(* For a string of length L, tension T and mass per unit length μ, the wave velocity is given by *)

$v := \sqrt{T/\mu}$

(* and the frequency of the normal modes of oscillation are *)

$$\omega[n\_] := \frac{n\pi}{L}\sqrt{\frac{T}{\mu}}$$

(* a general solution to the wave equation with boundary conditions y(0) = y(L) = 0 can be written *)

$$y[x\_, t\_] := \sum_{n=1}^{M} a[n]\,\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\mathrm{Cos}[\omega[n]\,t]$$

(* The coefficients a[n] can be calculated from the initial conditions f(x) at t=0 *)

$$a[n\_] := \frac{2}{L}\int_0^L f[x]\,\mathrm{Sin}\left[\frac{n\pi x}{L}\right]dx$$

(* Consider the case where T = 1, μ = 1 and L = 1 in a suitable set of units. *)

T = 1

1

μ = 1

1

L = 1

1

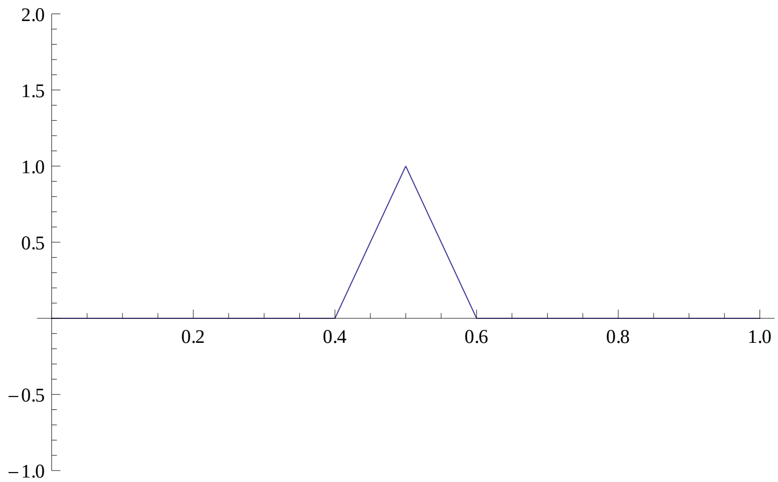
(* and consider the initial condition f(x) = 0 when x < 2/5 or x > 3/5 and some sort of triangular pulse in between. *)

In[22]:=
```
f[x_] = Piecewise[{{0, x < 2 / 5}, {10 * (x - 2 / 5), x > 2 / 5 && x < 1 / 2},
   {2 + 10 * (2 / 5 - x), x > 1 / 2 && x < 3 / 5}, {0, x > 3 / 5}}]
```

Out[22]=
$$\begin{cases} 0 & x < \frac{2}{5} \\ 10\left(-\frac{2}{5}+x\right) & x > \frac{2}{5}\ \&\&\ x < \frac{1}{2} \\ 2+10\left(\frac{2}{5}-x\right) & x > \frac{1}{2}\ \&\&\ x < \frac{3}{5} \\ 0 & \text{True} \end{cases}$$

In[23]:= **Plot[f[x], {x, 0, 1}, PlotRange → {-1, 2}]**

Out[23]=



In[20]:=

```
(* We will evaluate coefficients only up to M = 30 *)
M = 30
(* The coefficients a[n] are... *)
```

Out[20]= 30

In[24]:= **coef = Table[a[n], {n, M}]**

Out[24]=

$$\left\{-\frac{10\left(-4+\sqrt{2\left(5+\sqrt{5}\right)}\right)}{\pi^2},\ 0,\ \frac{10\left(-4+\sqrt{2\left(5-\sqrt{5}\right)}\right)}{9\pi^2},\ 0,\ \frac{8}{5\pi^2},\ 0,\ -\frac{10\left(4+\sqrt{2\left(5-\sqrt{5}\right)}\right)}{49\pi^2},\ 0,\right.$$

$$\frac{10\left(4+\sqrt{2\left(5+\sqrt{5}\right)}\right)}{81\pi^2},\ 0,\ -\frac{10\left(4+\sqrt{2\left(5+\sqrt{5}\right)}\right)}{121\pi^2},\ 0,\ \frac{10\left(4+\sqrt{2\left(5-\sqrt{5}\right)}\right)}{169\pi^2},\ 0,\ -\frac{8}{45\pi^2},$$

$$0,\ -\frac{10\left(-4+\sqrt{2\left(5-\sqrt{5}\right)}\right)}{289\pi^2},\ 0,\ \frac{10\left(-4+\sqrt{2\left(5+\sqrt{5}\right)}\right)}{361\pi^2},\ 0,\ -\frac{10\left(-4+\sqrt{2\left(5+\sqrt{5}\right)}\right)}{441\pi^2},\ 0,$$

$$\left.\frac{10\left(-4+\sqrt{2\left(5-\sqrt{5}\right)}\right)}{529\pi^2},\ 0,\ \frac{8}{125\pi^2},\ 0,\ -\frac{10\left(4+\sqrt{2\left(5-\sqrt{5}\right)}\right)}{729\pi^2},\ 0,\ \frac{10\left(4+\sqrt{2\left(5+\sqrt{5}\right)}\right)}{841\pi^2},\ 0\right\}$$

In[25]:= `z[x_, t_] = y[x, t]`

Out[25]=

$$-\frac{10\left(-4+\sqrt{2\left(5+\sqrt{5}\right)}\right)\text{Cos}[\pi\, t]\,\text{Sin}[\pi\, x]}{\pi^2}+\frac{10\left(-4+\sqrt{2\left(5-\sqrt{5}\right)}\right)\text{Cos}[3\pi\, t]\,\text{Sin}[3\pi\, x]}{9\pi^2}+\frac{8\,\text{Cos}[5\pi\, t]\,\text{Sin}[5\pi\, x]}{5\pi^2}-$$

$$\frac{10\left(4+\sqrt{2\left(5-\sqrt{5}\right)}\right)\text{Cos}[7\pi\, t]\,\text{Sin}[7\pi\, x]}{49\pi^2}+\frac{10\left(4+\sqrt{2\left(5+\sqrt{5}\right)}\right)\text{Cos}[9\pi\, t]\,\text{Sin}[9\pi\, x]}{81\pi^2}-$$

$$\frac{10\left(4+\sqrt{2\left(5+\sqrt{5}\right)}\right)\text{Cos}[11\pi\, t]\,\text{Sin}[11\pi\, x]}{121\pi^2}+\frac{10\left(4+\sqrt{2\left(5-\sqrt{5}\right)}\right)\text{Cos}[13\pi\, t]\,\text{Sin}[13\pi\, x]}{169\pi^2}-$$

$$\frac{8\,\text{Cos}[15\pi\, t]\,\text{Sin}[15\pi\, x]}{45\pi^2}-\frac{10\left(-4+\sqrt{2\left(5-\sqrt{5}\right)}\right)\text{Cos}[17\pi\, t]\,\text{Sin}[17\pi\, x]}{289\pi^2}+$$

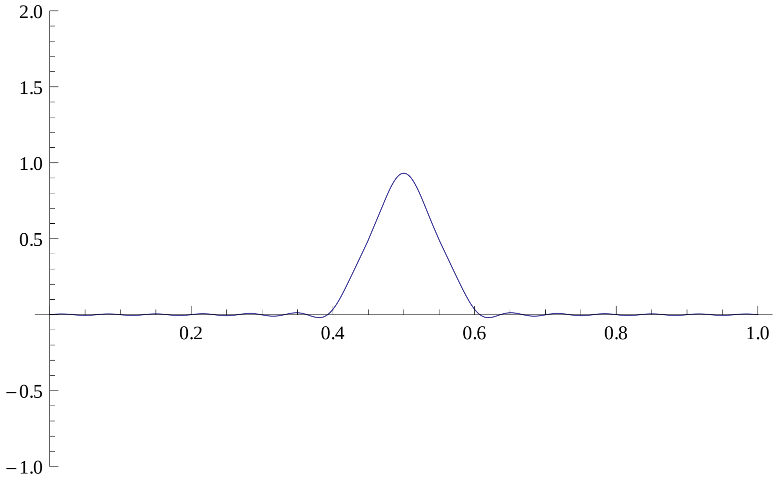
$$\frac{10\left(-4+\sqrt{2\left(5+\sqrt{5}\right)}\right)\text{Cos}[19\pi\, t]\,\text{Sin}[19\pi\, x]}{361\pi^2}-$$

$$\frac{10\left(-4+\sqrt{2\left(5+\sqrt{5}\right)}\right)\text{Cos}[21\pi\, t]\,\text{Sin}[21\pi\, x]}{441\pi^2}+$$

$$\frac{10\left(-4+\sqrt{2\left(5-\sqrt{5}\right)}\right)\text{Cos}[23\pi\, t]\,\text{Sin}[23\pi\, x]}{529\pi^2}+\frac{8\,\text{Cos}[25\pi\, t]\,\text{Sin}[25\pi\, x]}{125\pi^2}-$$

$$\frac{10\left(4+\sqrt{2\left(5-\sqrt{5}\right)}\right)\text{Cos}[27\pi\, t]\,\text{Sin}[27\pi\, x]}{729\pi^2}+\frac{10\left(4+\sqrt{2\left(5+\sqrt{5}\right)}\right)\text{Cos}[29\pi\, t]\,\text{Sin}[29\pi\, x]}{841\pi^2}$$

In[26]:=
```
Plot[z[x, 0], {x, 0, 1}, PlotRange → {-1, 2}]
```

Out[26]=


In[27]:=
```
Export["fourier_4.avi",
 Table[Plot[z[x, t], {x, 0, 1}, PlotRange → {-1, 2}], {t, 0, 2 , 0.01}]]
```

Out[27]= fourier_4.avi