PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 10 – Examples

Spring 2014 Semester

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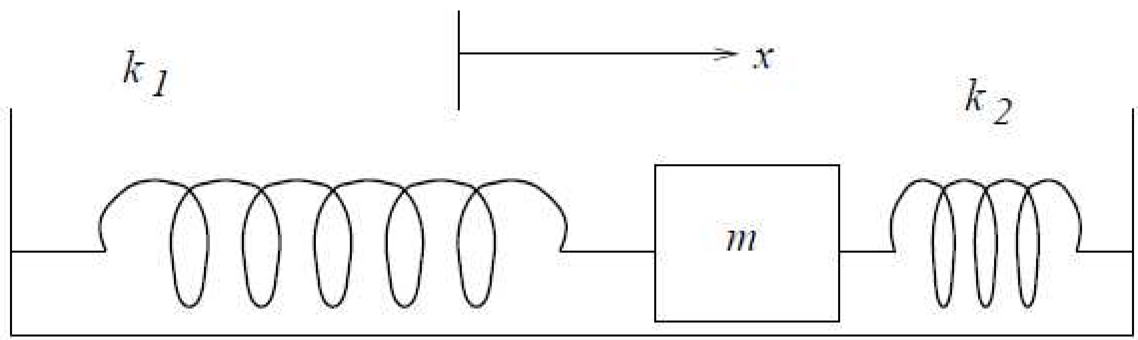
Question #1

4. Consider an object of mass m that is connected to two springs, with spring constants k_1 and k_2 as shown:



where x measures the displacement of the object from its equilibrium position. Determine the angular frequency, ω, with which the object will oscillate.

Question #1

- How to approach this problem?
  - There are two springs so there will be two forces.
  - What coordinate system should we use to describe the motion?
  - Can we express the forces in terms of the chosen coordinates?
  - How is the net force similar to the case with just one spring, which we know how to solve?

Question #2

1. *(French 3-10)* A metal rod, 0.5 m long, has a rectangular cross section of area 2 mm^2.
(a) With the rod vertical and a mass of 60 kg hung from the bottom, there is an extension of 0.25 mm. What is Young's modulus in N/m^2 for the material of the rod?

- How to approach this problem?
  - What is the definition of Young's modulus?

$$\frac{F}{A} = \frac{Y}{L}\Delta z$$

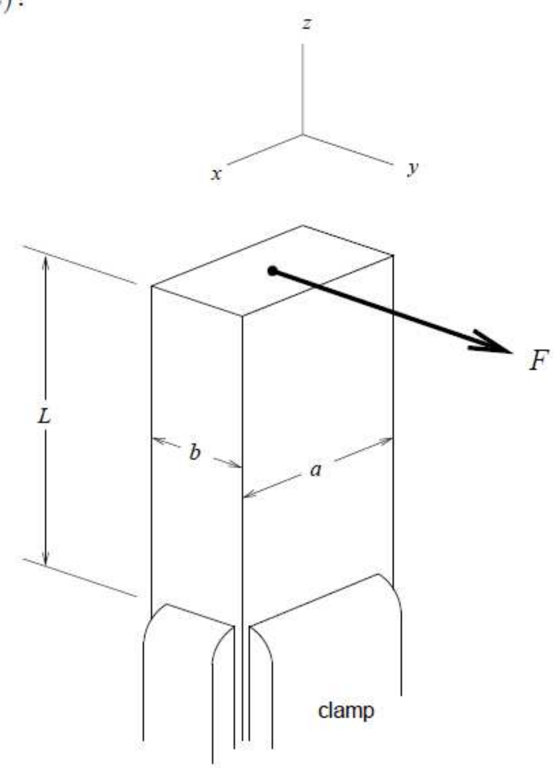
  - Do we have enough information to use this definition?
    - We are given A, L, Δz, but not F explicitly.
    - But we know that $F = mg$ (assume it is on Earth).
  - Yes! Use the definition to solve for Y.

Question #3

(b) The rod is firmly clamped at the bottom as shown in the figure below, and at the top a force F is applied in the y-direction as shown (parallel to the edge of length b). The result is a static deflection, y, given by

$$y = \frac{4L^3}{Yab^3}F$$

If the force F is removed and a msss m, which is much greater than the mass of the rod, is attached to the top en dof the rod, what is the ratio of the frequencies of vibration in the y and x directions (*i.e.*, parallel to the edges of length b and a)?



Question #3

- Do you understand the geometry?
  - It talks about a large mass, m, which is attached to the top of the rod (not shown in the diagram).
  - Should we ignore the mass of the rod?
    - We don't have much choice… we weren't given its mass or density.
  - What force is applied to the mass?
    - Newton's 3rd law: equal and opposite to the force that deflects the rod.

$$F = -Y\frac{ab^3}{4L^3}y = m\ddot{y}$$

  - Is this the same as a simple mass-spring system?

$$m\ddot{y} + ky = 0 \;\; \Rightarrow \;\; \omega = \sqrt{k/m}$$

Question #3

- What about oscillations in the x-direction?
  - What is the only difference between x and y?
    - The width b is measured in the y-direction
    - The width a is measured in the x-direction
    - Interchange $x \leftrightarrow y$ and $a \leftrightarrow b$

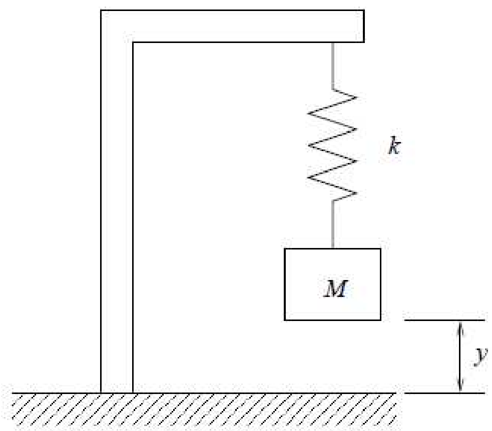
$$F = -Y \frac{ba^3}{4L^3} x = m\ddot{x}$$

  - Ratio of frequencies is

$$\frac{\omega_y}{\omega_x} = \sqrt{\frac{ab^3}{ba^3}} = \frac{b}{a}$$

Question #4

2. (*French, 4-6*) Imagine a simple seismograph consisting of a mass M hung from a spring on a rigid framework attached to the earth, as shown:



The spring force and the damping force depend on the displacement and velocity relative to the earth's surface, but this is not an inertial reference frame if its surface is moving which would be the case in the event of an earthquake. Instead, if we define the position of the earth's surface to be η in an inertial reference frame, then the mass would have a position $z = \eta + y$ in this reference frame which would then satisfy Newton's second law: $F = M\ddot{z}$.

(a) Using y to denote the displacement of M relative to the earth and η to denote the displacement of the earth's surface itself, show tha the equation of motion is

$$\ddot{y} + \gamma\dot{y} + \omega_0^2 y = -\frac{d^2\eta}{dt^2}$$

Question #4

- What creates the force in this problem?
  - A change in the length of the spring…
    - What changes the length of the spring?
    - The motion of the earth that supports the apparatus…
  - The reference frame in the diagram is not an inertial reference frame… during an earthquake, the surface of the earth is oscillating with an amplitude

$$\eta(t) = C\cos\omega t$$

Question #4

- Newton's laws only apply in inertial reference frames.
  - Express the position of the mass using inertial coordinates:
    $$z(t) = \eta(t) + y(t)$$
  - Newton's second law:
    $$F = -k(y - y_0) = M\ddot{z} = M\ddot{\eta} + M\ddot{y}$$
  - We can ignore the y_0 term – it doesn't affect the amplitude or frequency.
    $$M\ddot{y} + b\dot{y} + ky = -M\ddot{\eta}$$
  - The driving "force" is just $M\ddot{\eta}$
    $$\frac{d^2\eta}{dt^2} = -C\,\omega^2 \cos \omega t$$

Question #4

- How similar is this to the forced harmonic oscillator problem we studied in class?

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos\omega t$$
$$M\ddot{y} + b\dot{y} + ky = CM\omega^2 \cos\omega t$$
$$F_0 \to CM\omega^2$$

- Amplitude of steady-state oscillations:

$$A(\omega) = \frac{F_0}{k} \frac{\omega_0/\omega}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2}\right]^{1/2}}$$

$$A(\omega) = \frac{M}{k} \frac{C\omega_0\omega}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2}\right]^{1/2}} = C \frac{\omega/\omega_0}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2}\right]^{1/2}}$$