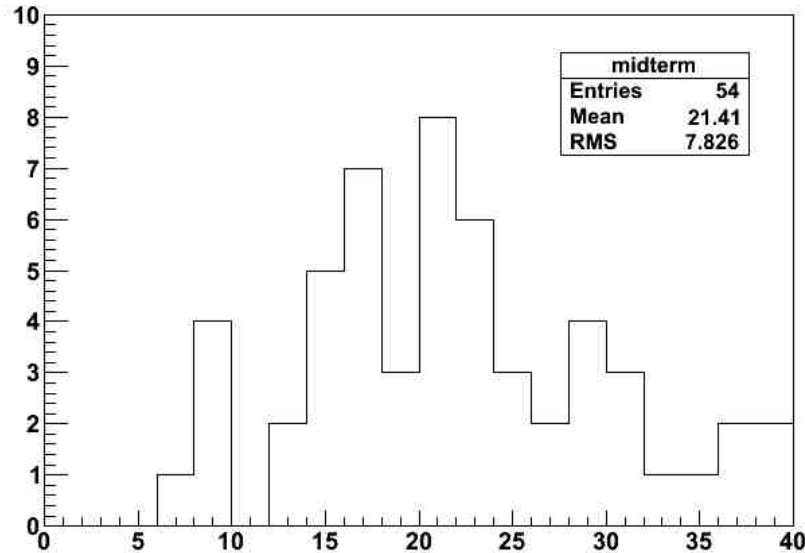# Physics 42200
Waves & Oscillations

Lecture 24 – Propagation of Light

Spring 2014 Semester

Matthew Jones

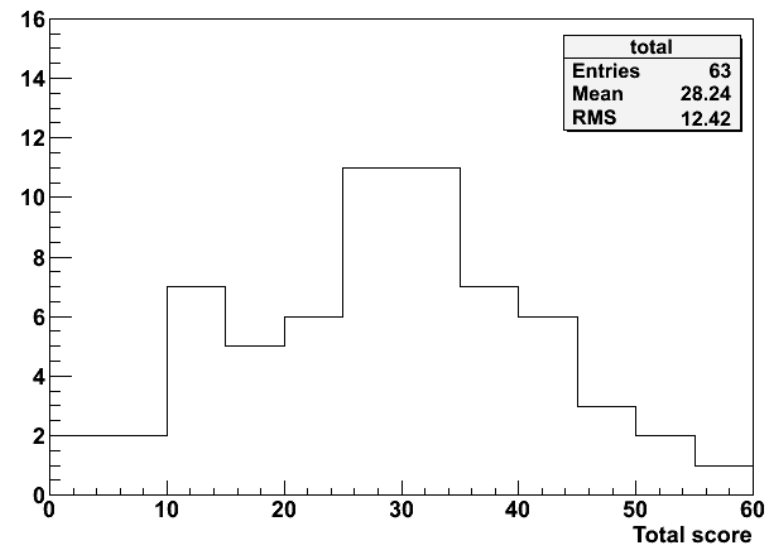# Midterm Grades

Spring 2014 midterm
(out of 40)
Mean is 53.5 ± 1.3%

Spring 2013 midterm
(out of 60)
Mean is 47.1 ± 1.3%

Geometric Optics

- Wave equation in free space:
$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2}$$

- Impedance of free space: $Z_0 = \mu_0 c = 377\ \Omega$

- Free space is isotropic, so $\rho = 0$ and $\tau = 1$

- Waves propagate in the forward direction according to the wave equation.

- What about wave propagation in transparent media?

Geometric Optics

- Huygens' principle: Light is continuously re-emitted in all directions from all points on a wave front
  - Further developed by Fresnel and Kirchhoff
  - Destructive interference except in the forward direction
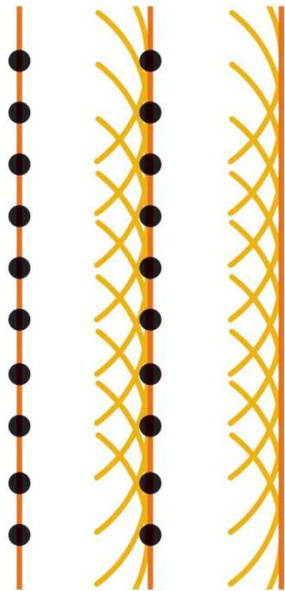
Forward Propagation
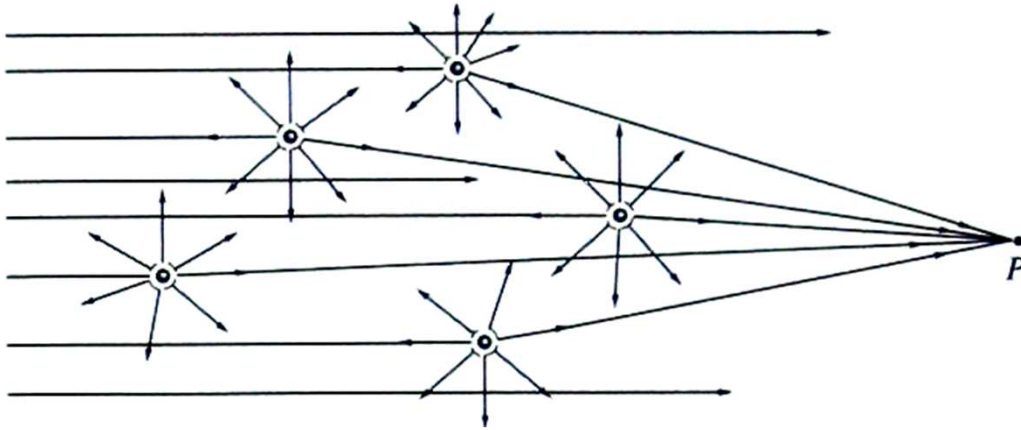
- Remember the forced harmonic oscillator problem:

$$m\ddot{x} + b\dot{x} + kx = F_0 e^{i\omega t}$$

$$A = \frac{F_0/m}{\sqrt{\left((\omega_0)^2 - \omega^2\right)^2 + (\omega\gamma)^2}}$$

$$\delta = \tan^{-1}\left(\frac{\omega\gamma}{(\omega_0)^2 - \omega^2}\right)$$

- When the driving force is much greater than the resonant frequency, $\delta \to 180°$

- Electrons bound to molecules act as tiny oscillators that are driven by the electric field.
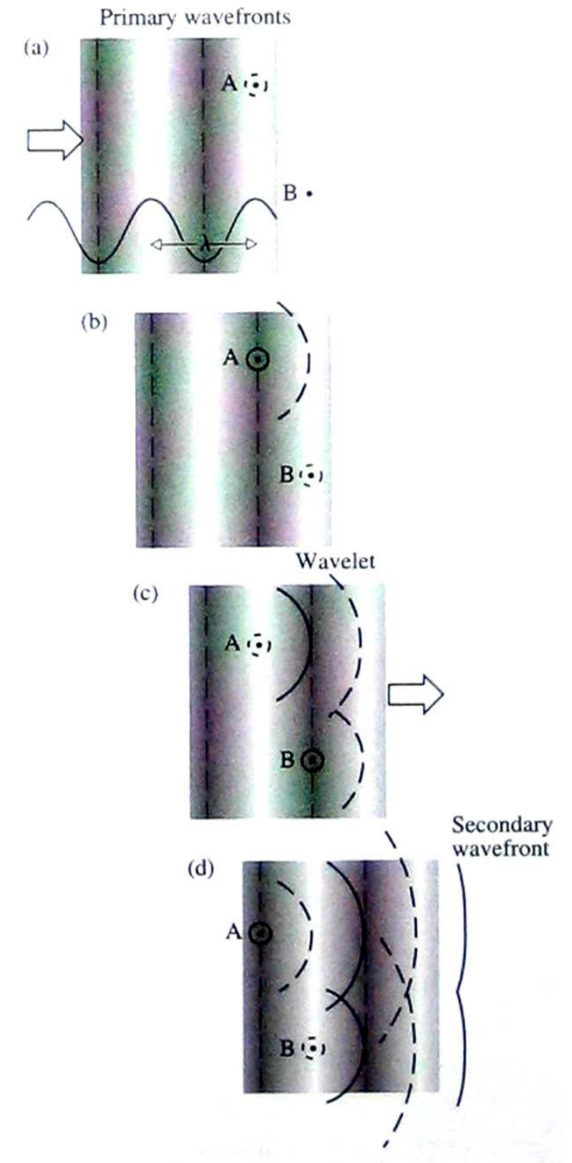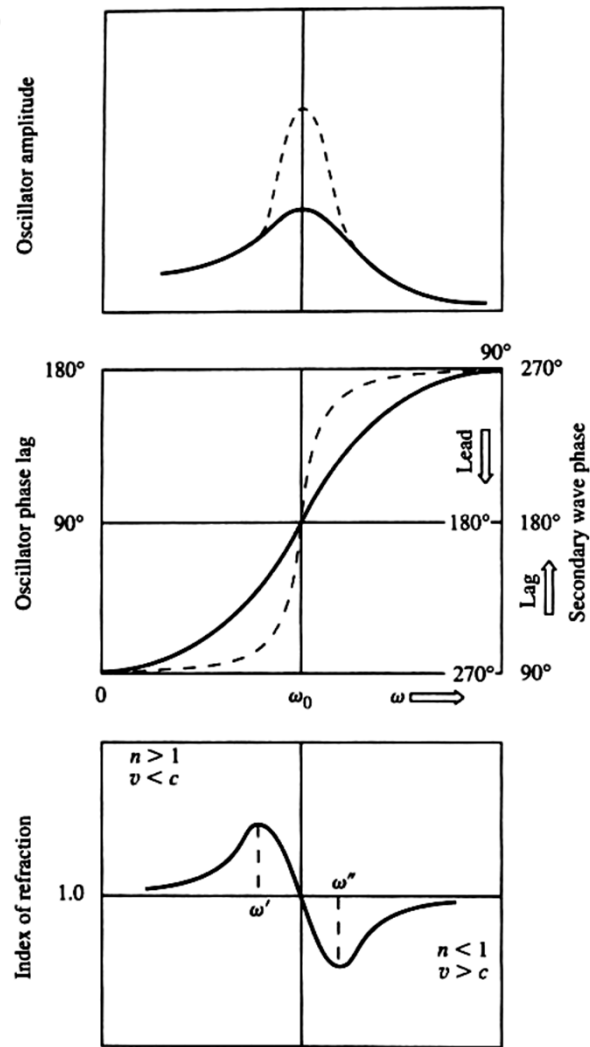
Forward Propagation

At point P the scattered waves are more or less in-phase:

- constructive interference of wavelets scattered in forward direction.
- Destructive interference of wavelets and incident light in the backward direction.

True for low and high density substance

Primary wavefronts

(a)

A

B

(b)

A

B

Wavelet

(c)

A

B

Secondary wavefront

(d)

A

B

Forward Propagation

- At lower frequencies the phase shift is less than 180°
- The phase lag results in the scattered waves having a shorter wavelength

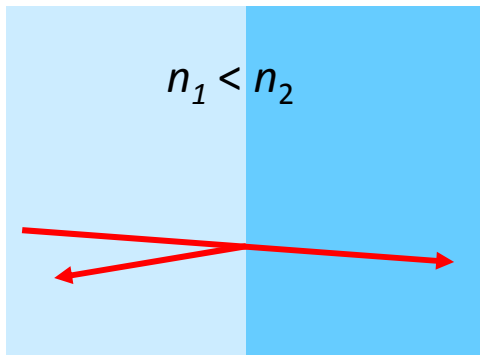$$\lambda' = \frac{v}{f} < \frac{c}{f}$$

- Index of refraction:

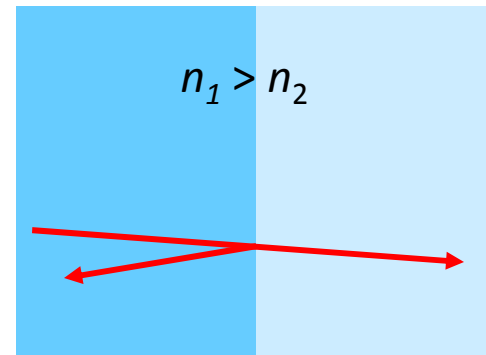$$n = \frac{c}{v}$$

- Applies to the *phase* velocity

Reflection

- Energy is reflected from an interface between materials with different indices of refraction

- Reflection coefficient:

$$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{1/n_2 - 1/n_1}{1/n_2 + 1/n_1} = \frac{n_1 - n_2}{n_1 + n_2}$$
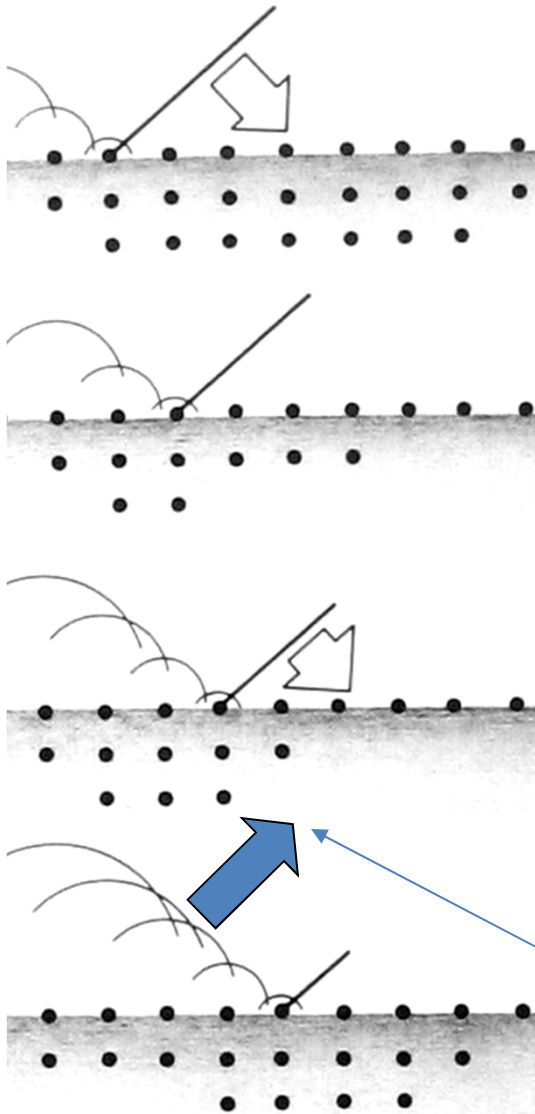
$n_1 < n_2$

$n_1 > n_2$

External reflection:
180° phase shift

Internal reflection:
No phase shift

Reflection: Microscopic View

When an incident plane wave front strikes the surface at some angle it does not reach all the atoms along the surface simultaneously.

Each consequent atom will scatter at slightly different phase, while the spherical wave created by previous atom had a chance to move away some distance.

The resulting reflected wave front created as a superposition of all scattered wavelets will emerge also at an angle to the surface.

Reflection: Constructive Interference

For constructive interference spherical waves created by the atoms on the surface must arrive in-phase.

Consider two atoms on the surface.
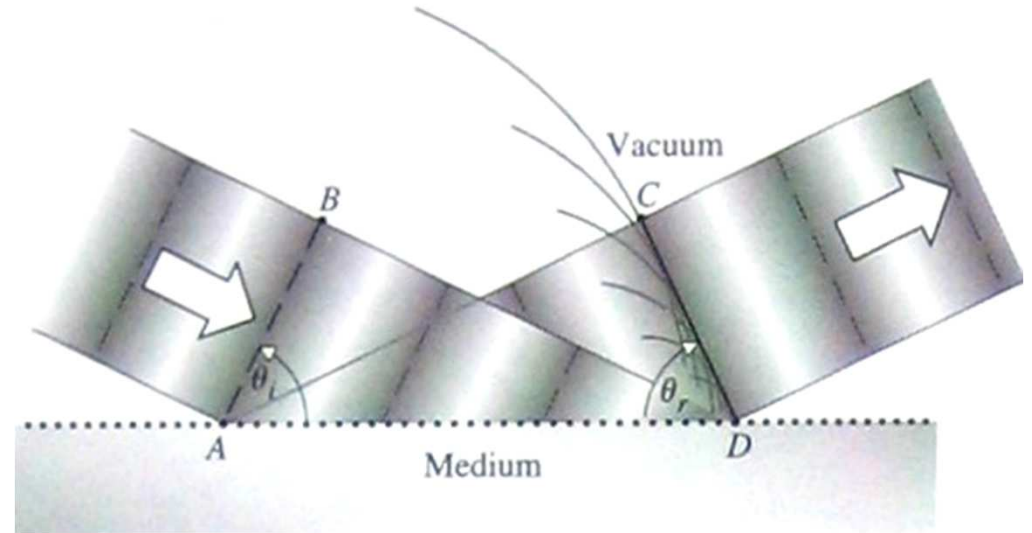
Wave function depends only on $\vec{k} \cdot \vec{r} - \omega t$: $E = f(\vec{k} \cdot \vec{r} - \omega t)$

Wave phase along the incident wavefront is the same.

Scattered wavefront: Points C and D must also have the same phase.

$$E_C = f(\xi + k \cdot \overline{AC} - \omega t)$$

$$E_D = f(k \cdot \overline{BD} + \xi - \omega t)$$

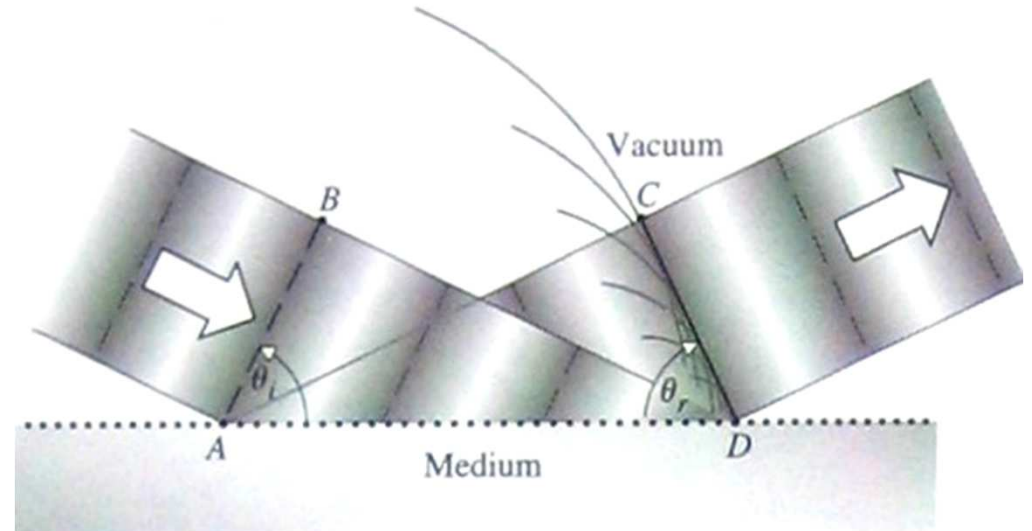$\implies \overline{AC} = \overline{BD}$

phase shift due to scattering atom

The Angle of Reflection

Triangles ABD and ACD:

$$\overline{AC} = \overline{BD}$$

$$\overline{AD} = \overline{AD}$$

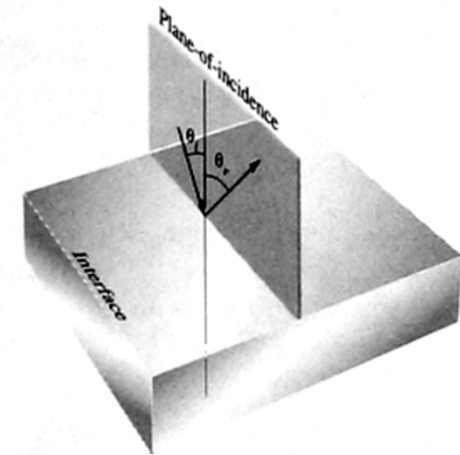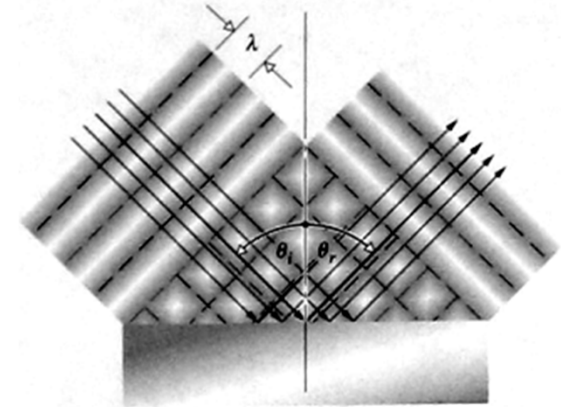$$\angle B = \angle C = 90°$$

The Law of Reflection (1st part):
The angle-of-incidence equals the angle-of-reflection:
$$\theta_r = \theta_i$$

Rays and the Law of Reflection

A ray is a line drawn in space along the direction of flow of the radiant energy. Rays are straight and they are perpendicular to the wavefront
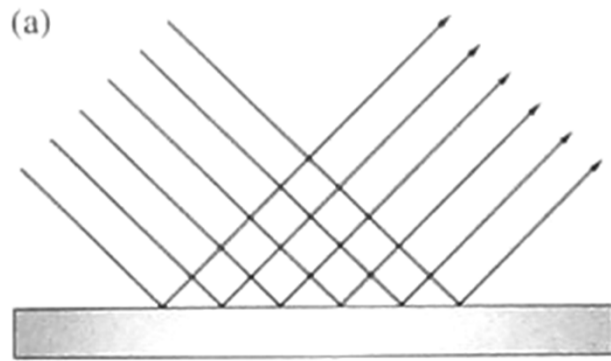
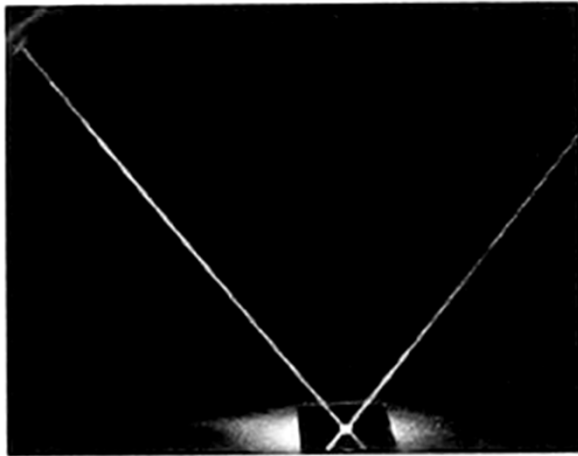Conventionally we talk about rays instead of wavefronts

The Law of Reflection
1. The angle-of-incidence equals the angle-of-reflection $\theta_r = \theta_i$
2. The incident ray, the perpendicular to the surface and the reflected ray all lie in a plane (**plane-of-incidence**)
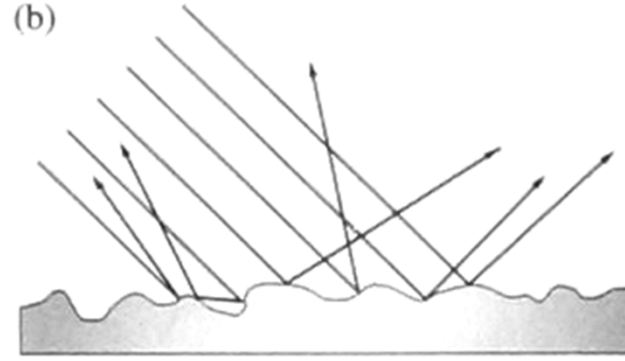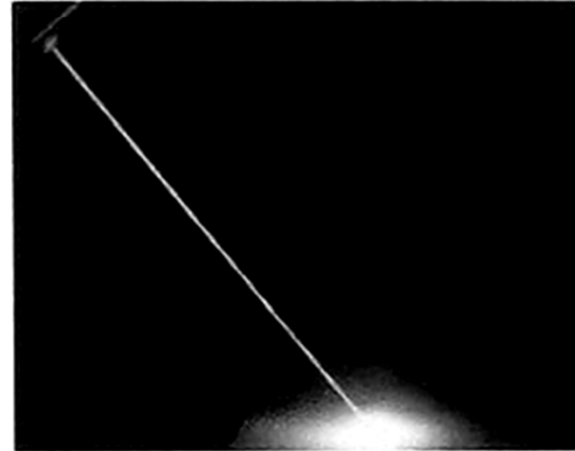
Specular and Diffuse Reflection

(a)

Specular

(b)

Diffuse

Smooth surface:
specular reflection
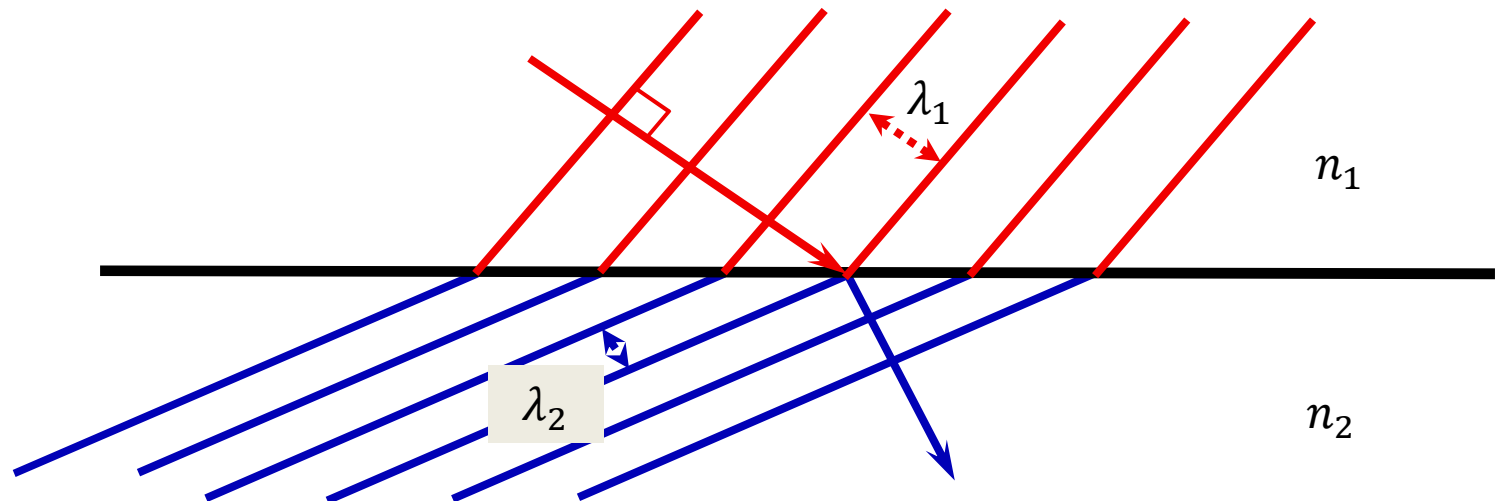
Rough surface:
diffuse reflection

Refraction

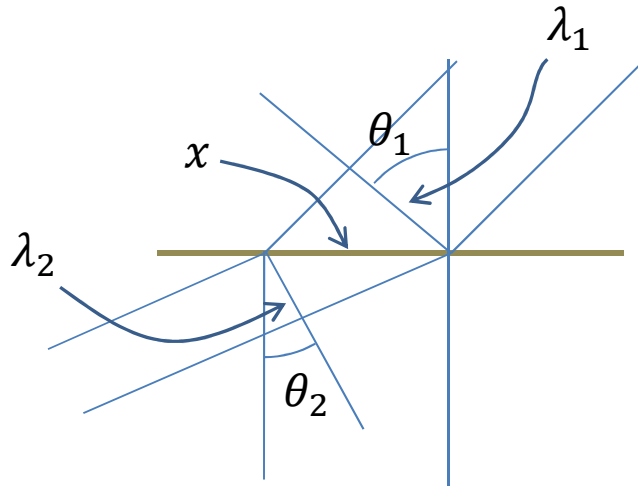- The speed changes in different media, but the frequency remains the same
  - Wavelength is shorter in the "slow" medium:
$$\lambda = \frac{v}{f} = \frac{c}{nf}$$
- The phases must be constant everywhere on a wave front:

Refraction

$$x\cos(90^o - \theta_1) = x\sin\theta_1 = \lambda_1$$
$$x\cos(90^o - \theta_2) = x\sin\theta_2 = \lambda_2$$

$$x = \frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$$
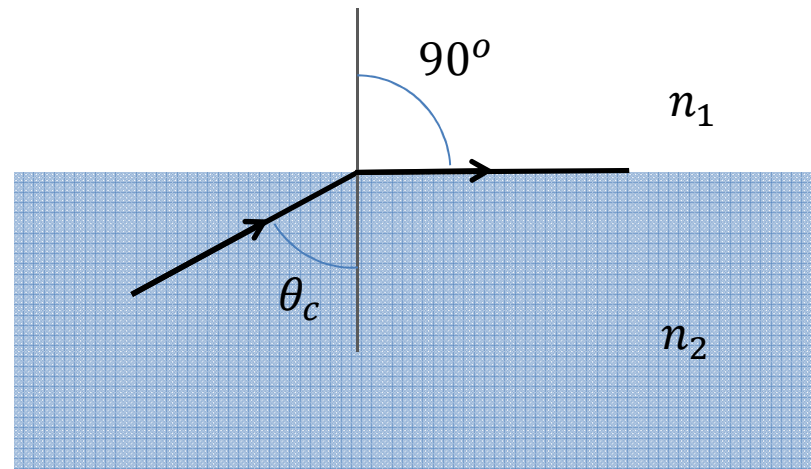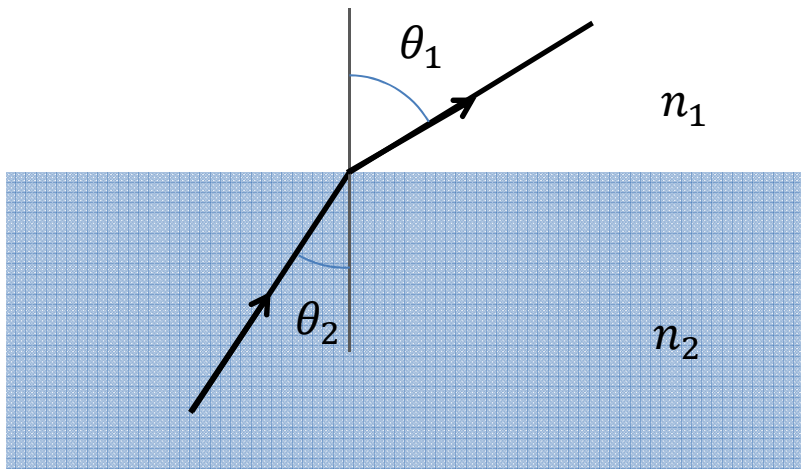
$$\lambda = \frac{c}{nf}$$

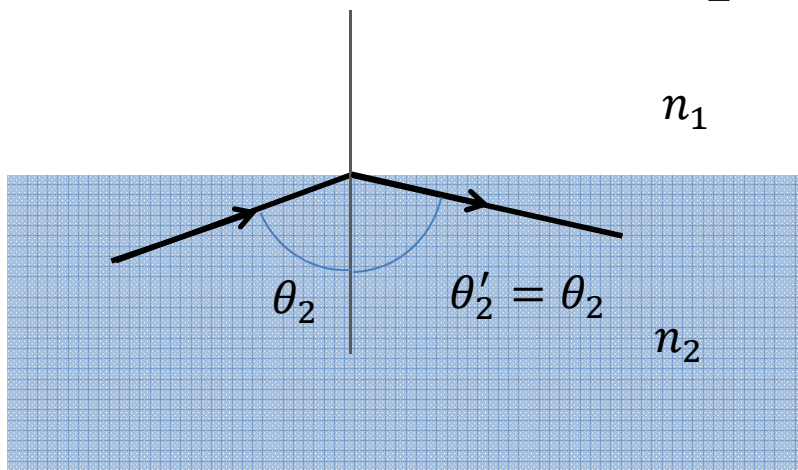$$\frac{c}{f}\frac{1}{n_1\sin\theta_1} = \frac{c}{f}\frac{1}{n_2\sin\theta_2}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

- When light enters a medium with a larger index of refraction, it bends towards the direction that is normal to the surface.

Total Internal Reflection

- What happens when $\theta_1 \geq 90^o$?

Critical angle:
$$n_2 \sin \theta_c = n_1$$
$$\sin \theta_c = \frac{n_1}{n_2}$$

Geometric Optics

- When the wavelength of light is much smaller than the dimensions of objects it interacts with, we can ignore its wave nature.

- Multiple paths by which light can reach a given point – phases are random (incoherent).

- We are generally not concerned with polarization.

- Treat light as rays propagating in straight lines

Geometric Optics

- Under these conditions, the only physical principles we need to describe the propagation of light are:
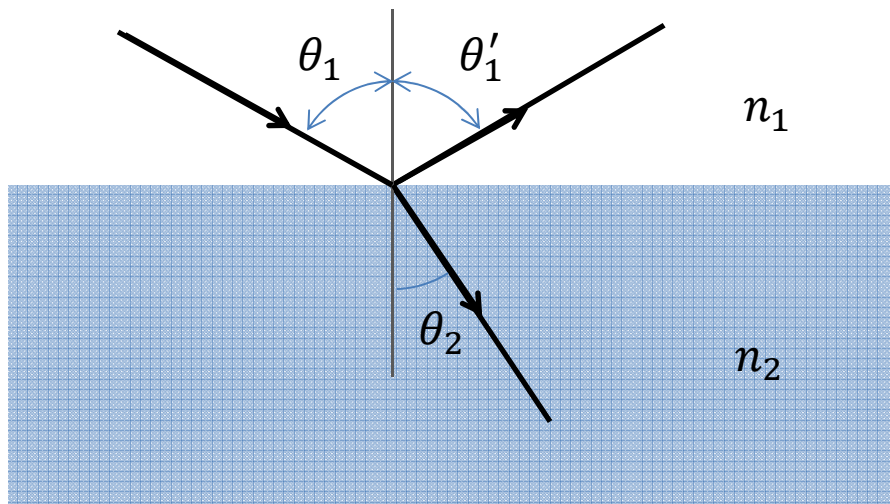

Reflection:
$$\theta_1' = \theta_1$$

Refraction:
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
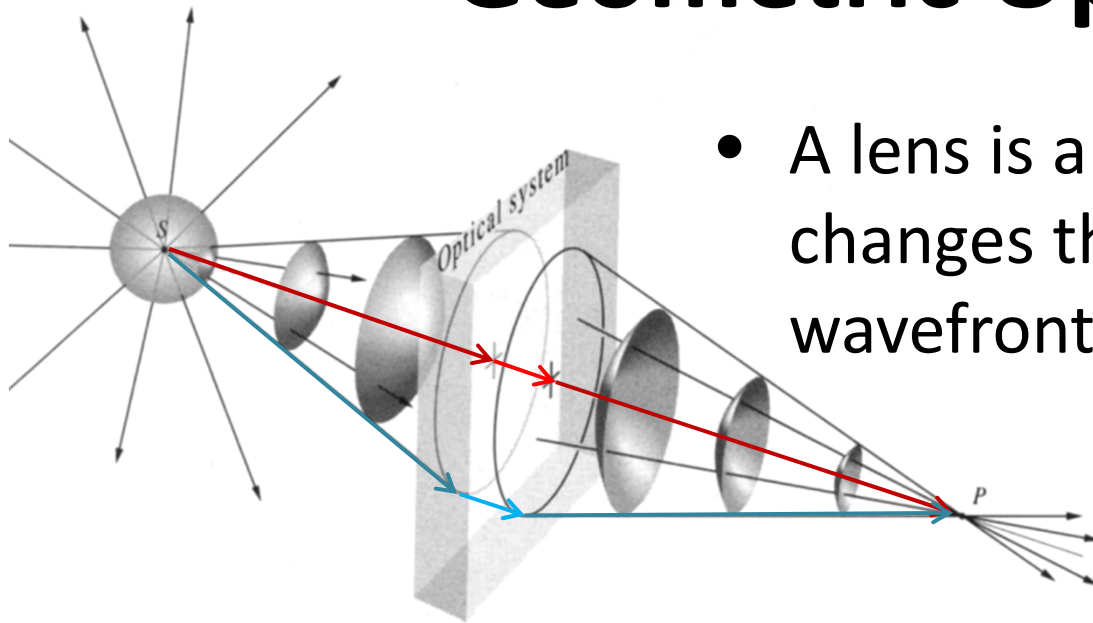
Geometric Optics

- Each point on an illuminated surface is a source of spherical waves
  - Rays diverge from that point
  - We perceive an image as the collection of points from which the rays emerge
- An optical system can cause the rays to diverge from a different point
  - We perceive this point as an image of the original object

Geometric Optics
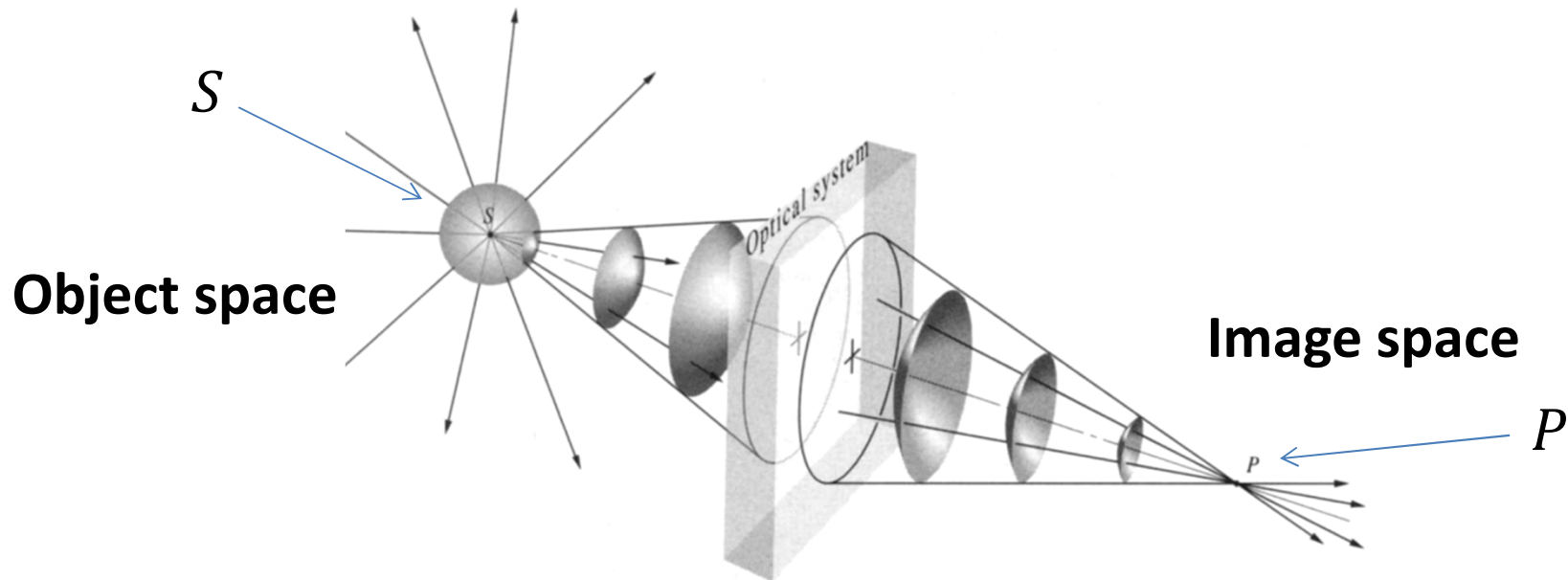
- A point from which a portion of the spherical wave diverges is a focus of the bundle of rays
- A point to which the portion of the spherical wave converges is also a focus of the bundle of rays
- The paths are reversible
- P and S are *conjugate points*

Geometric Optics

- A lens is a refractive device that changes the curvature of the wavefront

- All points on the wavefront have the same *optical path length (OPL)*

$$t = \frac{1}{c}\sum_{i=1}^{N} n_i s_i \rightarrow OPL = \int_{S}^{P} n(s)ds$$

Geometric Optics

- Typical problems in geometric optics:
  - Given an optical system, what are the properties of the image that is formed (if any)?
  - What configuration of optical elements (if any) will produce an image with certain desired characteristics?
- No new physical principles: the laws of reflection and refraction are all we will use
- We need a method for analyzing these problems in a systematic an organized way
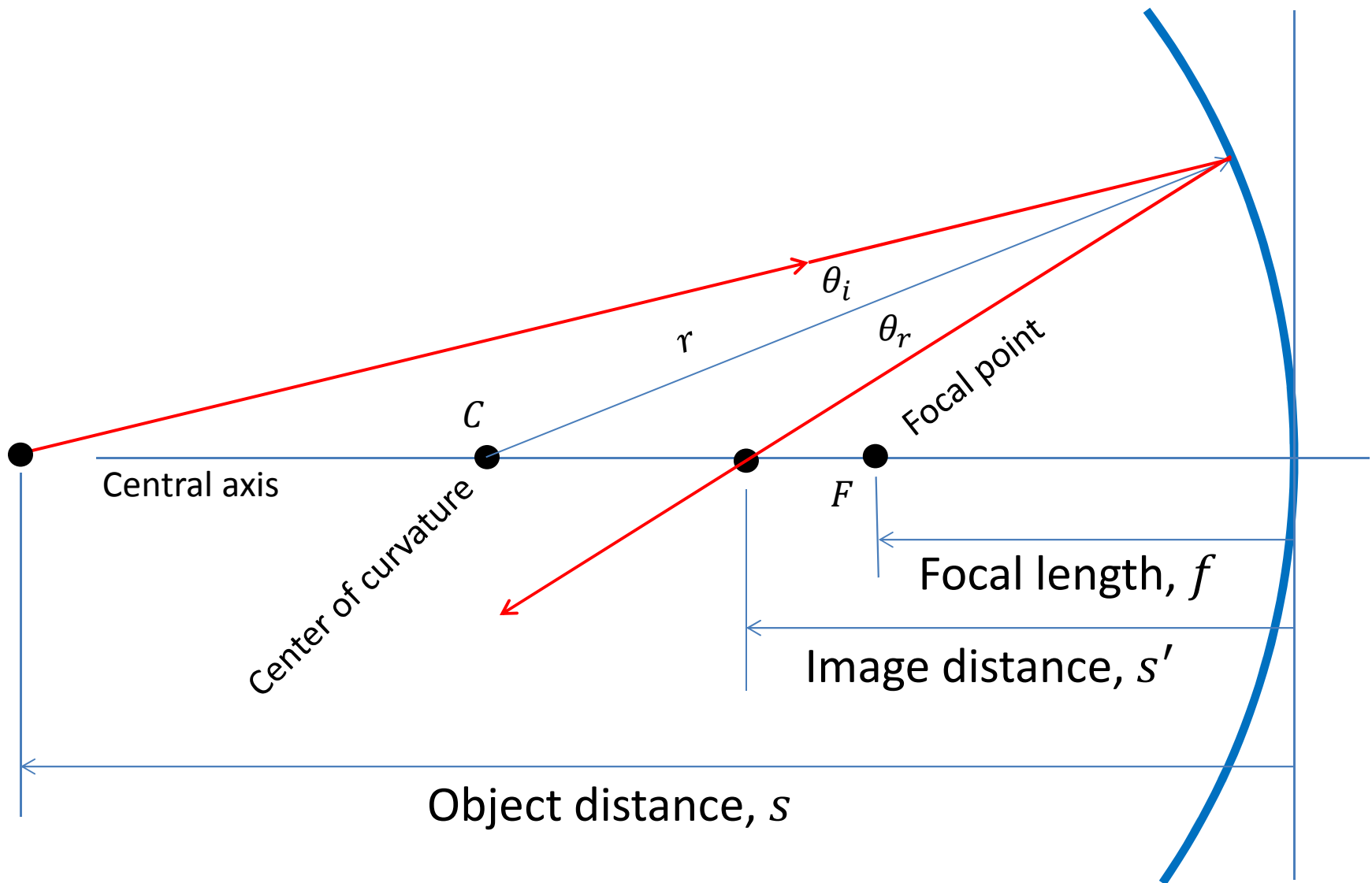
Types of Images

- **Real Image**: light emanates from points on the image
- **Virtual Image**: light *appears* to emanate from the image
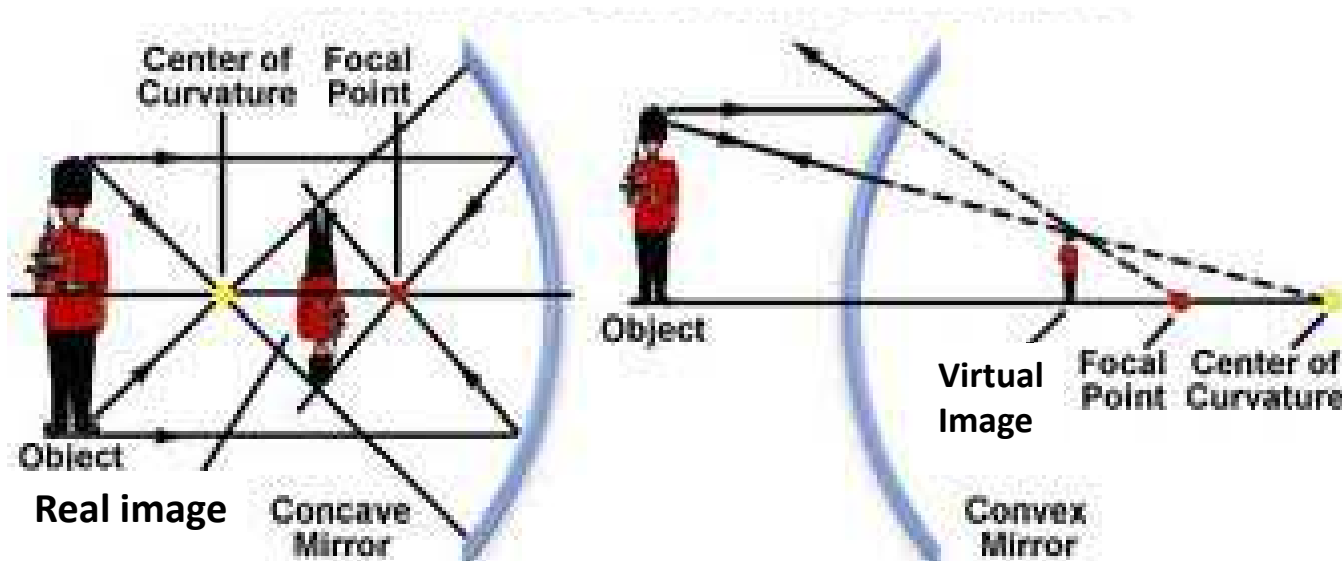
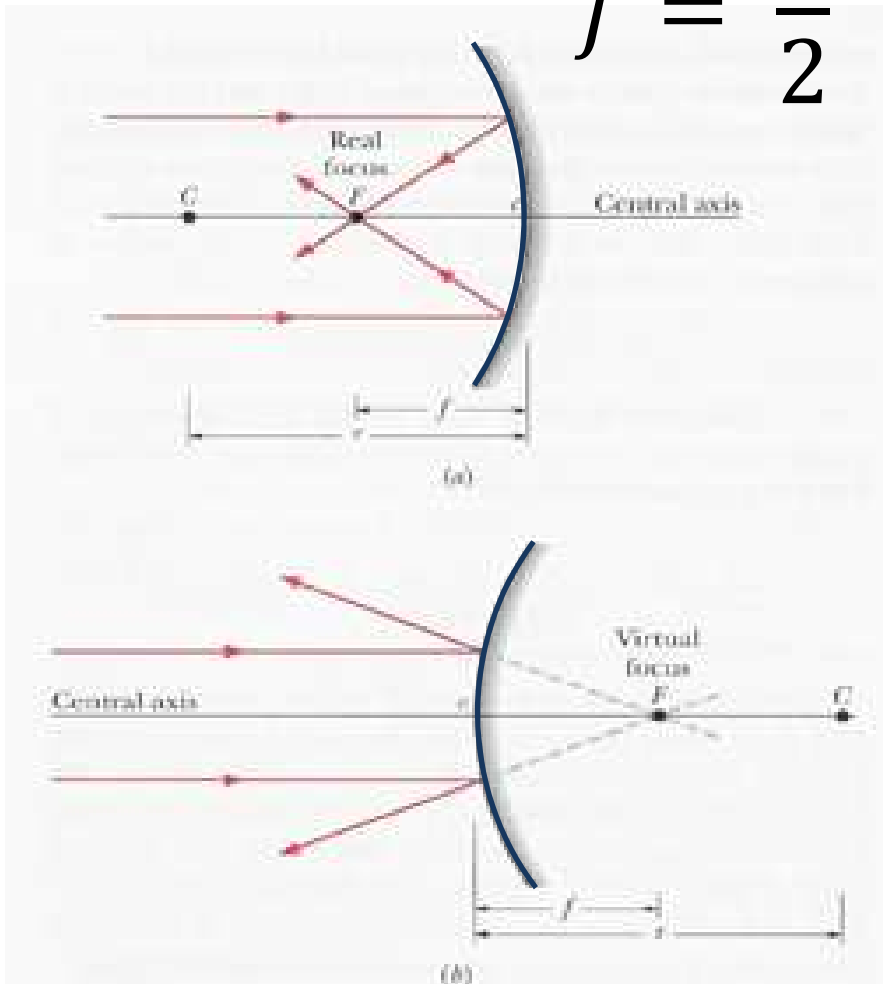# Spherical Mirrors
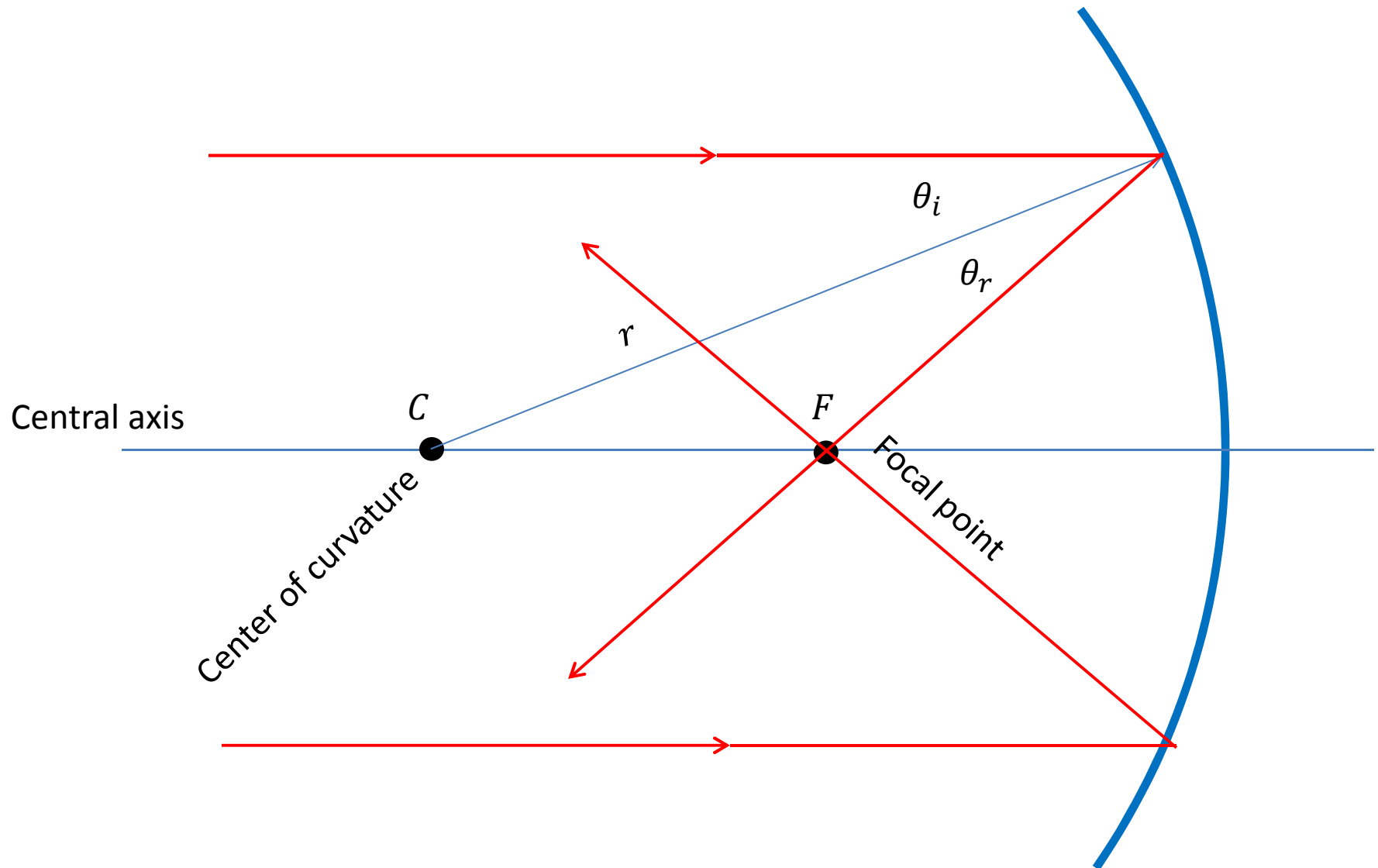
Spherical Mirrors

Focal Points of Spherical Mirrors
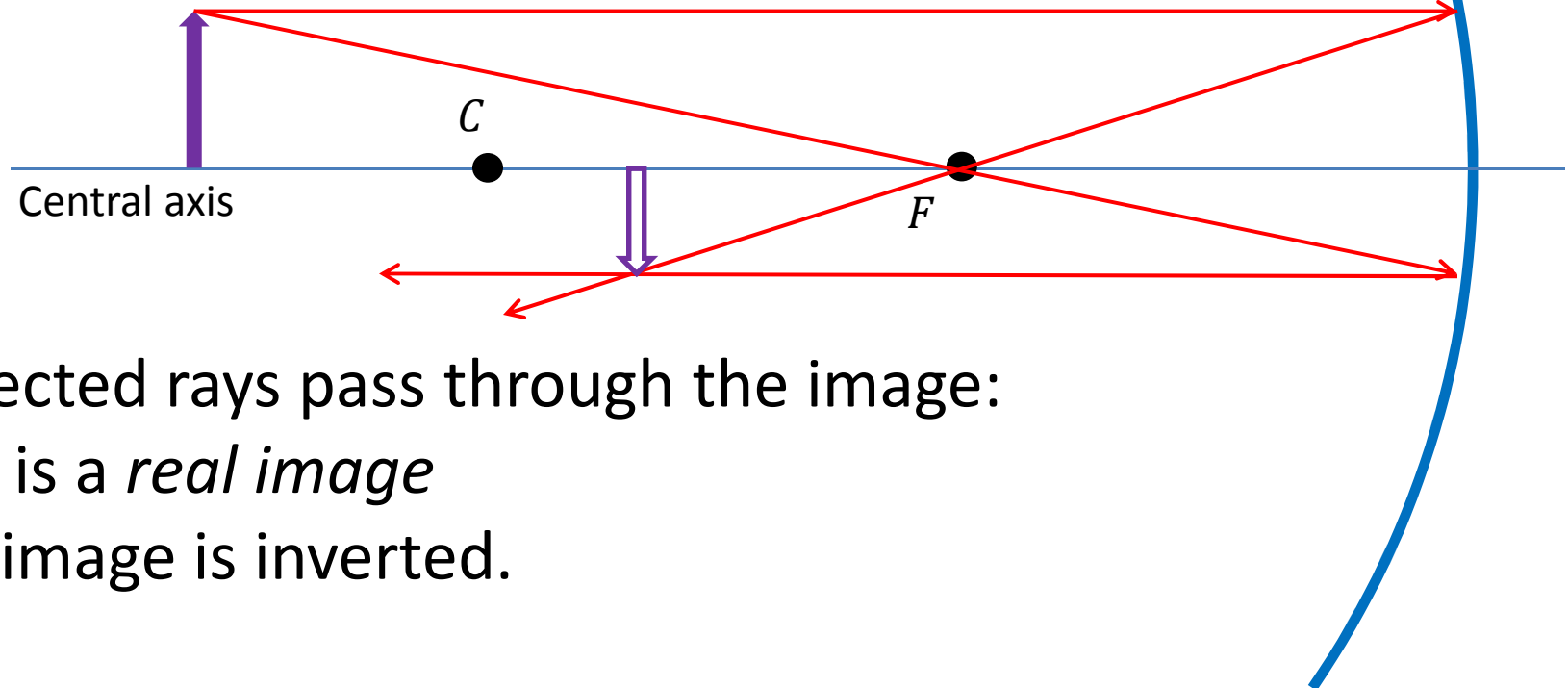
$$f = \frac{r}{2}$$

Sign convention:

- Concave:
  - Radius of curvature, $r > 0$
  - Focal length, $f > 0$
- Convex:
  - Radius of curvature, $r < 0$
  - Focal length, $f < 0$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Properties of Images

1. Ray parallel to central axis reflected through focal point
2. Ray through focal point reflected parallel to central axis.

Central axis

C

F

Reflected rays pass through the image:
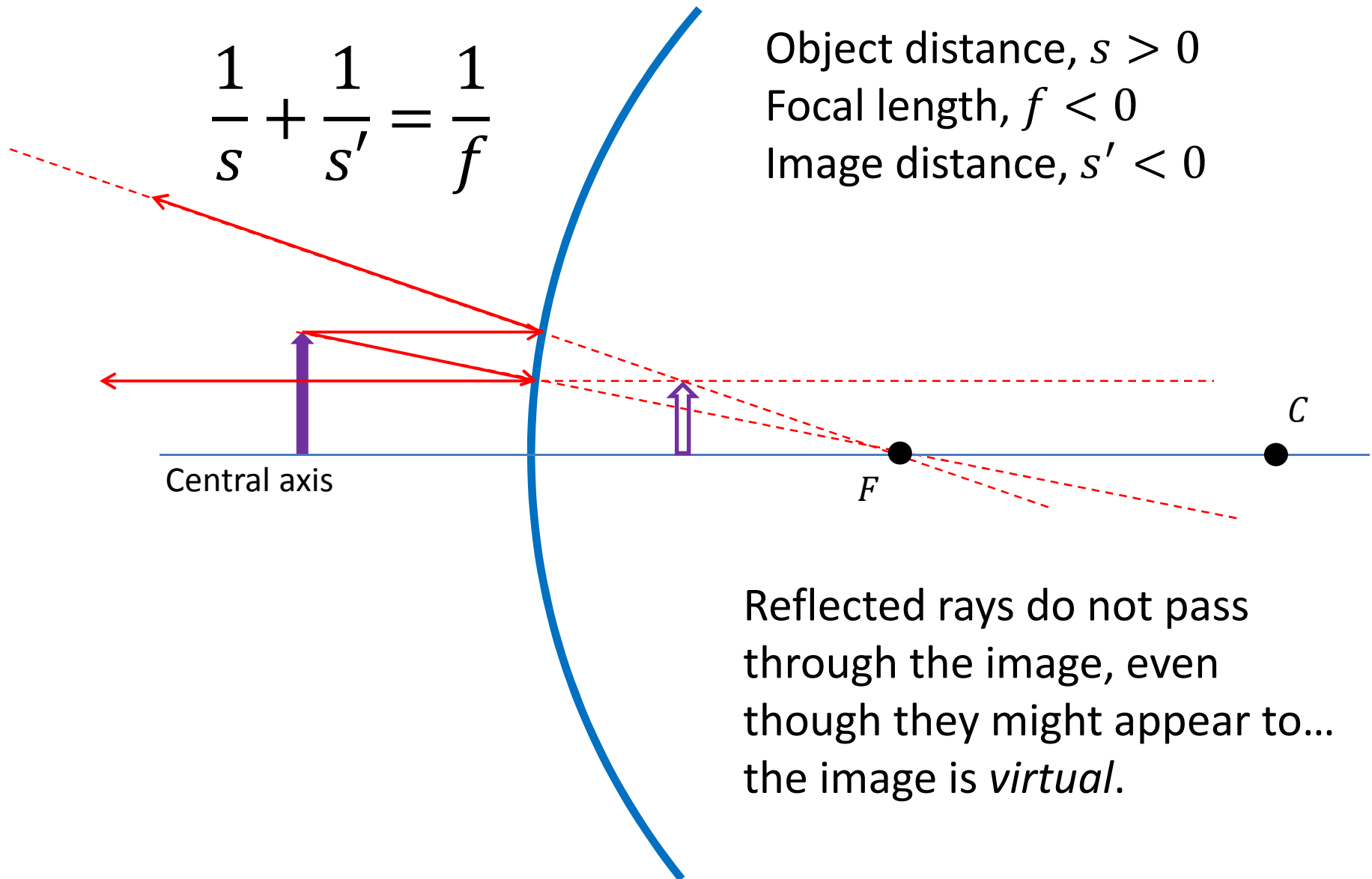it is a *real image*
The image is inverted.

Properties of Images

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Object distance, $s > 0$
Focal length, $f < 0$
Image distance, $s' < 0$

Central axis

F

C

Reflected rays do not pass through the image, even though they might appear to... the image is *virtual*.

Magnification

- Using these sign conventions, the magnification is

$$m = -\frac{n_1 s_i}{n_2 s_o}$$

- Ratio of image height to object height
- Sign indicates whether the image is inverted