1. Focal length of a planar-convex lens in air:

Let $R_1$ be the radius of curvature of the front surface and $R_2$ be the radius of curvature of the back surface. The thin lens equation is

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{s_o} - \frac{1}{s_i} = (\frac{n_e - 1}{R_1} - \frac{1}{R_2})$$

When $R_1 \to \infty$, $R_2 \to -50$ mm, $s_o \to \infty$ the image is formed at the focal point, $s_i = f$.

Thus, $\frac{1}{f} = (\frac{n_e - 1}{R_2})$

(a) $f = \frac{-R_2}{n - 1} = \frac{50 \text{ mm}}{1.5 - 1} = \frac{50 \text{ mm}}{0.5} = 100$ mm.

(b) If the lens was in water then the thin lens equation becomes

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{n_e - n_w}{n_w} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = \frac{n_e - n_w}{n_w} \left(\frac{-1}{R_2}\right), \quad f = \frac{-R_2 n_w}{n_e - n_w} = \frac{(50 \text{ mm})(1.33)}{1.5 - 1.33} = 391$ mm.
2. For a biconcave lens, the thin lens equation can be written

\[
\frac{1}{s_o} + \frac{1}{s_i} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

In this case, we are given the object position,

\[s_o = 8 \text{ cm}\]

the index of refraction, \(n = 1.5\), and the radii of curvature:

\[R_1 = -20 \text{ cm} \]
\[R_2 = +10 \text{ cm} \]

The image distance is then

\[s_i = \left( (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) - \frac{1}{s_o} \right)^{-1}
\]

\[= \left( (1.5-1) \left( \frac{1}{-20 \text{ cm}} - \frac{1}{10 \text{ cm}} \right) - \frac{1}{8 \text{ cm}} \right)^{-1}
\]

\[= -5 \text{ cm} \]

measured from the vertex of the lens.

The transverse magnification is

\[M_T = -\frac{s_i}{s_o} = \frac{5 \text{ cm}}{8 \text{ cm}} = 5/8 = 0.625
\]

The image is reduced in size but is upright.

The image is a virtual image.

If the object has a height of 1 cm, then the image will have a height of 6.25 mm.
When the thickness of the lens is taken into consideration, the thick lens equation can be used to calculate the focal length:

\[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2} \right) \]

\[ = (1.5 - 1) \left( \frac{1}{-20\text{cm}} - \frac{1}{10\text{cm}} + \frac{(1.5-1)(5\text{cm})}{(1.5)(-20\text{cm})(10\text{cm})} \right) \]

\[ S_0 \ f = -12.632 \text{ cm} \]

The principal planes are located at distances

\[ h_1 = -\frac{f(n-1)d}{nR_2} = -\frac{(-12.632\text{cm})(1.5-1)(5\text{cm})}{(1.5)(10\text{cm})} \]

\[ = 2.105 \text{ cm} \]

\[ h_2 = -\frac{f(n-1)d}{nR_1} = -\frac{(-12.632\text{cm})(1.5-1)(5\text{cm})}{(1.5)(-20\text{cm})} \]

\[ = -1.053 \text{ cm} \]

The position of the object, measured with respect to the first principal plane, is

\[ S_0' = S_0 + h_1 = 8\text{ cm} + 2.105 \text{ cm} = 10.105 \text{ cm} \]

The image position, measured with respect to the second principal plane is obtained from

\[ \frac{1}{S_i'} + \frac{1}{S_0'} = \frac{1}{f} \]

\[ S_0 = S_i' = \left( \frac{1}{f} - \frac{1}{S_0'} \right)^{-1} = \left( \frac{1}{-12.632\text{cm}} - \frac{1}{10.105\text{cm}} \right)^{-1} = -5.614 \text{ cm} \]
The image position measured with respect to the second vertex is

\[ s_i = s_i' + h_2 \]

\[ = -5.614 \text{ cm} + (-1.053 \text{ cm}) \]

\[ = -6.667 \text{ cm} \]

The transverse magnification is

\[ M_T = \frac{-s_i'}{s_i'} = -\frac{5.614 \text{ cm}}{10.105 \text{ cm}} = 0.556 \]
3. The following diagram shows the geometry for a single refracting surface:

![Diagram]

From this diagram we see that \( y_0 = s_0 \tan \theta_i \) and \( y_i = s_i \tan \theta_t \), but in the small angle approximation, \( \tan \theta \approx \theta \).

Thus, \( M_T = \frac{y_i}{y_0} = -\frac{s_i \theta_t}{s_0 \theta_i} \)

However, from Snell's law, \( n_1 \theta_i = n_2 \theta_t \)

So \( \frac{\theta_t}{\theta_i} = \frac{n_1}{n_2} \).

Therefore, \( M_T = -\frac{n_1 s_i}{n_2 s_0} \)
4. The parallel rays of the laser beams correspond to object and image distances at infinity. The geometry of the lens system is as follows:

There are similar triangles so \( \frac{d_1}{f_1} = \frac{d_2}{f_2} \).

Thus, \( f_2 = \frac{d_2}{d_1} f_1 = \frac{8 \text{ mm}}{1 \text{ mm}} \cdot \frac{50.0 \text{ mm}}{1 \text{ mm}} = 400 \text{ mm} \).

The separation between the lenses must be \( l = f_1 + f_2 = (50 \text{ mm}) + (400 \text{ mm}) = 450 \text{ mm} \).