(* For a string of length L, tension T and mass per unit length μ, the wave velocity is given by *)

$$v := \sqrt{T/\mu}$$

(* and the frequency of the normal modes of oscillation are *)

$$\omega[n\_] := \frac{n\pi}{L}\sqrt{\frac{T}{\mu}}$$

(* a general solution to the wave equation with boundary conditions y(0) = y(L) = 0 can be written *)

$$y[x\_, t\_] := \sum_{n=1}^{M} a[n]\,\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\mathrm{Cos}[\omega[n]\,t]$$

(* The coefficients a[n] can be calculated from the initial conditions f(x) at t=0 *)

$$a[n\_] := \frac{2}{L}\int_0^L f[x]\,\mathrm{Sin}\left[\frac{n\pi x}{L}\right] dx$$

(* Consider the case where T = 1, μ = 1 and L = 1 in a suitable set of units. *)

```
T = 1
```

1

```
μ = 1
```

1

```
L = 1
```

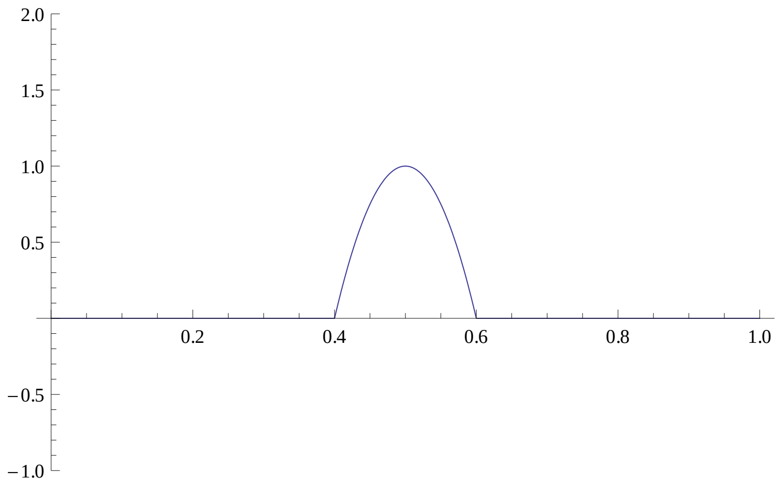
1

(* and consider the initial condition f(x) = 0 when x < 2/5 or x > 3/5 and f(x) = 1-10*(x-1/2)^2 *)

```
f[x_] = Piecewise[{{0, x < 2/5}, {1 - 100 * (x - 1/2)^2, x > 2/5 && x < 3/5}, {0, x > 3/5}}]
```

$$\begin{cases} 0 & x < \frac{2}{5} \\ 1 - 100\left(-\frac{1}{2} + x\right)^2 & x > \frac{2}{5} \,\&\&\, x < \frac{3}{5} \\ 0 & \text{True} \end{cases}$$

```
Plot[f[x], {x, 0, 1}, PlotRange → {-1, 2}]
```




```
(* We will evaluate coefficients only up to M = 30 *)
M = 30
(* The coefficients a[n] are... *)
```

30

```
coef = Table[a[n], {n, M}]
```

$$\left\{-\frac{20\left(10-10\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\,\pi\right)}{\pi^3},\ 0,\ \frac{20\left(-10-10\sqrt{5}+3\sqrt{2\left(5-\sqrt{5}\right)}\,\pi\right)}{27\,\pi^3},\ 0,\ \frac{32}{5\,\pi^3},\right.$$

$$0,\ -\frac{20\left(10+10\sqrt{5}+7\sqrt{2\left(5-\sqrt{5}\right)}\,\pi\right)}{343\,\pi^3},\ 0,\ \frac{20\left(-10+10\sqrt{5}+9\sqrt{2\left(5+\sqrt{5}\right)}\,\pi\right)}{729\,\pi^3},\ 0,$$

$$-\frac{20\left(10-10\sqrt{5}+11\sqrt{2\left(5+\sqrt{5}\right)}\,\pi\right)}{1331\,\pi^3},\ 0,\ \frac{20\left(-10-10\sqrt{5}+13\sqrt{2\left(5-\sqrt{5}\right)}\,\pi\right)}{2197\,\pi^3},\ 0,\ \frac{32}{135\,\pi^3},$$

$$0,\ -\frac{20\left(10+10\sqrt{5}+17\sqrt{2\left(5-\sqrt{5}\right)}\,\pi\right)}{4913\,\pi^3},\ 0,\ \frac{20\left(-10+10\sqrt{5}+19\sqrt{2\left(5+\sqrt{5}\right)}\,\pi\right)}{6859\,\pi^3},\ 0,$$

$$-\frac{20\left(10-10\sqrt{5}+21\sqrt{2\left(5+\sqrt{5}\right)}\,\pi\right)}{9261\,\pi^3},\ 0,\ \frac{20\left(-10-10\sqrt{5}+23\sqrt{2\left(5-\sqrt{5}\right)}\,\pi\right)}{12\,167\,\pi^3},\ 0,\ \frac{32}{625\,\pi^3},$$

$$\left.0,\ -\frac{20\left(10+10\sqrt{5}+27\sqrt{2\left(5-\sqrt{5}\right)}\,\pi\right)}{19\,683\,\pi^3},\ 0,\ \frac{20\left(-10+10\sqrt{5}+29\sqrt{2\left(5+\sqrt{5}\right)}\,\pi\right)}{24\,389\,\pi^3},\ 0\right\}$$

```
(* Ugh! That was messy... good thing we didn't have to do this by hand... *)
z[x_, t_] = y[x, t]
```

$$-\frac{20\left(10-10\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\ \pi\right)\text{Cos}[\pi\,t]\ \text{Sin}[\pi\,x]}{\pi^3}+$$

$$\frac{20\left(-10-10\sqrt{5}+3\sqrt{2\left(5-\sqrt{5}\right)}\ \pi\right)\text{Cos}[3\,\pi\,t]\ \text{Sin}[3\,\pi\,x]}{27\,\pi^3}+$$

$$\frac{32\,\text{Cos}[5\,\pi\,t]\ \text{Sin}[5\,\pi\,x]}{5\,\pi^3}-\frac{20\left(10+10\sqrt{5}+7\sqrt{2\left(5-\sqrt{5}\right)}\ \pi\right)\text{Cos}[7\,\pi\,t]\ \text{Sin}[7\,\pi\,x]}{343\,\pi^3}+$$

$$\frac{20\left(-10+10\sqrt{5}+9\sqrt{2\left(5+\sqrt{5}\right)}\ \pi\right)\text{Cos}[9\,\pi\,t]\ \text{Sin}[9\,\pi\,x]}{729\,\pi^3}-$$

$$\frac{20\left(10-10\sqrt{5}+11\sqrt{2\left(5+\sqrt{5}\right)}\ \pi\right)\text{Cos}[11\,\pi\,t]\ \text{Sin}[11\,\pi\,x]}{1331\,\pi^3}+$$

$$\frac{20\left(-10-10\sqrt{5}+13\sqrt{2\left(5-\sqrt{5}\right)}\ \pi\right)\text{Cos}[13\,\pi\,t]\ \text{Sin}[13\,\pi\,x]}{2197\,\pi^3}+$$

$$\frac{32\,\text{Cos}[15\,\pi\,t]\ \text{Sin}[15\,\pi\,x]}{135\,\pi^3}-\frac{20\left(10+10\sqrt{5}+17\sqrt{2\left(5-\sqrt{5}\right)}\ \pi\right)\text{Cos}[17\,\pi\,t]\ \text{Sin}[17\,\pi\,x]}{4913\,\pi^3}+$$

$$\frac{20\left(-10+10\sqrt{5}+19\sqrt{2\left(5+\sqrt{5}\right)}\ \pi\right)\text{Cos}[19\,\pi\,t]\ \text{Sin}[19\,\pi\,x]}{6859\,\pi^3}-$$

$$\frac{20\left(10-10\sqrt{5}+21\sqrt{2\left(5+\sqrt{5}\right)}\ \pi\right)\text{Cos}[21\,\pi\,t]\ \text{Sin}[21\,\pi\,x]}{9261\,\pi^3}+$$

$$\frac{20\left(-10-10\sqrt{5}+23\sqrt{2\left(5-\sqrt{5}\right)}\ \pi\right)\text{Cos}[23\,\pi\,t]\ \text{Sin}[23\,\pi\,x]}{12\,167\,\pi^3}+$$

$$\frac{32\,\text{Cos}[25\,\pi\,t]\,\text{Sin}[25\,\pi\,x]}{625\,\pi^3} - \frac{20\left(10 + 10\sqrt{5} + 27\sqrt{2\left(5 - \sqrt{5}\right)}\,\pi\right)\text{Cos}[27\,\pi\,t]\,\text{Sin}[27\,\pi\,x]}{19\,683\,\pi^3} +$$

$$\frac{20\left(-10 + 10\sqrt{5} + 29\sqrt{2\left(5 + \sqrt{5}\right)}\,\pi\right)\text{Cos}[29\,\pi\,t]\,\text{Sin}[29\,\pi\,x]}{24\,389\,\pi^3}$$

```
Plot[z[x, 0], {x, 0, 1}, PlotRange → {-1, 2}]
```