PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 25 – Propagation of Light

Spring 2014 Semester

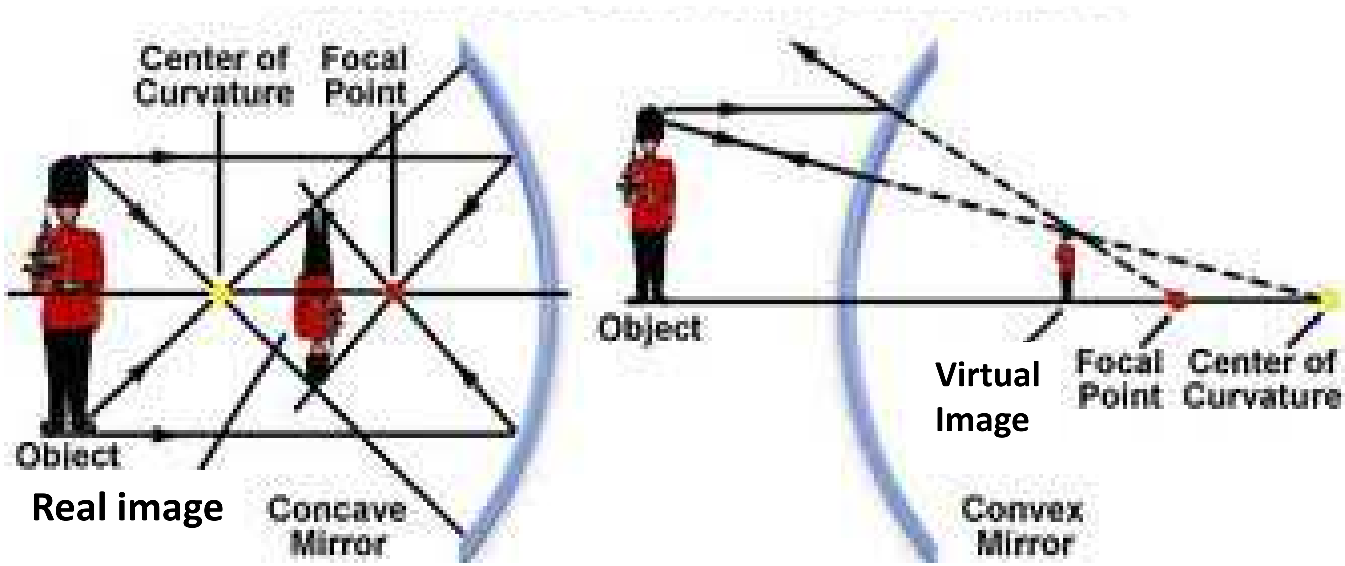
Matthew Jones

Geometric Optics

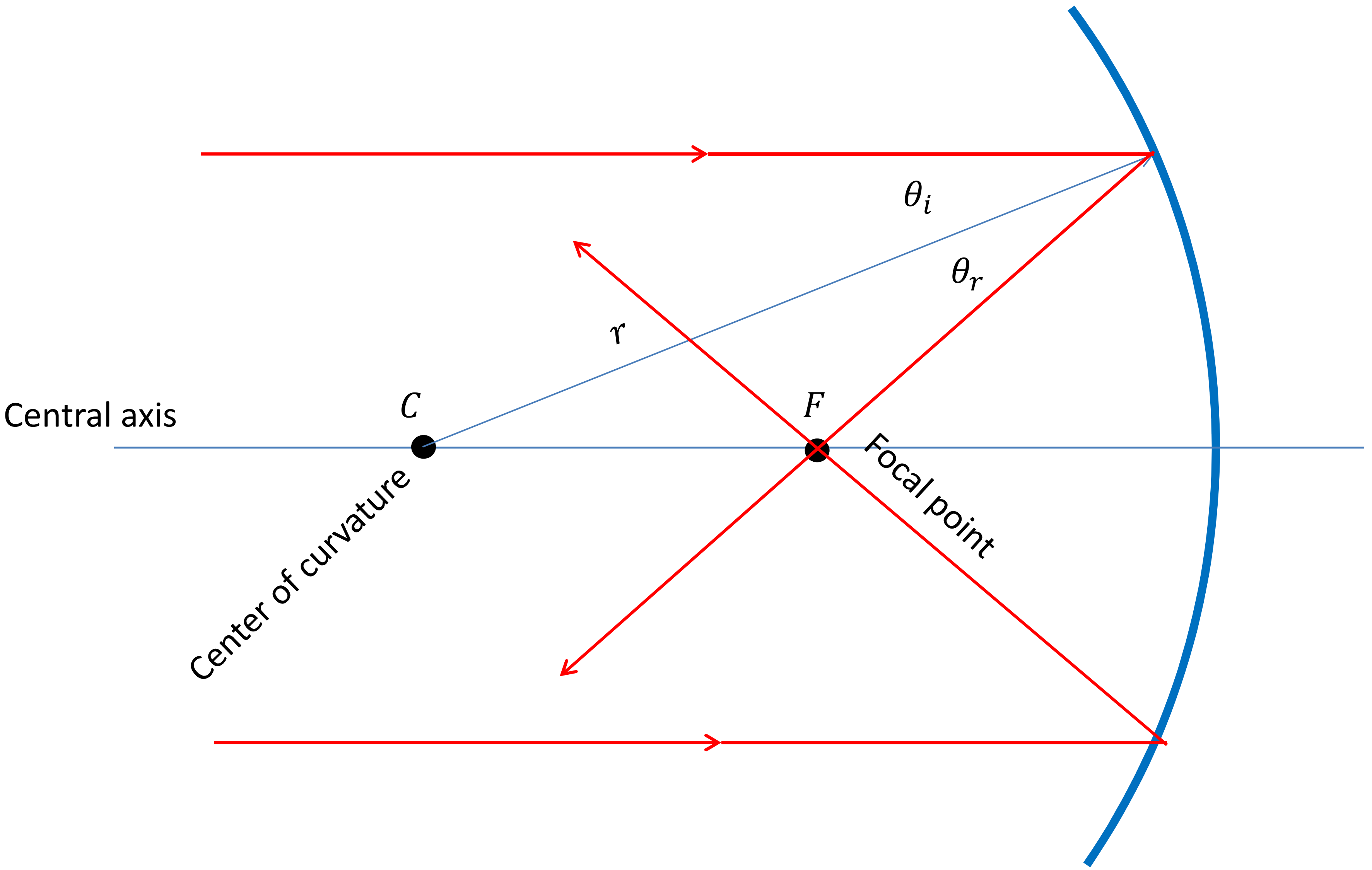
- Typical problems in geometric optics:
  - Given an optical system, what are the properties of the image that is formed (if any)?
  - What configuration of optical elements (if any) will produce an image with certain desired characteristics?
- No new physical principles: the laws of reflection and refraction are all we will use
- We need a method for analyzing these problems in a systematic an organized way

Types of Images

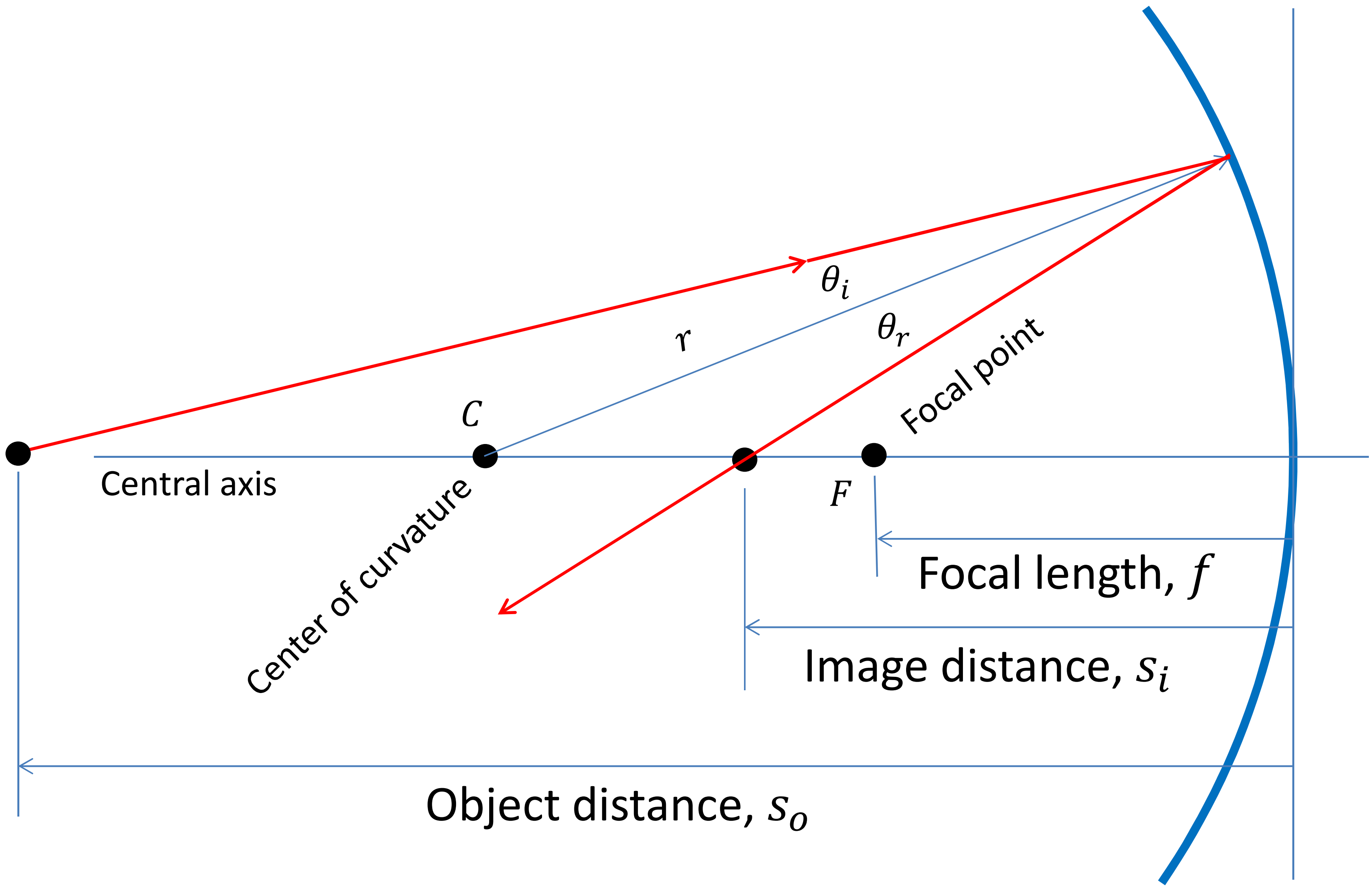
- **Real Image**: light emanates from points on the image
- **Virtual Image**: light *appears* to emanate from the image



Spherical Mirrors

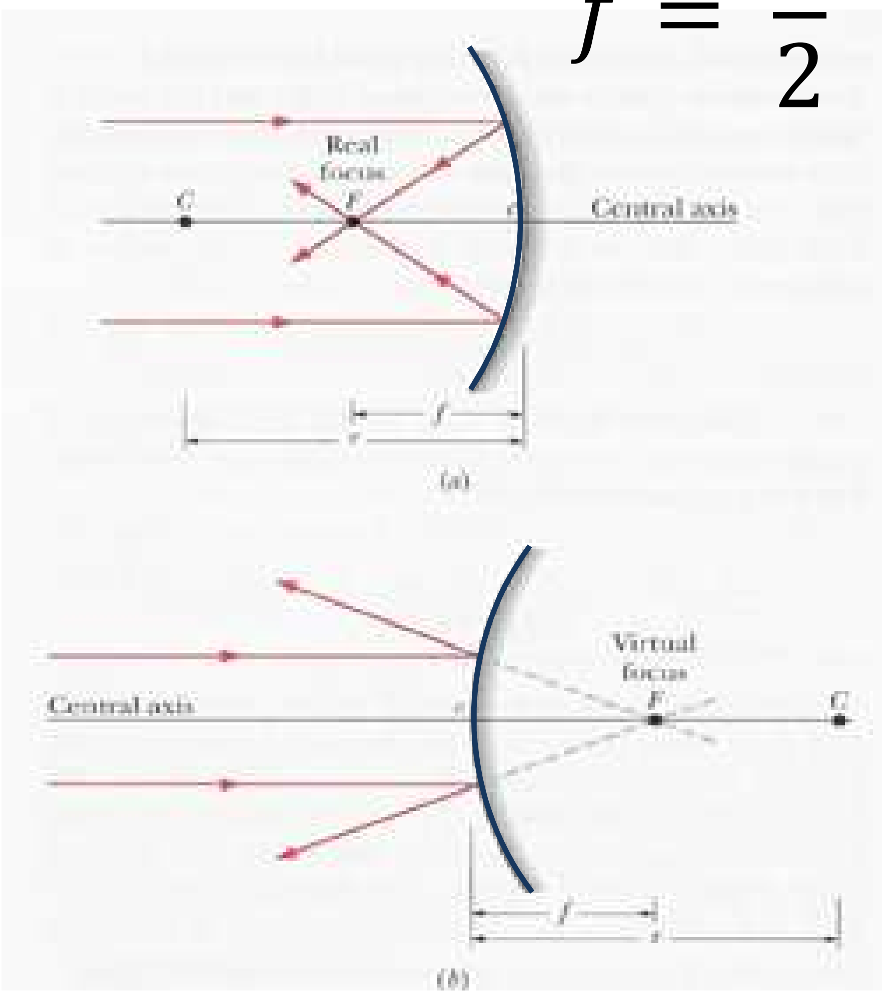


Spherical Mirrors



Focal Points of Spherical Mirrors

$$f = \frac{r}{2}$$



Sign convention:

- Concave:
  - Radius of curvature, $r > 0$
  - Focal length, $f > 0$
- Convex:
  - Radius of curvature, $r < 0$
  - Focal length, $f < 0$

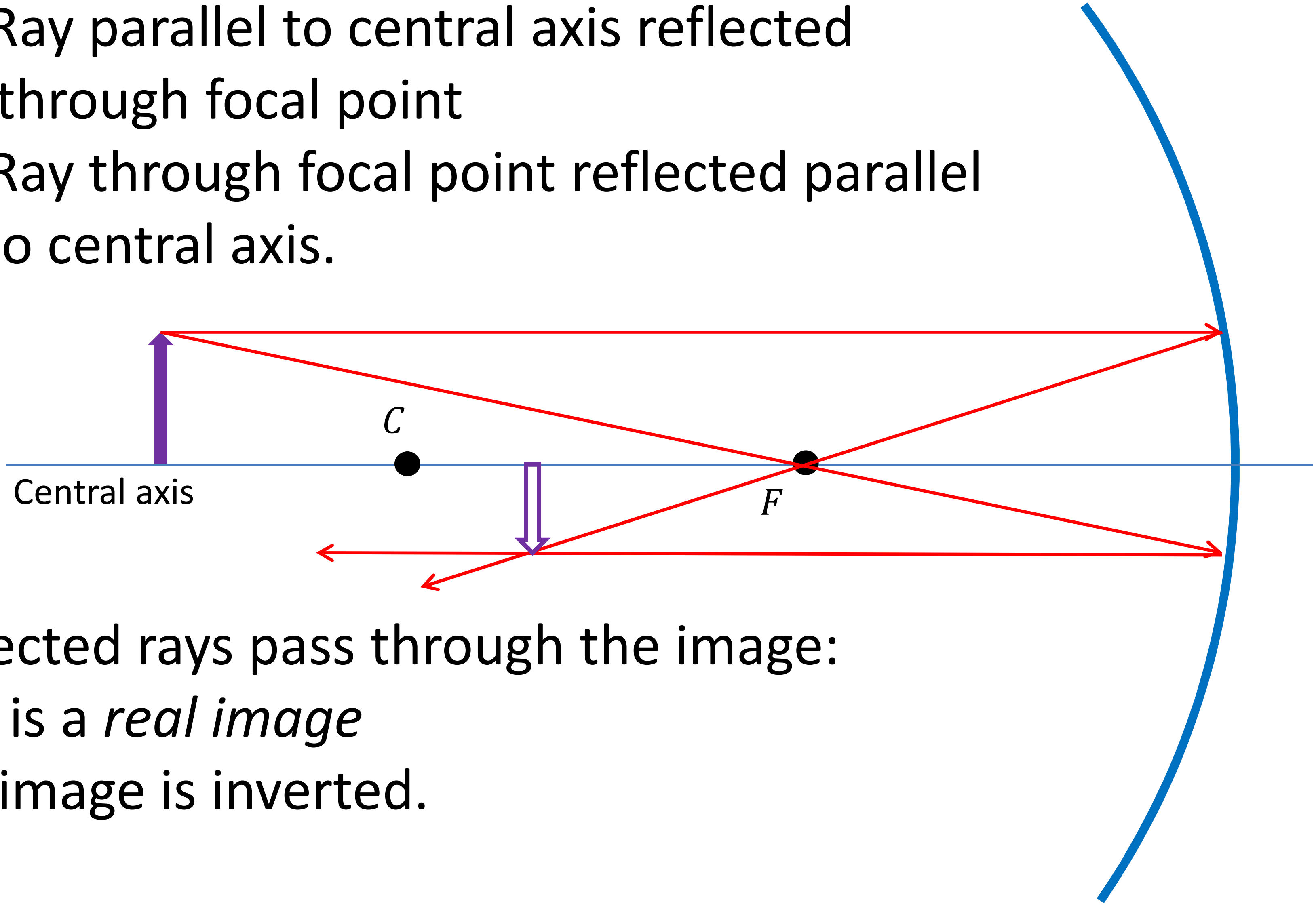
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Sign Conventions

- Be careful about sign conventions!
- There is nothing physical about making $r < 0$ for convex mirrors and $r > 0$ for concave mirrors.
- Different books use different conventions.
- Make sure you know what sign conventions are used in any formulas you make use of.
- This is also true in many other fields of physics.

Properties of Images

1. Ray parallel to central axis reflected through focal point
2. Ray through focal point reflected parallel to central axis.



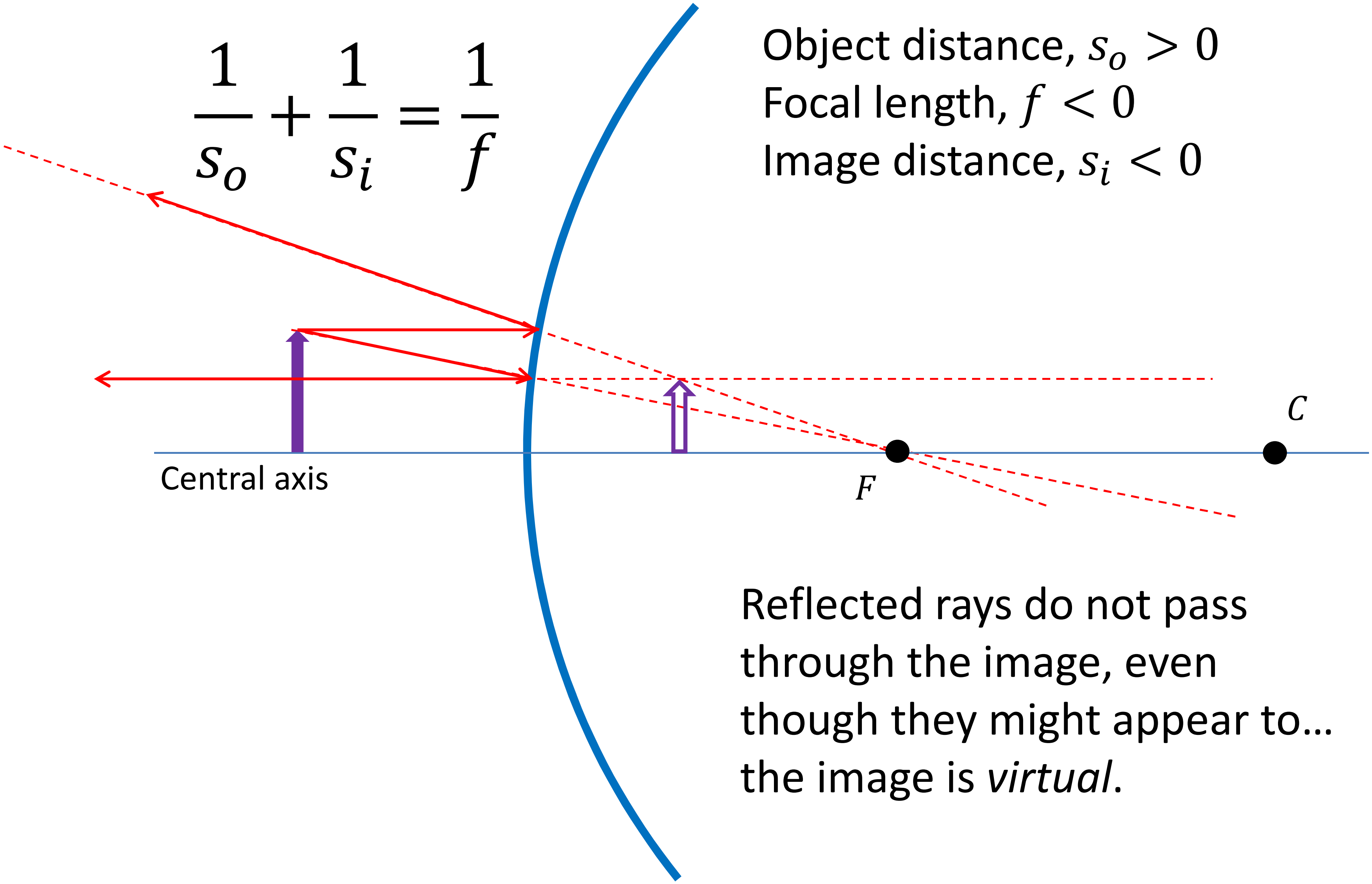
Reflected rays pass through the image: it is a *real image*

The image is inverted.

Properties of Images

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

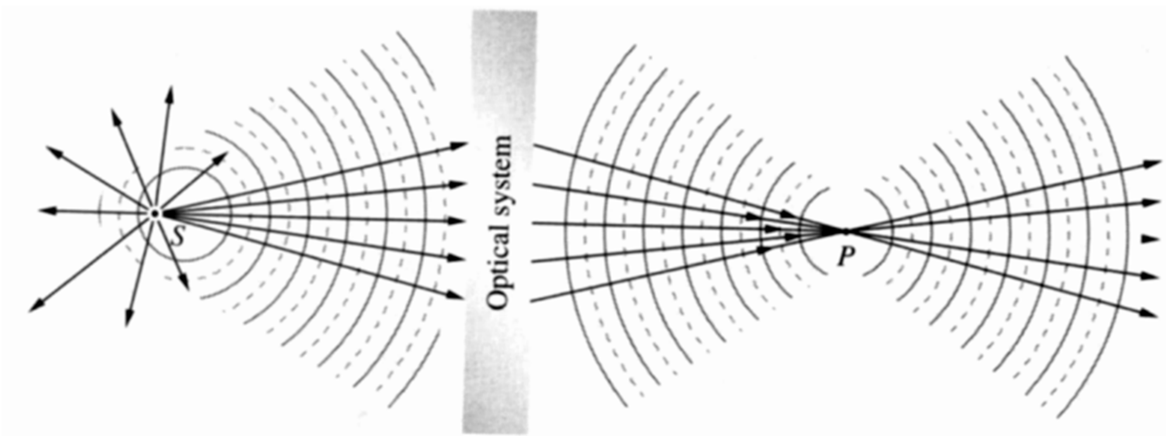
Object distance, $s_o > 0$
Focal length, $f < 0$
Image distance, $s_i < 0$

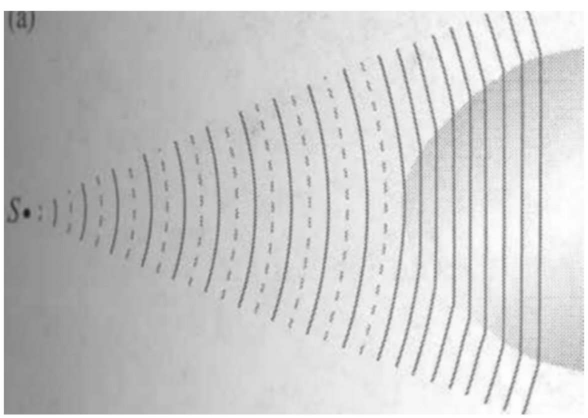


Reflected rays do not pass through the image, even though they might appear to… the image is *virtual*.

Lenses

- Insert a transparent object with $n > 1$ that is thicker in the middle and thinner at the edges

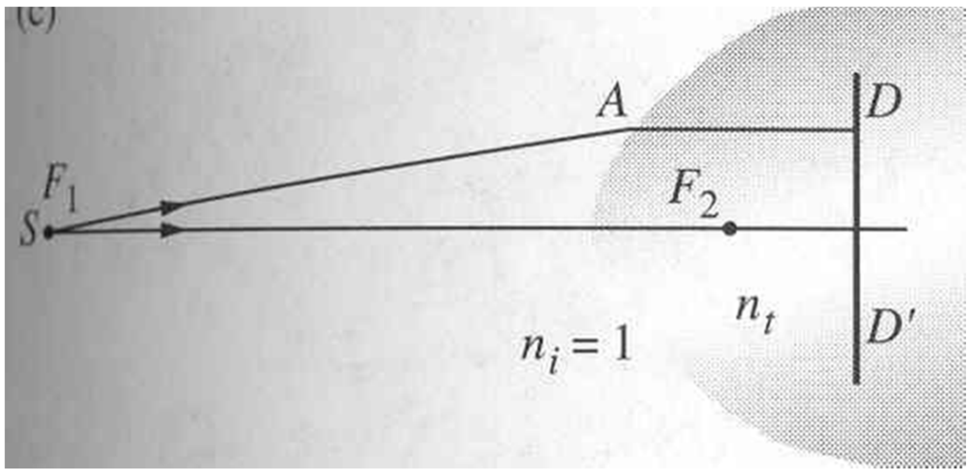




Spherical waves can be turned into plane waves.

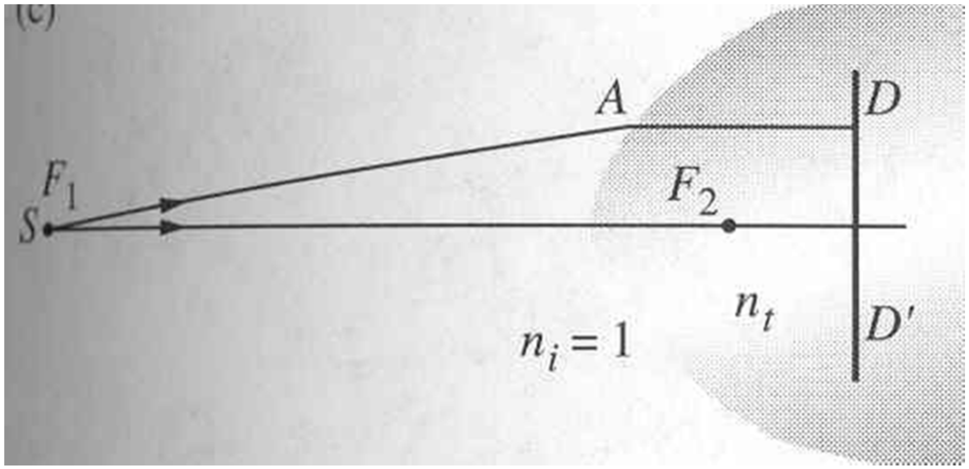
Aspherical Surfaces

- What shape of surface will change spherical waves to plane waves?



- Time to travel from S to plane DD' must be equal for all points A on the surface.

Aspherical Surfaces

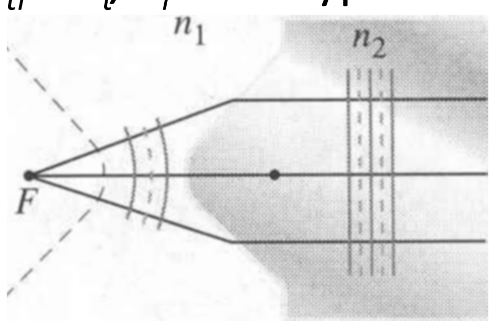


$$\frac{\overline{F_1 A}}{v_i} + \frac{\overline{AD}}{v_t} = \frac{n_i(\overline{F_1 A})}{c} + \frac{n_t(\overline{AD})}{c}$$

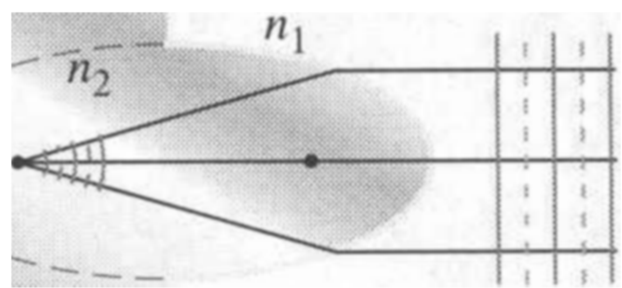
$$\overline{F_1 A} + \frac{n_t}{n_i}\,\overline{AD} = \text{constant}$$

- This is the equation for a hyperbola if $n_t/n_i > 1$ and the equation for an ellipse if $n_t/n_i < 1$.

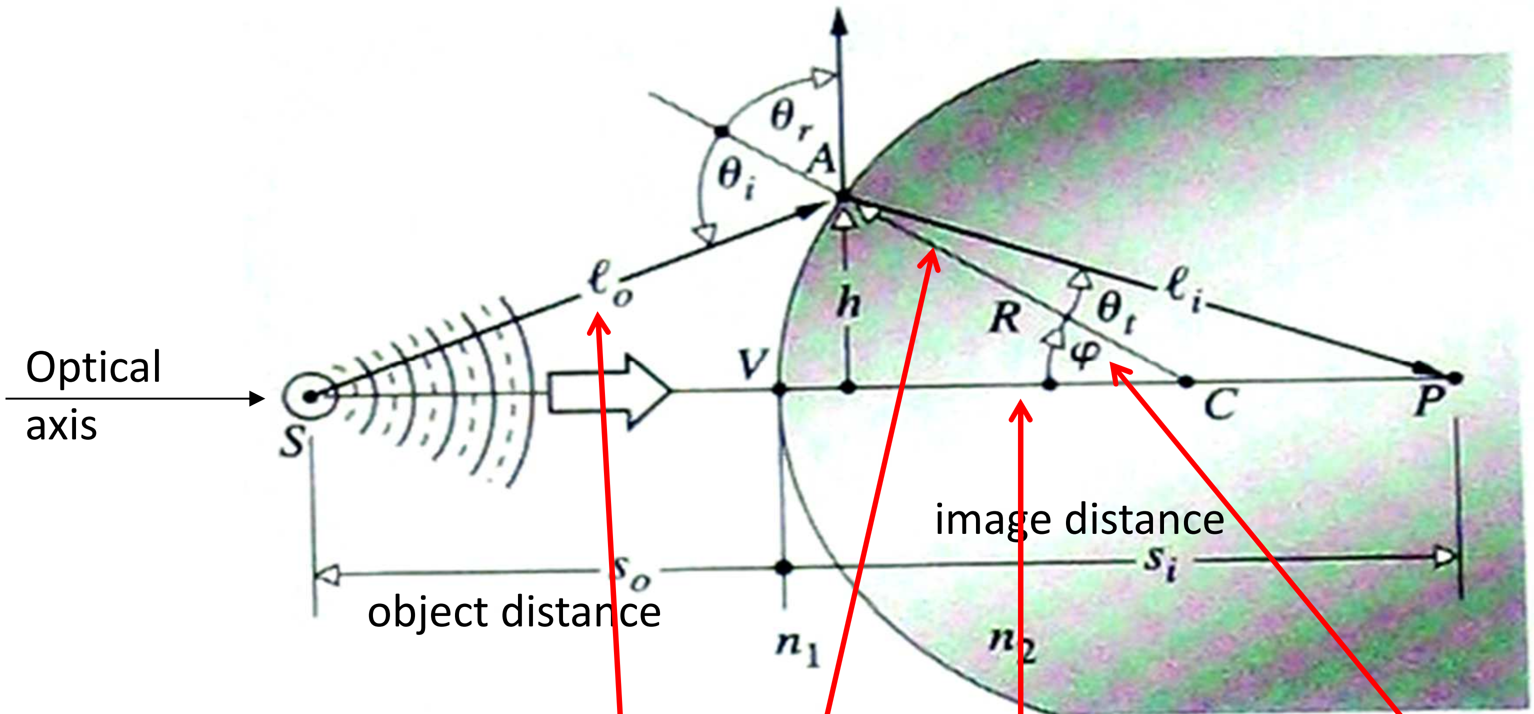
$n_{ti} \equiv n_t/n_i > 1$ - hyperbola



$n_{ti} \equiv n_t/n_i < 1$ - ellipsoid



Spherical Lens



- Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\ell_o = \sqrt{R^2 + (s_o + R)^2 - 2R(s_o + R)\cos\varphi}$$

$$\ell_i = \sqrt{R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\varphi}$$

Spherical Lens

Fermat's principle: *Light will travel on paths for which the optical path length is stationary* (ie, minimal, but possibly maximal)

$$\ell_o = \sqrt{R^2 + (s_o + R)^2 - 2R(s_o + R)\cos\varphi}$$

$$\ell_i = \sqrt{R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\varphi}$$

$$OPL = \frac{n_1 \ell_o}{c} + \frac{n_2 \ell_i}{c}$$

$$\frac{d(OPL)}{d\varphi} = \frac{n_1 R(s_o + R)\sin\varphi}{2\ell_o} - \frac{n_2 R(s_i - R)\sin\varphi}{2\ell_i} = 0$$

$$\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i} = \frac{1}{R}\left(\frac{n_2 s_i}{\ell_i} - \frac{n_1 s_o}{\ell_o}\right)$$

But P will be different for different values of φ…

Spherical Lens

$$\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i} = \frac{1}{R}\left(\frac{n_2 s_i}{\ell_i} - \frac{n_1 s_o}{\ell_o}\right)$$

- Approximations for small φ:

$$\cos\varphi = 1 \qquad \sin\varphi = \varphi$$

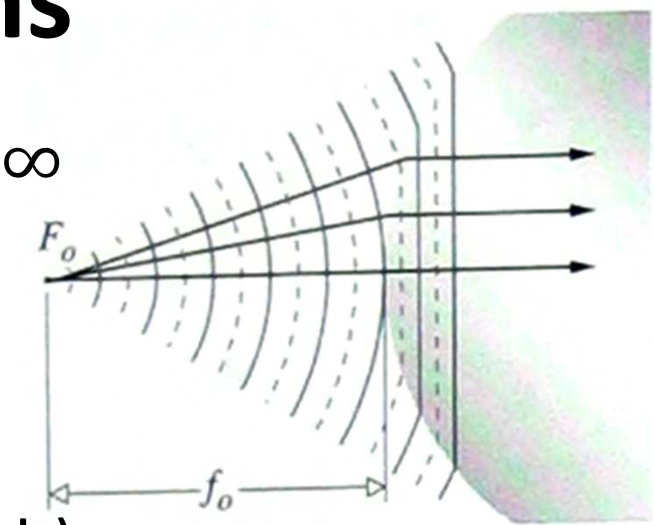
$$\ell_o = s_o \qquad \ell_i = s_i$$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

- The position of P is independent of the location of A over a small area close to the optical axis.
- **Paraxial rays**: rays that form small angles with respect to the optical axis.
- **Paraxial approximation**: consider paraxial rays only.

Spherical Lens

- For parallel transmitted rays, $s_i \to \infty$



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \to \frac{n_1}{f_o} = \frac{n_2 - n_1}{R}$$

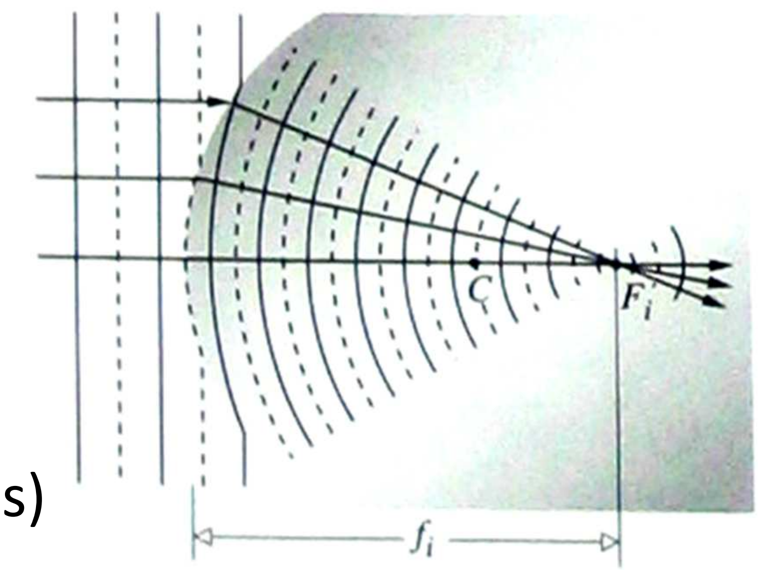
- First focal length (object focal length):

$$f_o = \frac{n_1}{n_2 - n_1} R$$

- Second focal length

(Image focal length)

$$f_i = \frac{n_2}{n_2 - n_1} R$$

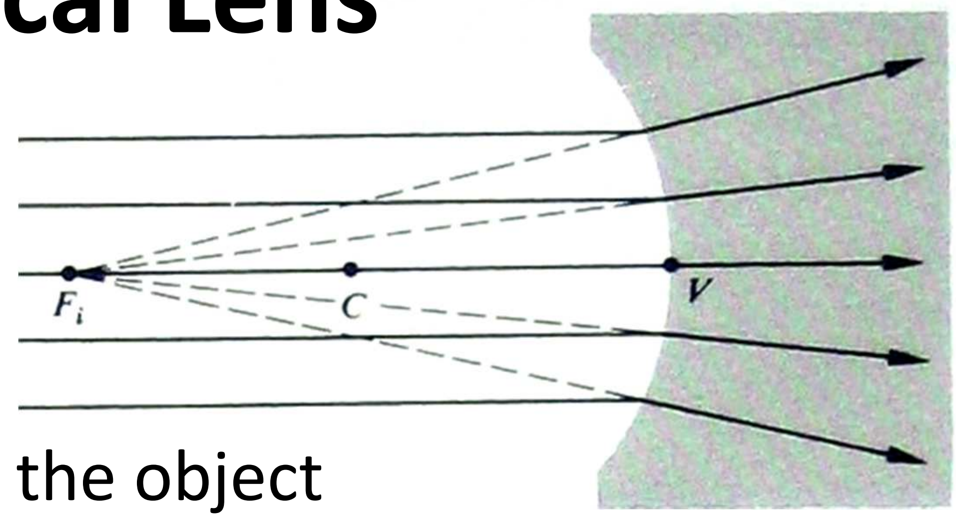


$R > 0, n_2 > n_1 \to f > 0$ (converging lens)

Spherical Lens

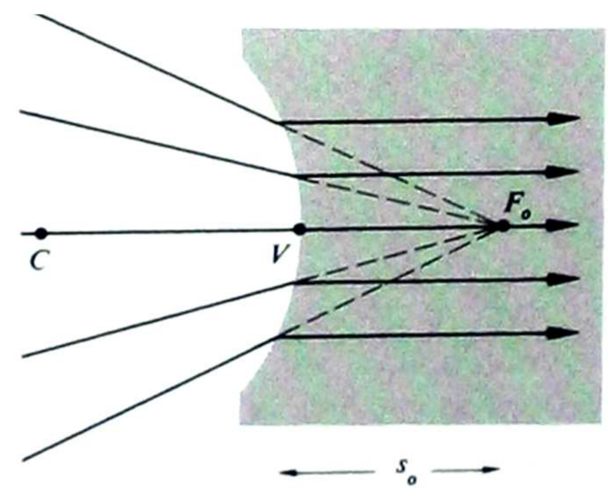
- When $R < 0$:

$$f_i = \frac{n_1}{n_2 - n_1} R$$



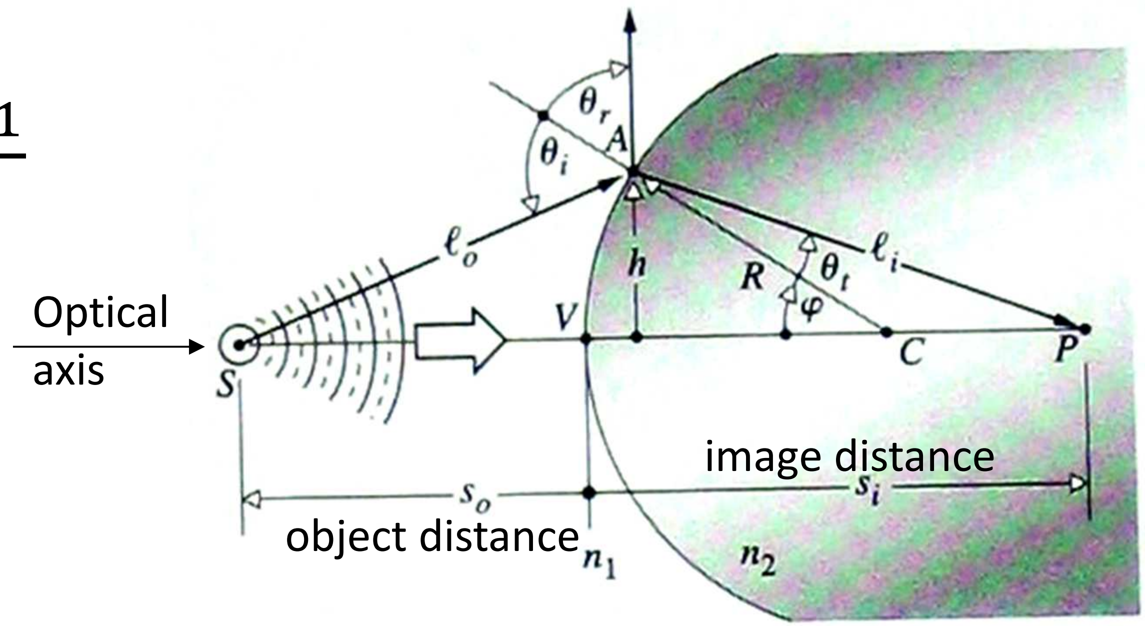
A virtual image appears on the object side.

$$f_o = \frac{n_2}{n_2 - n_1} R$$



Sign Conventions

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$



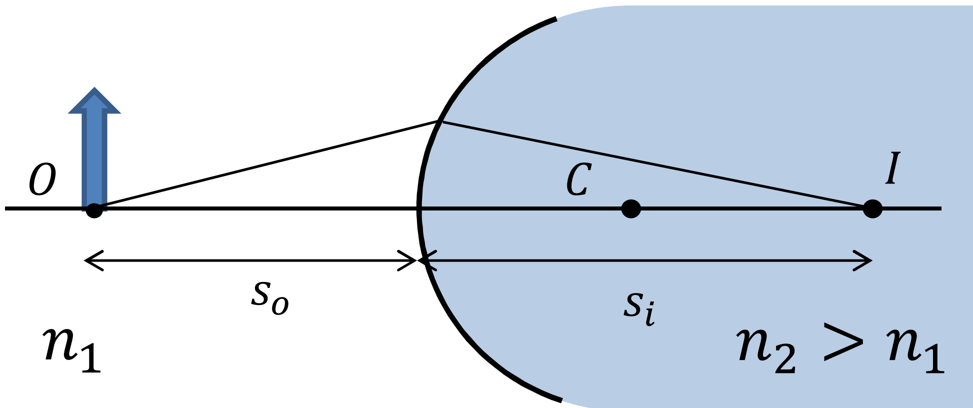
- Assuming light enters from the left:

$s_o, f_o > 0$ when left of vertex, V

$s_i, f_i > 0$ when right of vertex, V

$R > 0$ if C is on the right of vertex, V

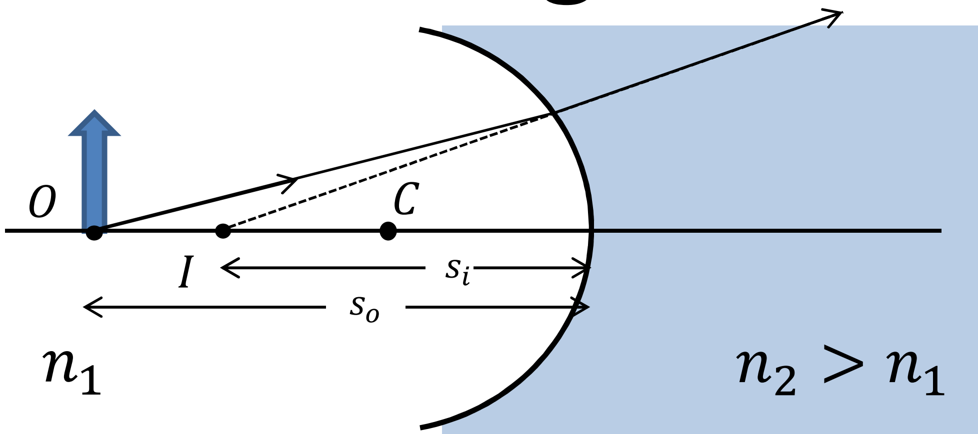
Sign Conventions



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

- Convex surface:
  - s_o is positive for objects on the incident-light side
  - s_i is positive for images on the refracted-light side
  - R is positive if C is on the refracted-light side

Sign Conventions



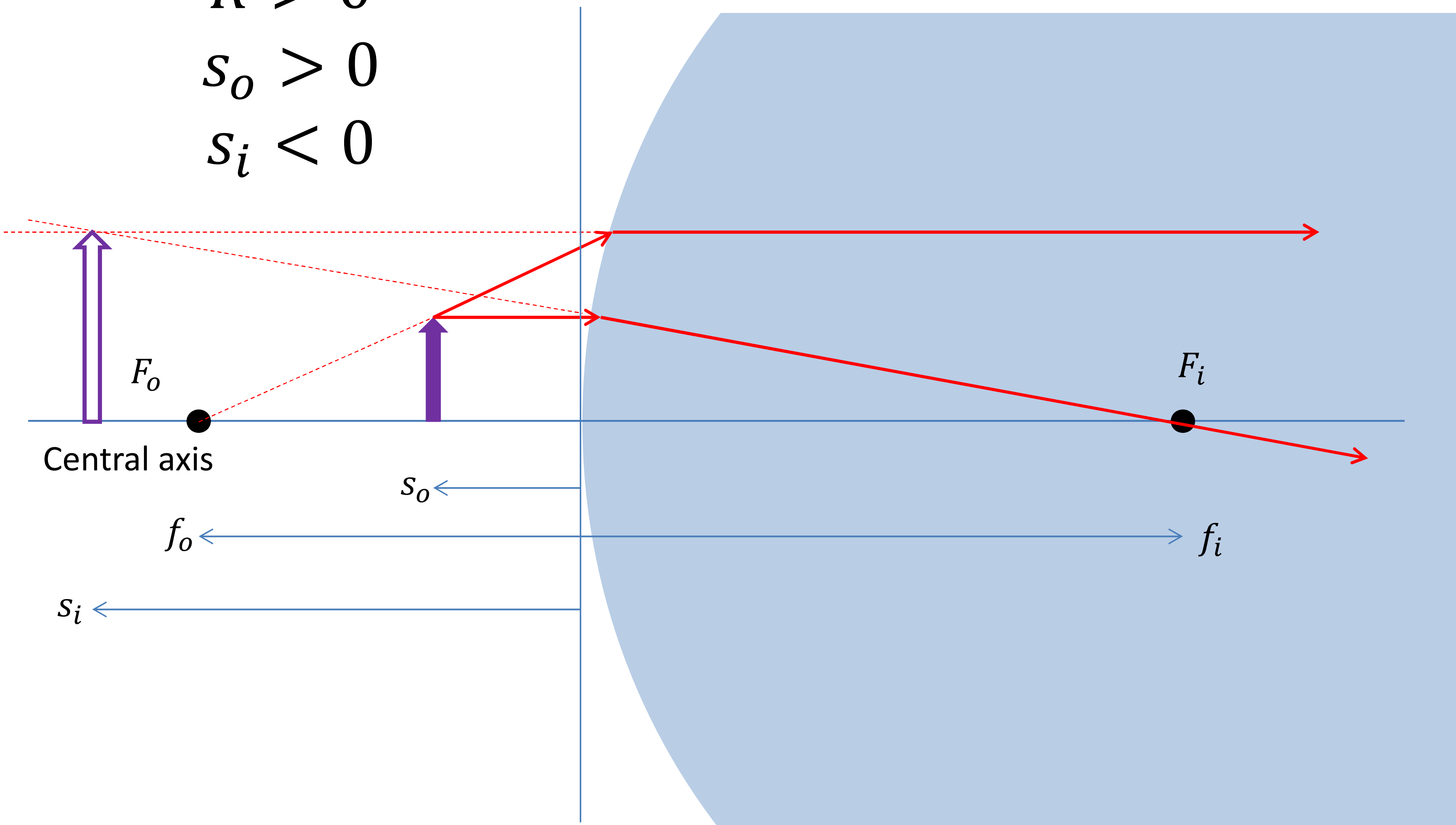
$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

(same formula)

- Concave surface:
  - s_o is positive for objects on the incident-light side
  - s_i is negative for images on the incident-light side
  - R is negative if C is on the incident-light side

Spherical Lens

$$R > 0$$
$$s_o > 0$$
$$s_i < 0$$



Magnification

- Using these sign conventions, the magnification is

$$m = -\frac{n_1 s_i}{n_2 s_o}$$

- Ratio of image height to object height
- Sign indicates whether the image is inverted