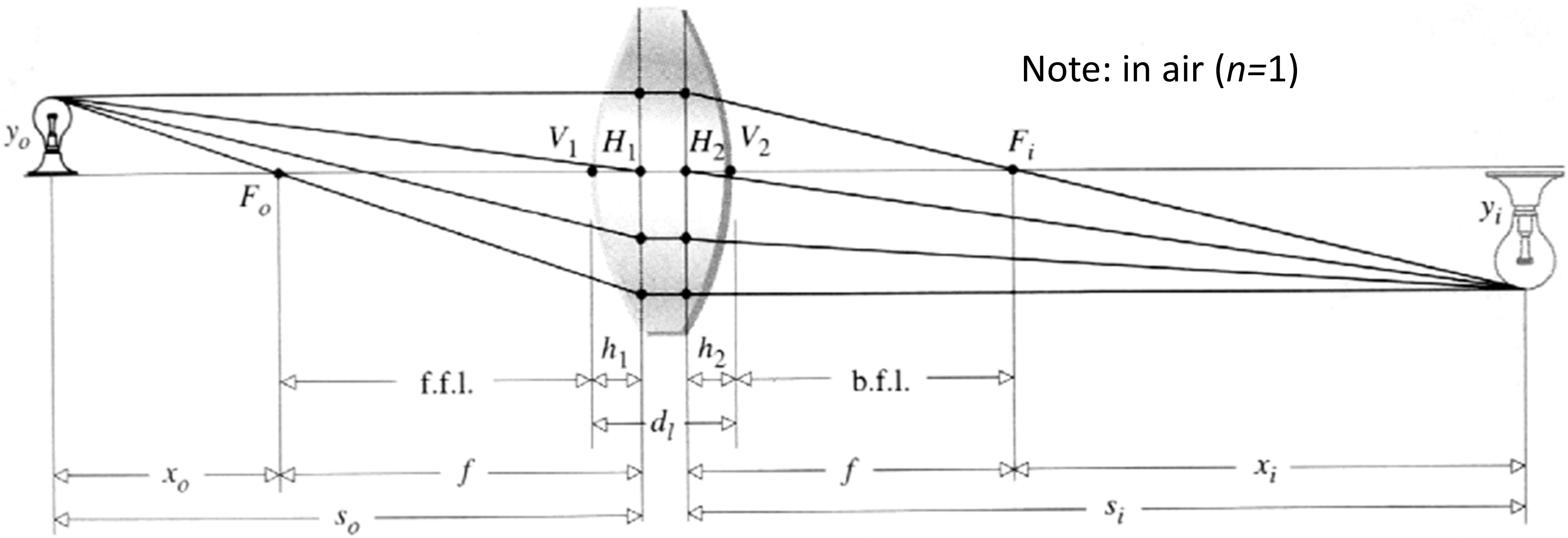
Thick Lens: equations

Note: in air (n=1)



$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

$$x_o x_i = f^2$$

effective focal length:

$$\frac{1}{f}=(n_l-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n_l-1)d_l}{n_l R_1 R_2}\right]$$

Principal planes:

$$h_1=-\frac{f(n_l-1)d_l}{n_l R_2}$$

$$h_2=-\frac{f(n_l-1)d_l}{n_l R_1}$$

Magnification:

$$M_T\equiv\frac{y_i}{y_o}=-\frac{s_i}{s_o}=-\frac{x_i}{f}=-\frac{f}{x_o}$$

Thick Lens Calculations

1. Calculate focal length

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

2. Calculate positions of principal planes

$$h_1 = -\frac{f(n-1)d}{nR_2}$$

$$h_2 = -\frac{f(n-1)d}{nR_1}$$

3. Calculate object distance, s_o, measured from principal plane
4. Calculate image distance:

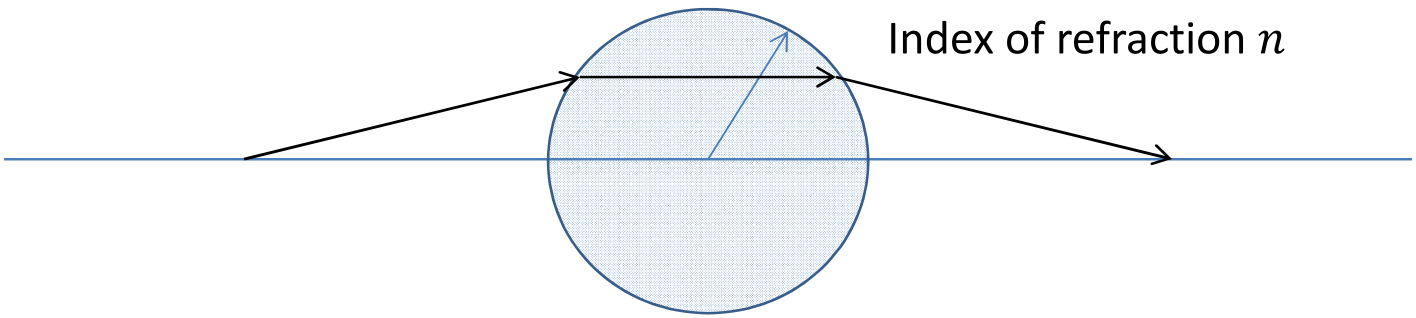
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

5. Calculate magnification, $m_T = -s_i/s_o$

Thick Lens Equation

- How can we check that our understanding of this is correct?
- Are we using a consistent sign convention?
- Do we understand how the distances h_1 and h_2 are defined?
- Let's apply this to a non-trivial example that we can solve in three different ways…
  - From our basic knowledge of optics
  - Using the thick-lens equation
  - Using ray-tracing

Simple Example



- Sphere with radius R (by definition R>0).
- Object placed at the focal point of first refracting surface:

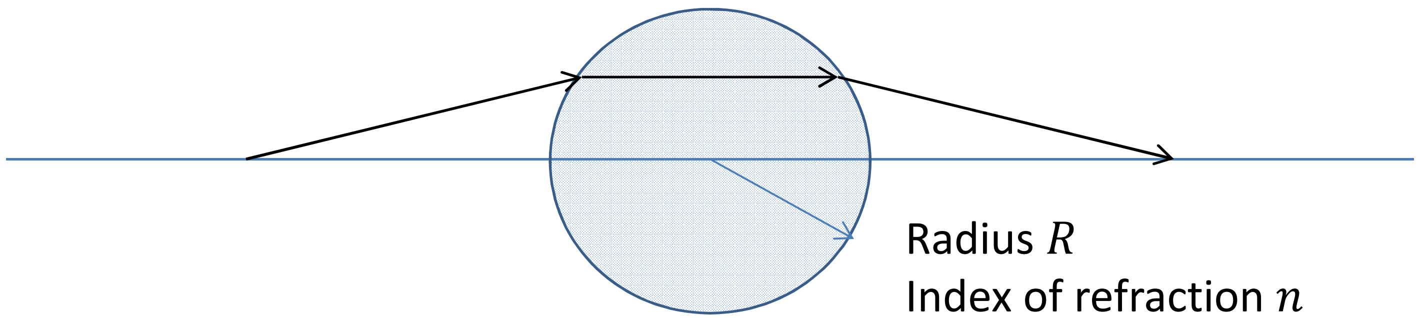
$$\frac{1}{s_o} + \frac{n}{s_i} = \frac{n-1}{R_1}$$

Convex: $R_1 > 0$

$$\frac{1}{f} = \frac{n-1}{R}$$

$$f = \frac{R}{n-1}$$

Simple Example



- Sphere with radius R (by definition R>0).
- Image formed at the focal point of the second surface:

$$\frac{n}{s_o} + \frac{1}{s_i} = \frac{1-n}{R_2}$$

Concave: $R_2 < 0$

$$\frac{1}{s_i} = \frac{n-1}{R}$$

$$s_i = \frac{R}{n-1} = s_o$$

Thick Lens Equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{n\,R_1 R_2}\right)$$

$$R_1 = R > 0$$

$$R_2 = -R < 0$$

$$d = 2R$$

$$\frac{1}{f} = (n-1)\left(\frac{2}{R} - \frac{2(n-1)}{nR}\right) = \frac{2(n-1)}{nR}$$

$$f = \frac{R}{2}\frac{n}{n-1}$$

Thick Lens Equation

- We also need to know the positions of the principle planes…

$$h_1 = -\frac{f(n-1)d}{n\,R_2}$$

$$= -\frac{R}{2}\frac{n}{n-1}\frac{(n-1)}{n(-R)}(2R)$$

$$= R$$

- Similarly,

$$h_2 = -\frac{f(n-1)d}{n\,R_1} = -R$$

Thick Lens Equation

- The first principal plane H_1 is located a distance h_1 from the first vertex.
- An object placed a distance s_o from the vertex would be located a distance

$$s_o' = s_o + h_1$$

from the first principal plane.

- Now use the thick lens equation:

$$\frac{1}{s_i'} + \frac{1}{s_o'} = \frac{1}{f} \Rightarrow s_i' = \frac{nR}{n-1}$$

Thick Lens Equation

- What is the distance from the second vertex?

$$s_i = s_i' + h_2$$

$$= \frac{nR}{n-1} - R$$

$$s_i = \frac{R}{n-1}$$

- This agrees with the previous calculation.
- What about ray tracing?

Tray Tracing Example

- Transfer matrix (distance d in medium n_1):

$$\begin{pmatrix} n_2\alpha_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ d/n_1 & 1 \end{pmatrix} \begin{pmatrix} n_1\alpha_1 \\ y_1 \end{pmatrix}$$

- Refraction matrix (spherical surface)

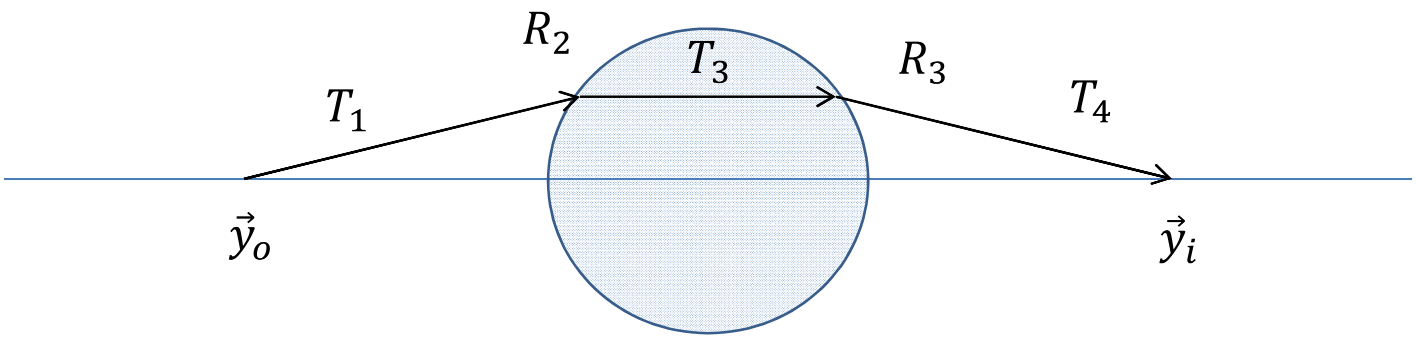
$$\begin{pmatrix} n_2\alpha_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & -D \\ 0 & 1 \end{pmatrix} \begin{pmatrix} n_1\alpha_1 \\ y_1 \end{pmatrix}$$

$$D = \frac{n_2 - n_1}{R}$$

- This example:

$$\vec{y}_o = T_5 R_4 T_3 R_2 T_1 \vec{y}_i$$

Ray Tracing Example



- Initial ray:

$$\vec{y}_o = \begin{pmatrix} \alpha \\ 0 \end{pmatrix}$$

- Final ray should cross the optical axis at a distance s_i from the second vertex.
- Multiply the matrices, solve for s_i…

Ray Tracing Example

- Use Mathematica…

In[10]:=
```
(*  Propagate ray to the image point *)
yi = tm[si, 1].y4
```

Out[10]=

$$\left\{\left\{\alpha + \frac{(1-n)\ so\alpha}{r} - \frac{(-1+n)\left(so\alpha + \frac{2\ r\left(\alpha + \frac{(1-n)\ so\alpha}{r}\right)}{n}\right)}{r}\right\},\right.$$

$$\left.\left\{so\alpha + \frac{2\ r\left(\alpha + \frac{(1-n)\ so\alpha}{r}\right)}{n} + si\left(\alpha + \frac{(1-n)\ so\alpha}{r} - \frac{(-1+n)\left(so\alpha + \frac{2\ r\left(\alpha + \frac{(1-n)\ so\alpha}{r}\right)}{n}\right)}{r}\right)\right\}\right\}$$

In[13]:=
```
(*  The condition for an image is that the ray crosses the optical axis
 at the image point.  So we need to solve for si as a function of so.  *)
solution = Solve[yi[[2]] == 0, {si}]
```

Out[13]=

$$\left\{\left\{si \rightarrow \frac{r\ (2\ r + 2\ so - n\ so)}{-2\ r + n\ r - 2\ so + 2\ n\ so}\right\}\right\}$$

Ray Tracing Example

- Use Mathematica…

In[10]:=
```
(*  Propagate ray to the image point *)
yi = tm[si, 1].y4
```

Out[10]=

$$\left\{\left\{\alpha+\frac{(1-n)\ so\alpha}{r}-\frac{(-1+n)\left(so\alpha+\frac{2\ r\left(\alpha+\frac{(1-n)\ so\alpha}{r}\right)}{n}\right)}{r}\right\},\left\{so\alpha+\frac{2\ r\left(\alpha+\frac{(1-n)\ so\alpha}{r}\right)}{n}+si\left(\alpha+\frac{(1-n)\ so\alpha}{r}-\frac{(-1+n)\left(so\alpha+\frac{2\ r\left(\alpha+\frac{(1-n)\ so\alpha}{r}\right)}{n}\right)}{r}\right)\right\}\right\}$$

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```

Out[13]=

$$\left\{\left\{si\to\frac{r\ (2\ r+2\ so-n\ so)}{-2\ r+n\ r-2\ so+2\ n\ so}\right\}\right\}$$

Ray Tracing Example

- Use Mathematica…
  - Object position was at the focal point of the first refracting surface:

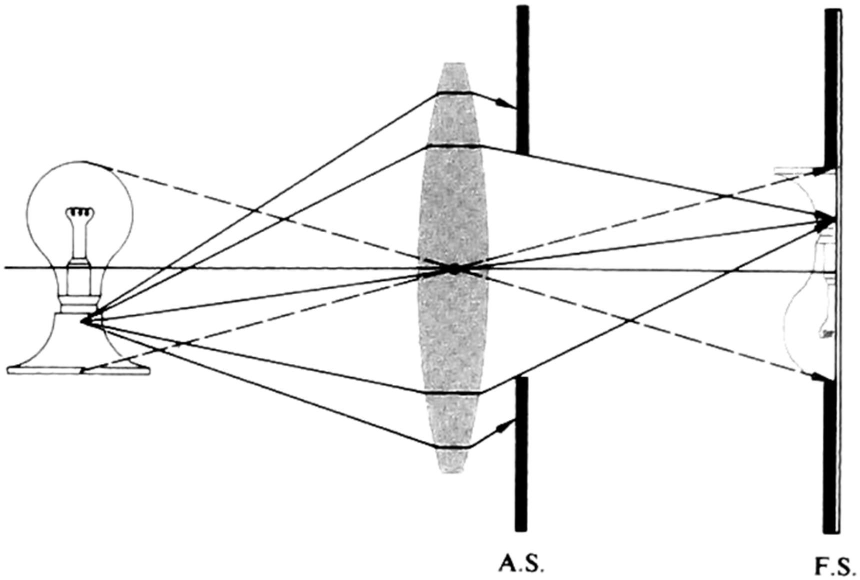
$$s_o = \frac{R}{n-1}$$

```
In[15]:=  FullSimplify[si /. solution/. {so → r / (n - 1)}]

Out[15]=  { r / (-1 + n) }
```

- It works!

Apertures and Stops



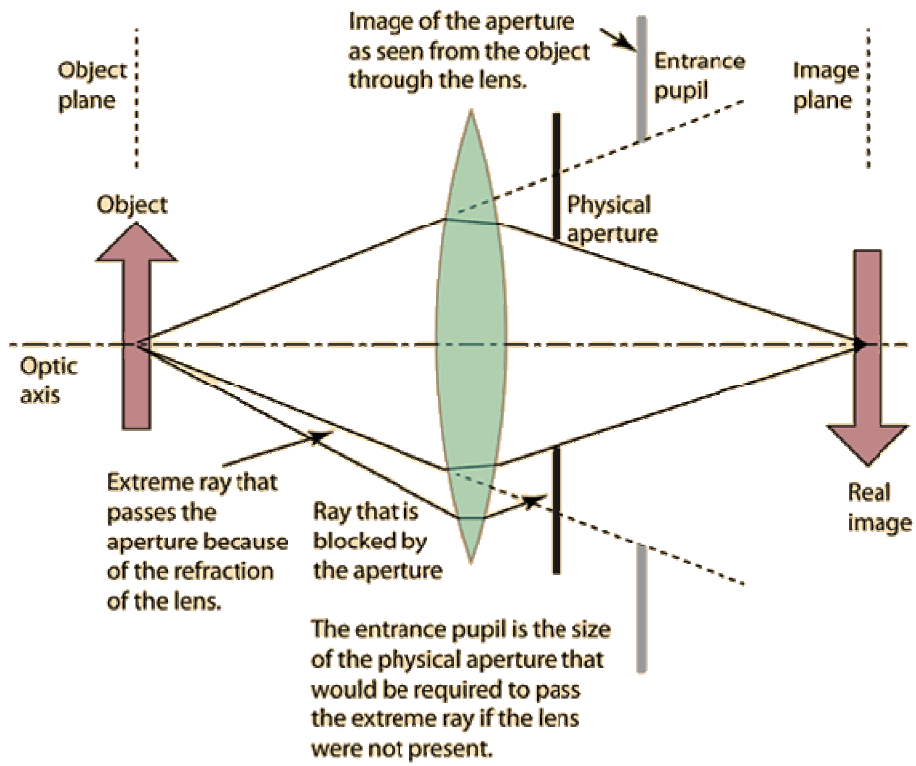
Field stop - an element limiting the size, or angular breadth of the image (for example film edge in camera)

Aperture stop - an element that determines the amount of light reaching the image

- Field stop determines the field of view and limits the size of objects that can be imaged.
- Aperture determines amount of light only

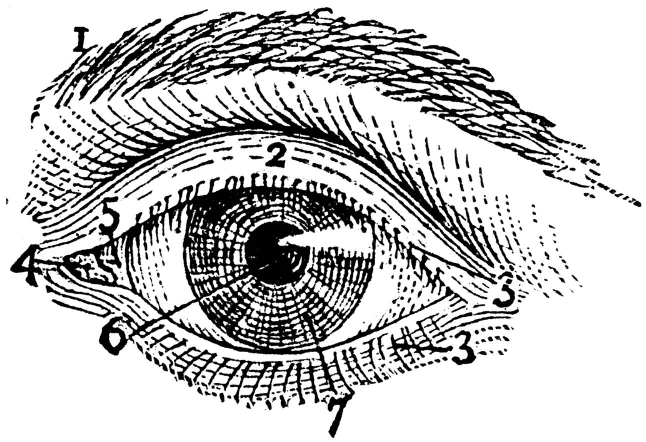
Entrance Pupil

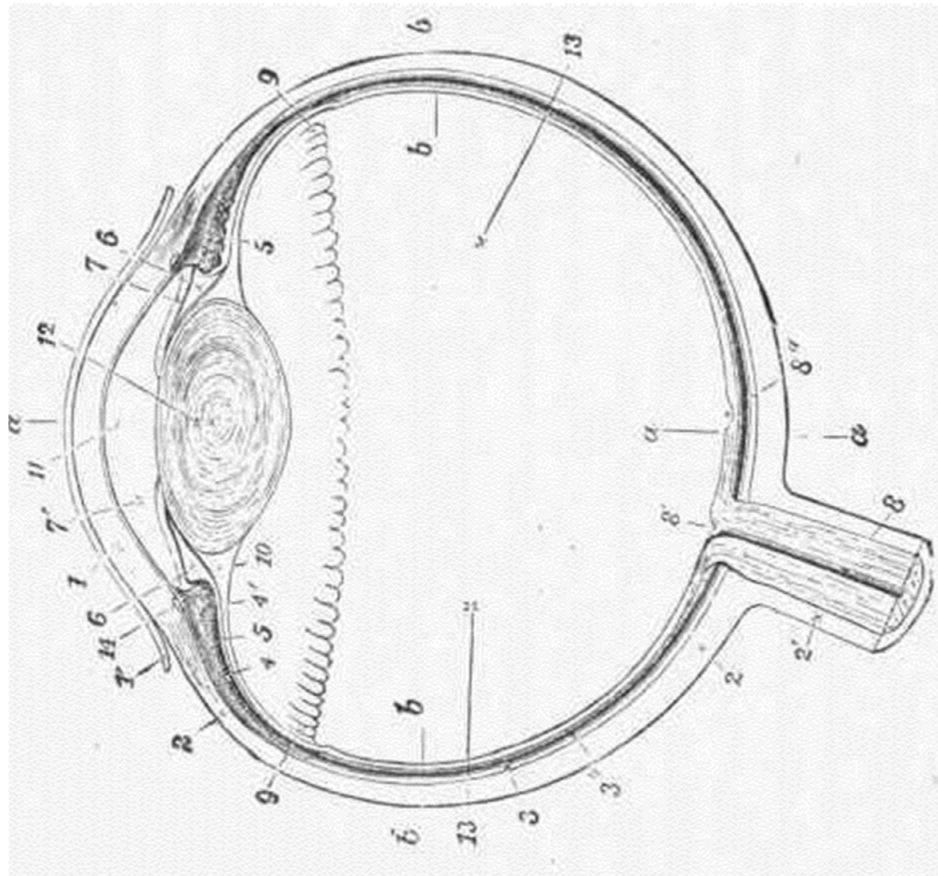
- How big does the aperture stop appear when viewed from the position of the object?



(Aperture stop is behind the lens)

Entrance Pupil

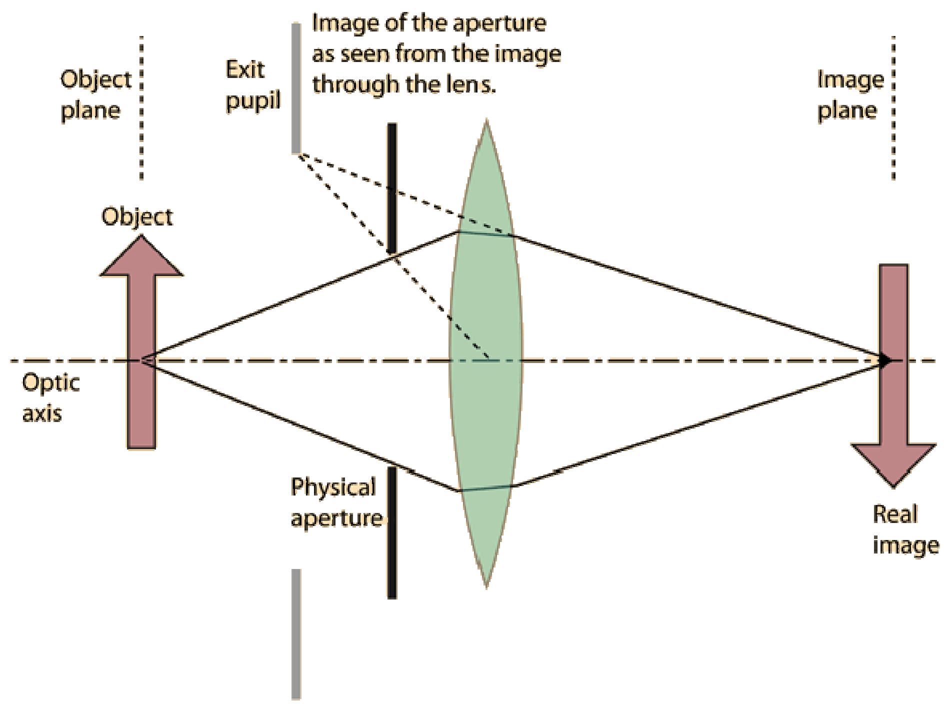




- If the cornea were removed, the pupil would appear smaller
- The cornea magnifies the image of the pupil

Exit Pupil

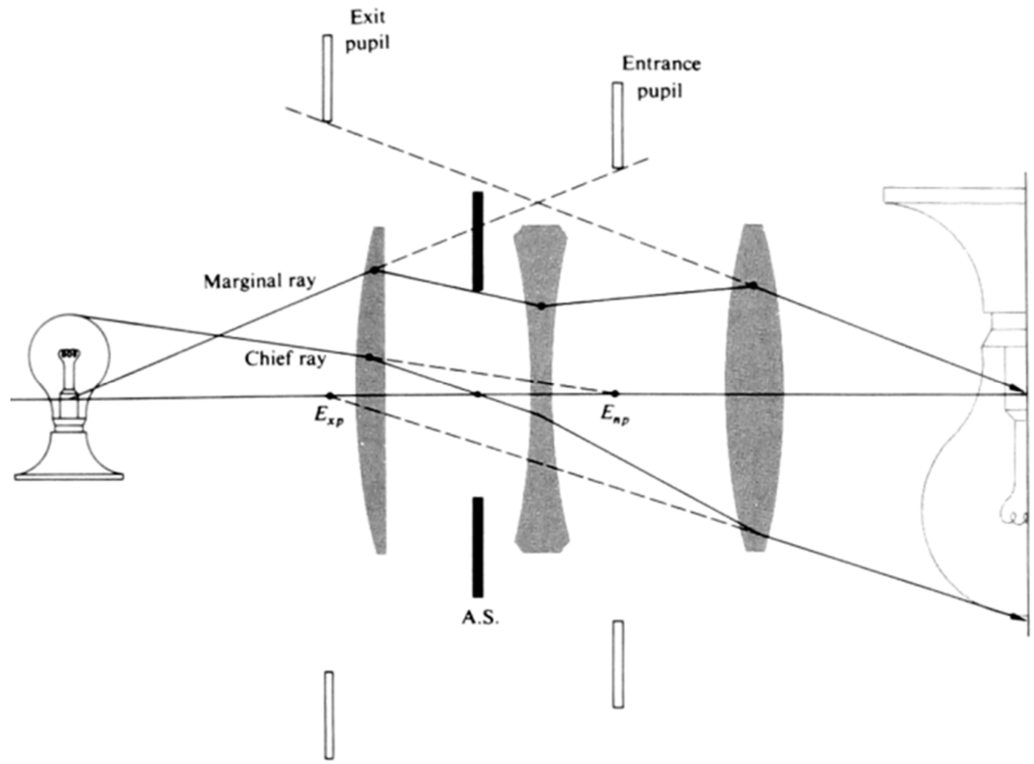
- How big does the aperture stop appear when viewed from the image plane?



(Aperture stop is in front of the lens)

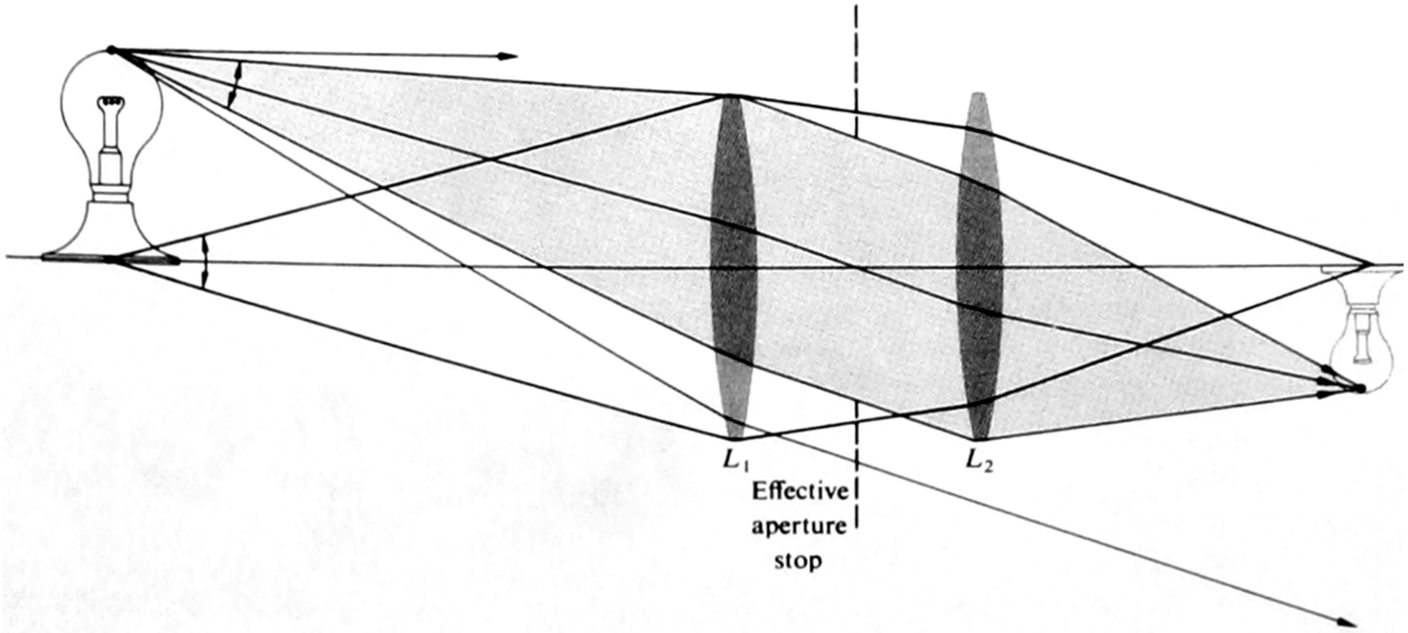
Chief and marginal rays

Marginal ray: the ray that comes from point on object and marginally passes the aperture stop



Chief ray: any ray from an object point that passes through the middle of the aperture stop
It is effectively the central ray of the bundle emerging from a point on an object that can get through the aperture.
Importance: aberrations in optical systems

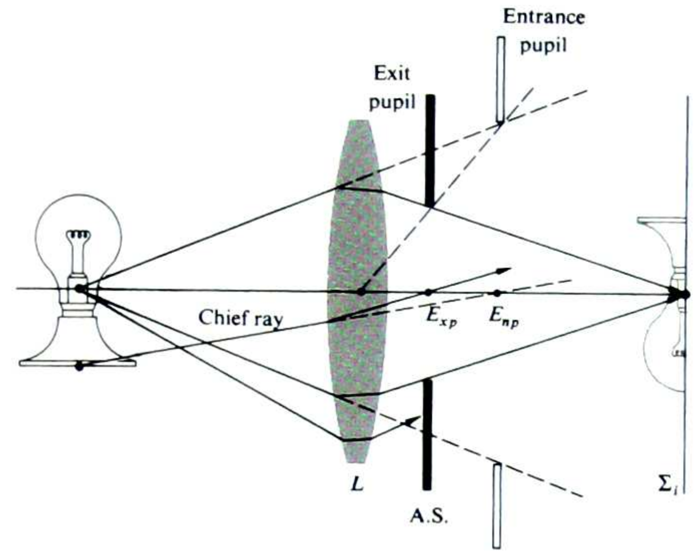
Vignetting



The cone of rays that reaches image plane from the top of the object is smaller than that from the middle. There will be less light on the periphery of the image - a process called **vignetting**

Example: entrance pupil of the eye can be as big as 8 mm. Telescopes are designed to have exit pupil of 8 mm for maximum brightness of the image

Relative Aperture



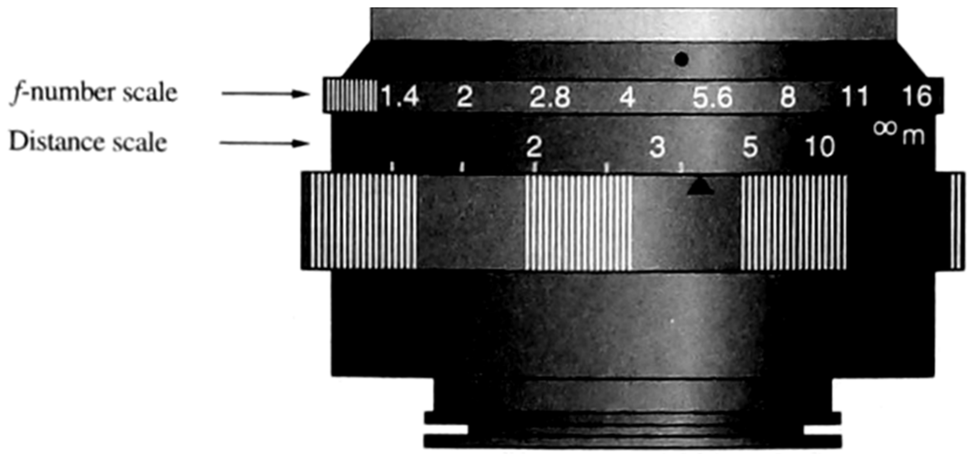
- The area of the entrance pupil determines how much light will reach the image plane.
- Pupils are typically circular: the area varies as the square of the diameter, D.
- The image area varies as the square of the lateral dimension, $A \sim f^2$
- Light intensity at the image plane varies as $(D/f)^2$
- (D/f) is called the *relative aperture*

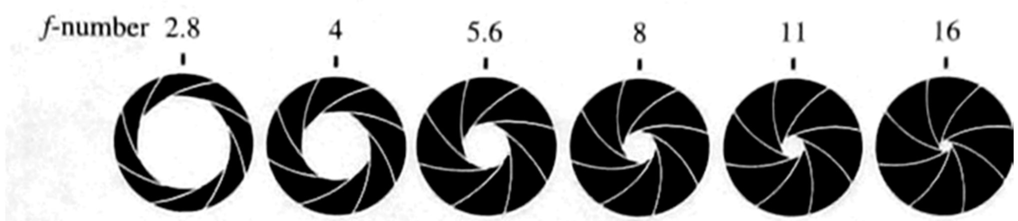
Relative Aperture

- Relative aperture: $f/D =$ (focal length/diameter)
- For optical equipment (camera lenses) this is usually labeled as $f/\#$
- Example:
  - $f = 50\,mm$
  - $D = 25\,mm$

  $\Big\}$ $f/D = 2$ denoted "$f/2$"
- This provides a standard way to reference the intensity of light shining on film or other photosensitive material.

f-number of a camera lens

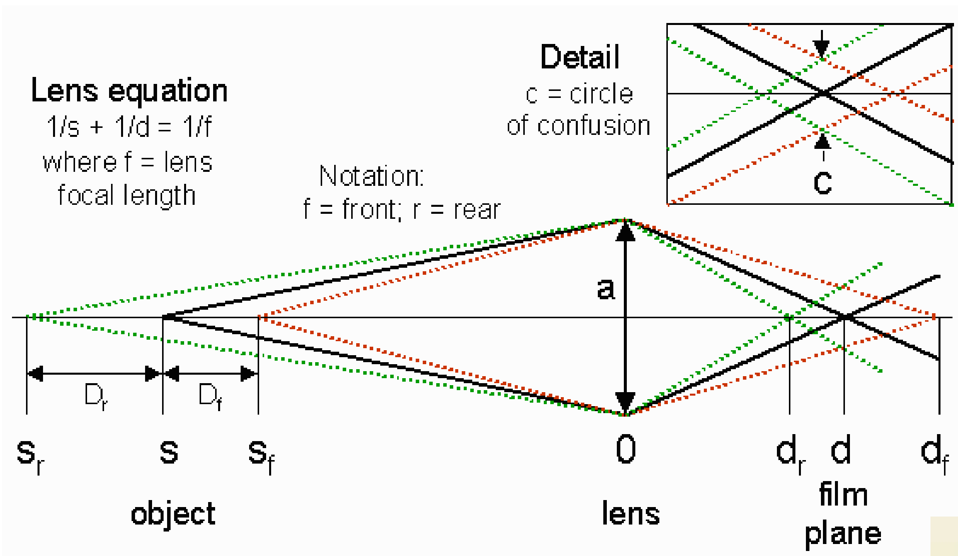




Change in neighboring numbers is $\sqrt{2}$

Intensity is ~$1/(f/\#)^2$: changing diaphragm from one label to another changes light intensity on film 2 times

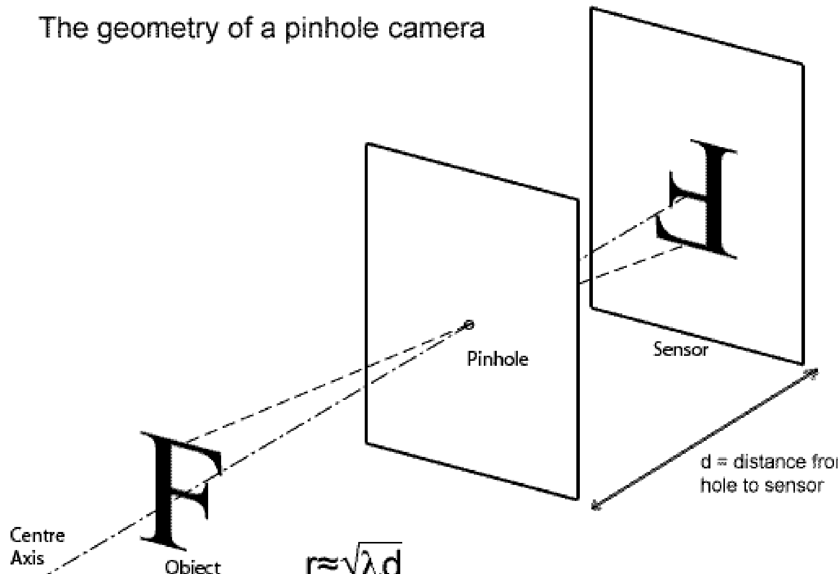
Depth of Field



Depth of Field

- Extreme case is the pinhole camera

The geometry of a pinhole camera



www.northlight-images.co.uk/article_pages/Canon_1ds_pinhole.html