

Physics 42200
Waves & Oscillations

Spring 2014 Semester

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Preliminary Information

- Course web page:
http://www.physics.purdue.edu/~mjones/phys42200_Spring2013
- Contact information:
 - E-mail: mjones@physics.purdue.edu
 - Office: PHYS 378
 - Phone: 765-496-2464
 - Office hours: Tuesday/Thursday 2 pm-3 pm or by appointment
- Textbooks:
 - A. P. French, Vibrations and Waves (required)
 - E. Hecht, Optics – 4th Edition (highly recommended)
- Lecture:
 - Not guaranteed to cover *all* the material you are responsible for on the homework or exams.
 - *May* cover material you are not responsible for but which will benefit your general education on the subject matter.
 - Read the text first, ask questions in class.

Preliminary Information

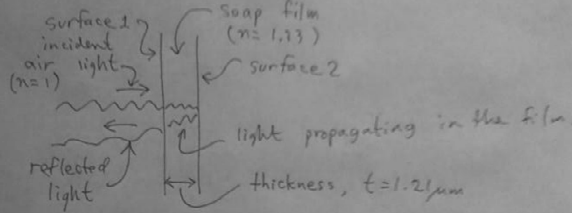
- Grading policy:
 - 30% homework
 - 30% midterm evening exam
 - 40% comprehensive final exam
- Homework policy:
 - Assigned on Friday, due the following Friday.
 - At most, one assignment will be dropped when computing final homework grade
 - Corollary #1: You can freely skip one assignment at your discretion
 - Corollary #2: No late homework will be accepted

Homework solutions must include complete, legible explanations of your work. It must be easy for the reader to follow your reasoning. If it takes longer for the grader to figure out your reasoning than it took you to write out your solution then something is wrong!

Assignments

Assignment x, question 4:

Bright light of wavelength 585 nm is incident perpendicularly on a soap film with index of refraction $n = 1.33$ and thickness $t = 1.21 \mu\text{m}$, and is suspended in air ($n = 1$). If light is reflected by the two surfaces of the film, will the reflected light interfere constructively or destructively?



Since the index of refraction of the soap film is greater than that of air, the light reflected from surface 1 will have a phase shift of π ($\frac{1}{2}$ a wavelength).

Inside the soap film, the wavelength is

$$\lambda' = \lambda/n = \frac{585 \text{ nm}}{1.33} = 440 \text{ nm}.$$

There is no phase shift when reflecting from surface 2 because air has a lower index of refraction than the film. The number of wavelengths in the total path length is $N = \frac{2t}{\lambda'} = \frac{2(1.21 \mu\text{m})}{0.440 \mu\text{m}} = 5.5$. Therefore, both reflected rays are in phase resulting in fully constructive interference.

← **Good!**

#4. $\frac{585}{1.33} = 440$

$$\frac{2 \times 1.21}{.440} = 5.5$$

Beam #1 & #2 are
shift \Rightarrow Fully constructive

##

BAD!
(But I've seen worse)

Assignments

- General advice:
 - In general, we don't care if you can get the right answer or not. You need to explain *why* it is correct.
 - Explain using words and algebra you arrive at a solution. Then substitute in numerical data, if applicable.
 - The algebra should be readable as part of a good English sentence: state what idea or information the formula is used to describe.
 - Symbols that are used without definitions or explanations are usually meaningless.
 - Diagrams can be extremely helpful. Make them large enough to convey the information they need to communicate.
 - Explain your logic! A bunch of formulas are not sufficient to explain the solution.
 - Re-iterate principles learned in class.
 - Your solution should read like an example found in a good text book.
 - Your solution should be useful for studying or reference long after it has been graded.

Oscillations and Waves

- Why study oscillations and waves?
 - A large fraction of all physical situations involve periodic or oscillatory behavior
 - Motion of the planets
 - Stable mechanical systems
 - Electrical systems
 - Fundamental forces
 - Periodic motion in continuous media
 - Wave propagation
 - Electromagnetic radiation (light/optics)
 - Quantum mechanics
 - Most fundamental description of nature
 - Quantum Mechanics + Special Relativity → Quantum Field Theory
 - Matter particles are quantized “waves” in an underlying “field”.

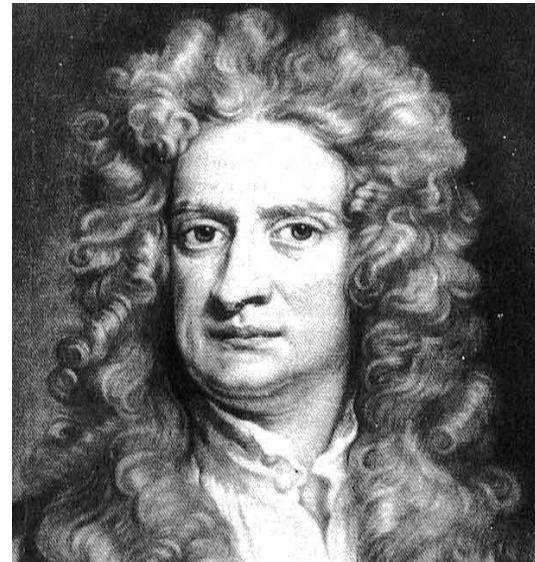
Early Studies of Oscillations



Robert Hooke
(1635-1703)



Christian Huygens
(1623-1697)



Isaac Newton
(1642-1727)

$$F(x) = -k x$$

How to Read

$$F(x) = -k x$$

- We are describing a force...
- The force depends on a displacement, x ...
- The magnitude of the force is proportional to x ...
- The direction of the force is opposite the direction of the displacement...
- This defines the k as the constant of proportionality.

The formula by itself, out of context, conveys very little meaning.

Nevertheless, algebra is very useful for solving equations.

Mechanics

Newton's second law:

Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.

The alteration of motion is ever proportional to the motive force impressed, and is made in the right line in which that force is impressed.

$$\sum \vec{F} = m\vec{a} = m \frac{d^2 \vec{x}}{dt^2}$$

Putting it together...

- Pick a suitable coordinate system so that

$$\vec{F}(x) = F(x)\hat{i}$$

- When the only force is described by Hooke's law,


$$F(x) = -k x$$

the resulting acceleration is given by Newton's law:

$$F(x) = -k x = m \frac{d^2 x}{dt^2} = m \ddot{x}$$

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

But what is the resulting acceleration?



More notation you
should get used to...

Oscillating Systems

- Differential equation:

$$m \frac{d^2 x}{dt^2} + kx = 0$$
$$\frac{d^2 x}{dt^2} + \omega^2 x = 0$$

- Solutions:

$$x(t) = \mathbf{A} \cos(\omega t + \boldsymbol{\varphi})$$
$$x(t) = \mathbf{A} \sin \omega t + \mathbf{B} \cos \omega t$$

- The solutions are periodic with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{k/m}$$

Oscillating Systems

- Potential Energy:

$$F(x) = -\frac{dV}{dx}$$

- Linear spring described by Hooke's Law:

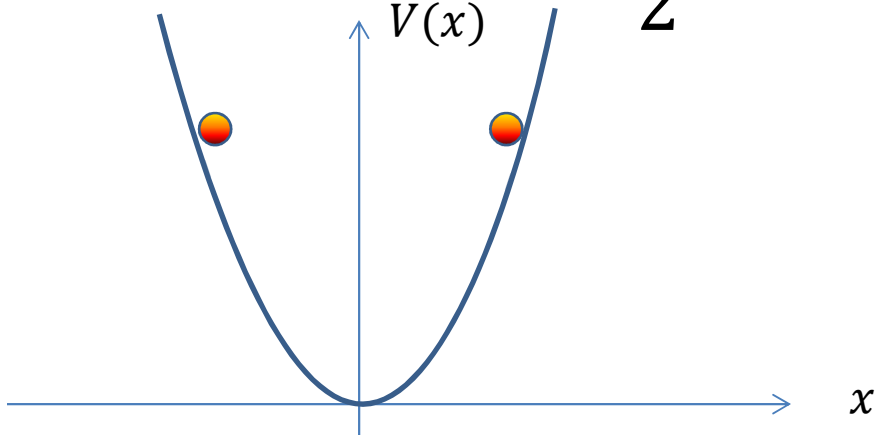
$$V(x) = \frac{1}{2} k x^2$$

- Energy conservation:

$$E = T + V = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 = \textit{const.}$$

Visualizing Potential Energy

- Gravitational potential:

$$V(x) = mg h(x) = \frac{1}{2} k x^2$$


- When the force depends on position, the motion will be like that of a marble rolling on a suitably shaped surface.

Other Dynamical Variables

- The position, x , is used to describe the “state” of the system:
 - If you know x and \dot{x} at one time, then you can predict what they will be at any other time.
- Other systems can be described using other dynamical variables:
 - Angles for rotating mechanical systems
 - Voltage or current in electrical systems
 - Electric or magnetic fields (*eg*, light!)

Other Oscillating Systems

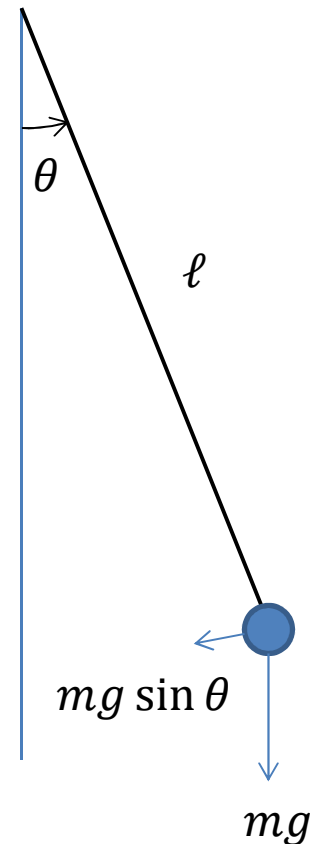
- A pendulum:

$$m\ell\ddot{\theta} = -mg \sin \theta \approx -mg\theta$$

$$\ddot{\theta} + \omega^2 \theta = 0,$$

$$\omega = \sqrt{g/\ell}$$

$$\theta(t) = \mathbf{A} \cos(\omega t + \mathbf{\varphi})$$



Other Oscillating Systems

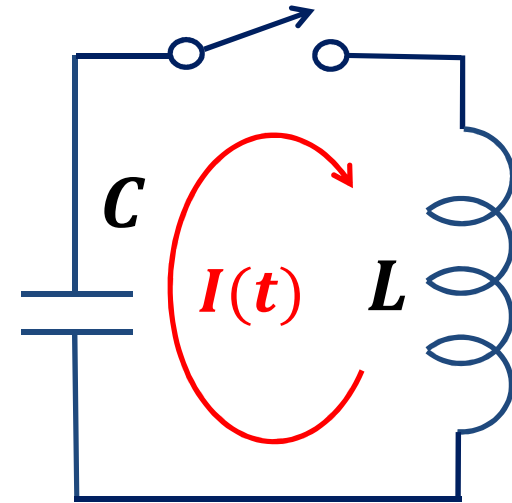
- An electric circuit:

$$\frac{1}{C}I + L \frac{d^2 I}{dt^2} = 0$$

$$\ddot{I} + \omega^2 I = 0,$$

$$\omega = \sqrt{1/LC}$$

$$I(t) = \mathbf{A} \cos(\omega t + \mathbf{\varphi})$$



Summary so far...

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

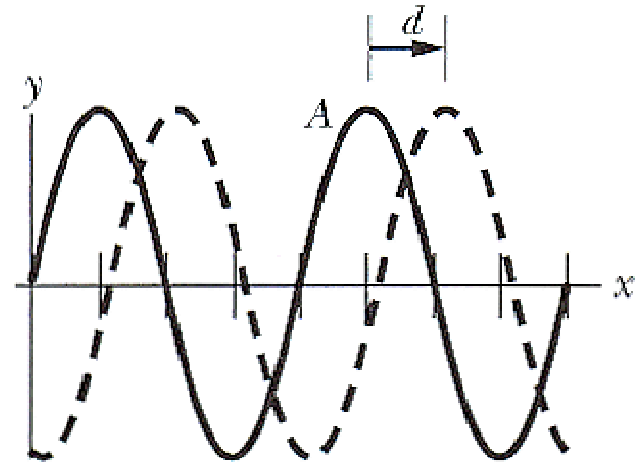
- The first half of the course is only concerned with the solutions to this type of differential equation.
- How to solve it is not very interesting.
- Of more importance is what the solutions tell us about the behavior of various physical systems.

Other Oscillating Systems

- Waves:
 - Amplitude is a function of both x and t
- Wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

- Solutions are of the form
 $y(x, t) = \mathbf{A} \cos(kx - \omega t + \mathbf{\varphi})$



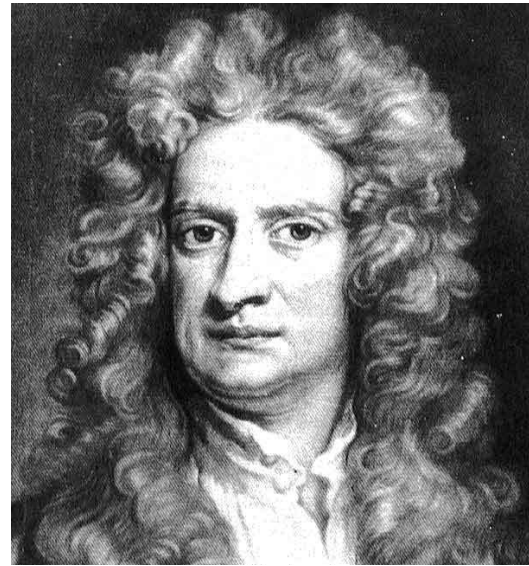
Is light a wave?

Descriptions of Light



Christian Huygens
(1623-1697)

Promoted the “wave
theory” of light.

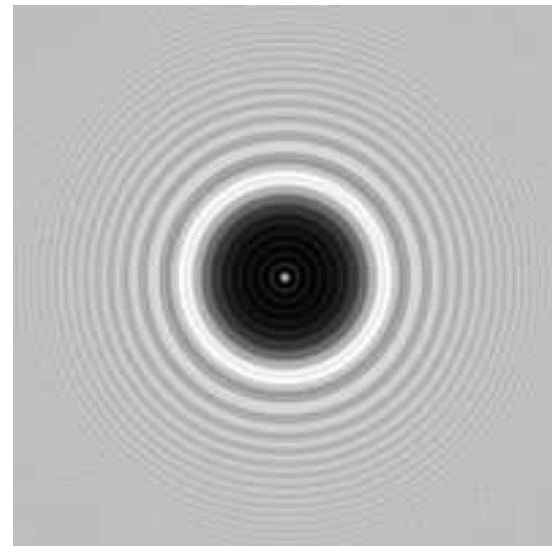
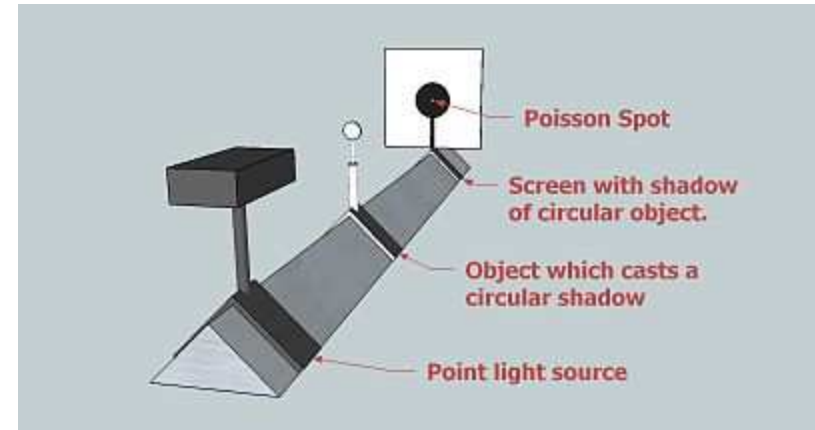


Isaac Newton
(1642-1727)

Described light as particles
that moved in straight lines.

Diffraction Experiments

- Newton rejected the wave theory for good reason, but assumed the waves would be *longitudinal* like sound.
- A century later it was postulated that light could be a *transverse* wave.
- Based on the wave theory, Poisson, Arago and Fresnel predicted that there should be a bright spot at the center of a circular shadow.



Electromagnetic Radiation

By 1864, Maxwell had introduced four equations that described all known electromagnetic phenomena.

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$$
$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\phi_e}{dt}$$



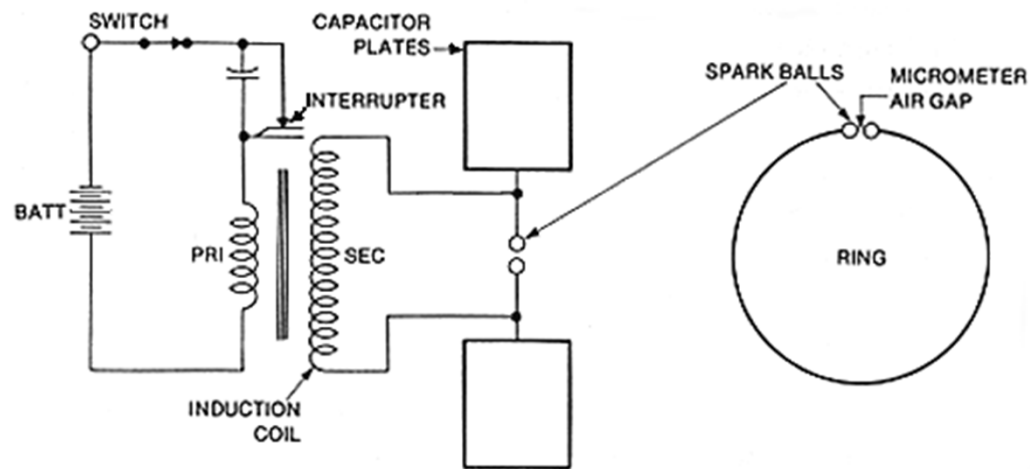
“An changing electric field induces a magnetic field and a changing magnetic field induces an electric field...”

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$
$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.998 \times 10^8 \text{ m/s}$$

Observation of Electromagnetic Radiation



Heinrich Hertz
(1857-1894)



"It's of no use whatsoever ... this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there."

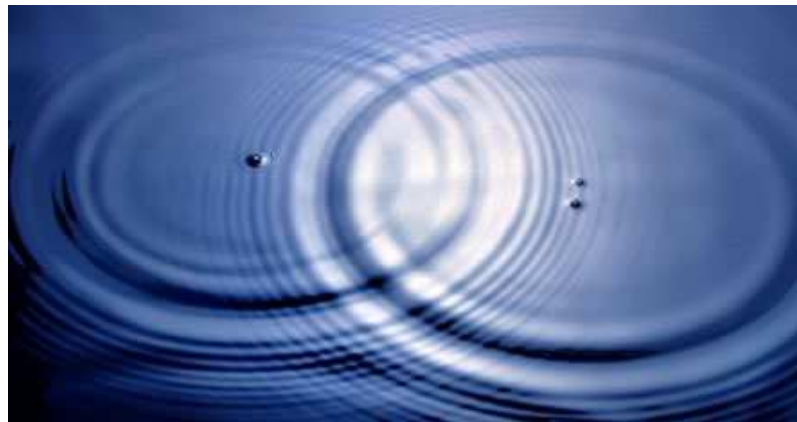
Asked about the ramifications of his discoveries, Hertz replied, "Nothing, I guess."

Modern Physics

- Maxwell's Equations could not explain:
 - Spectrum of light from blackbody radiation
 - Photoelectric effect
 - Speed of light is the same in all reference frames
- Quantum Mechanics:
 - Planck proposed that the energy of light was quantized, $E = h\nu$
 - Einstein explained the photoelectric effect: light was composed of particles with $E = h\nu$
 - Special theory of relativity: speed of light is the same in all reference frames.

Quantum Field Theory

- All particles are quantized excitations of a *field*.
 - Photons are quantized excitations of the electromagnetic field.
 - Electrons are quantized excitations of the “electron field”.
 - Explains particle creation and annihilation.



- A natural mathematical consequence: the Higgs field.
- Very likely to have been directly observed last summer.
- But in this course we will only consider classical waves.