```
In[1]:= (*  For a string of length L, tension T and mass per unit length μ,
        the wave velocity is given by  *)
        v := Sqrt[T / μ]
```

$$v := \sqrt{T/\mu}$$

```
In[2]:= (* and the frequency of the normal modes of oscillation are *)
```

$$\omega[n\_] := \frac{n\pi}{L}\sqrt{\frac{T}{\mu}}$$

```
In[3]:= (* a general solution to the wave equation with boundary conditions y(0) =
         y(L) = 0 can be written *)
```

$$y[x\_, t\_] := \sum_{n=1}^{M} a[n]\,\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\mathrm{Cos}[\omega[n]\,t]$$

```
In[4]:= (* The coefficients a[n] can be calculated from the initial conditions f(x) at t=0 *)
```

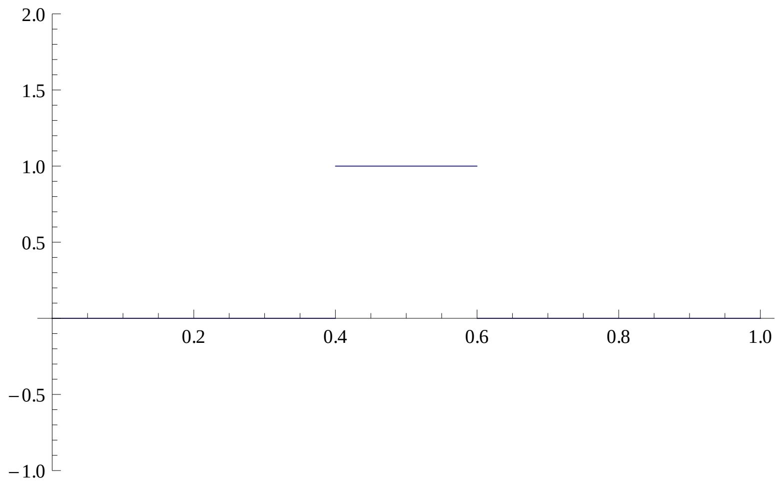
$$a[n\_] := \frac{2}{L}\int_0^L f[x]\,\mathrm{Sin}\left[\frac{n\pi x}{L}\right]dx$$

```
        (*  Consider the case where T = 1, μ = 1 and L = 1 in a suitable set of units.  *)

In[5]:= T = 1

Out[5]= 1

In[6]:= μ = 1

Out[6]= 1

In[7]:= L = 1

Out[7]= 1

        (* and consider the initial condition f(x) =
         0 when x < 1/4 or x > 3/4 and f(x) = 1 otherwise *)

In[8]:= f[x_] = Piecewise[{{0, x < 2 / 5}, {1, x > 2 / 5 && x < 3 / 5}, {0, x > 3 / 5}}]
```

$$\text{Out[8]=} \begin{cases} 0 & x < \frac{2}{5} \\ 1 & x > \frac{2}{5} \,\&\&\, x < \frac{3}{5} \\ 0 & \text{True} \end{cases}$$

In[9]:=
```
Plot[f[x], {x, 0, 1}, PlotRange → {-1, 2}]
```

Out[9]=




```
(* We will evaluate coefficients only up to M = 30 *)
M = 30
```

Out[10]= 30

```
(* The coefficients a[n] are... *)
```

In[11]:=
```
coef = Table[a[n], {n, M}]
```

Out[11]=

$$\left\{\frac{-1+\sqrt{5}}{\pi}, 0, \frac{-1-\sqrt{5}}{3\pi}, 0, \frac{4}{5\pi}, 0, \frac{-1-\sqrt{5}}{7\pi}, 0, \frac{-1+\sqrt{5}}{9\pi}, 0, \frac{-1+\sqrt{5}}{11\pi}, 0, \frac{-1-\sqrt{5}}{13\pi}, 0, \frac{4}{15\pi}, 0, \frac{-1-\sqrt{5}}{17\pi}, 0, \frac{-1+\sqrt{5}}{19\pi}, 0, \frac{-1+\sqrt{5}}{21\pi}, 0, \frac{-1-\sqrt{5}}{23\pi}, 0, \frac{4}{25\pi}, 0, \frac{-1-\sqrt{5}}{27\pi}, 0, \frac{-1+\sqrt{5}}{29\pi}, 0\right\}$$

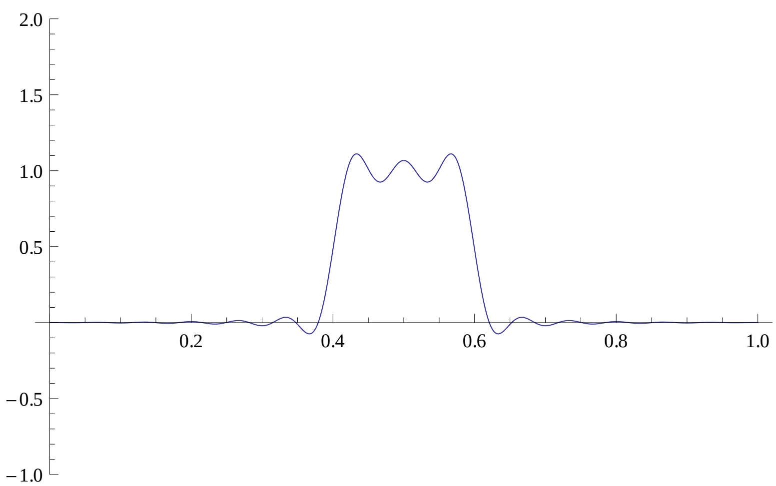
In[12]:=
```
z[x_, t_] = y[x, t]
```

Out[12]=

$$\frac{\left(-1+\sqrt{5}\right)\text{Cos}[\pi\,t]\,\text{Sin}[\pi\,x]}{\pi}+\frac{\left(-1-\sqrt{5}\right)\text{Cos}[3\pi\,t]\,\text{Sin}[3\pi\,x]}{3\pi}+\frac{4\,\text{Cos}[5\pi\,t]\,\text{Sin}[5\pi\,x]}{5\pi}+$$
$$\frac{\left(-1-\sqrt{5}\right)\text{Cos}[7\pi\,t]\,\text{Sin}[7\pi\,x]}{7\pi}+\frac{\left(-1+\sqrt{5}\right)\text{Cos}[9\pi\,t]\,\text{Sin}[9\pi\,x]}{9\pi}+$$
$$\frac{\left(-1+\sqrt{5}\right)\text{Cos}[11\pi\,t]\,\text{Sin}[11\pi\,x]}{11\pi}+\frac{\left(-1-\sqrt{5}\right)\text{Cos}[13\pi\,t]\,\text{Sin}[13\pi\,x]}{13\pi}+$$
$$\frac{4\,\text{Cos}[15\pi\,t]\,\text{Sin}[15\pi\,x]}{15\pi}+\frac{\left(-1-\sqrt{5}\right)\text{Cos}[17\pi\,t]\,\text{Sin}[17\pi\,x]}{17\pi}+$$
$$\frac{\left(-1+\sqrt{5}\right)\text{Cos}[19\pi\,t]\,\text{Sin}[19\pi\,x]}{19\pi}+\frac{\left(-1+\sqrt{5}\right)\text{Cos}[21\pi\,t]\,\text{Sin}[21\pi\,x]}{21\pi}+$$
$$\frac{\left(-1-\sqrt{5}\right)\text{Cos}[23\pi\,t]\,\text{Sin}[23\pi\,x]}{23\pi}+\frac{4\,\text{Cos}[25\pi\,t]\,\text{Sin}[25\pi\,x]}{25\pi}+$$
$$\frac{\left(-1-\sqrt{5}\right)\text{Cos}[27\pi\,t]\,\text{Sin}[27\pi\,x]}{27\pi}+\frac{\left(-1+\sqrt{5}\right)\text{Cos}[29\pi\,t]\,\text{Sin}[29\pi\,x]}{29\pi}$$

In[13]:=
```
Plot[z[x, 0], {x, 0, 1}, PlotRange → {-1, 2}]
```

Out[13]=



In[15]:=
```
Export["fourier_2.avi",
  Table[Plot[z[x, t], {x, 0, 1}, PlotRange → {-1, 2}], {t, 0, 2 , 0.01}]]
```

Out[15]= fourier_2.avi