Physics 422 - Spring 2014 - Final Exam, May 8th

Answer all questions in the exam booklets provided.
There are 8 questions – please answer any 6.
Explain your reasoning clearly but concisely.
Clearly indicate which work is to be graded.
Each question is of equal weight.
You can use two pages of your own notes/formulas.

Here are some useful trigonometric identities:

\[
\begin{align*}
\sin^2 \theta + \cos^2 \theta & = 1 \\
\cos^2 \theta - \sin^2 \theta & = \cos(2\theta) \\
\sin(\alpha \pm \beta) & = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\
\cos(\alpha \pm \beta) & = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\
\sin(2\theta) & = 2 \sin \theta \cos \theta \\
\cos(2\theta) & = 2 \cos^2 \theta - 1
\end{align*}
\]

1. Describe the polarization state (ie. unpolarized, linear, circular, elliptical) of light produced in the following ways:

(a) Unpolarized light reflecting from a mirror with \( \theta_i \neq 0 \).
(b) Unpolarized light reflecting from a dielectric surface with \( \theta_i \neq 0 \).
(c) Circular polarized light reflecting from a dielectric surface \( \theta_i \neq 0 \).
(d) Circular polarized light incident on a quarter-wave plate with \( \theta_i = 0 \).
(e) Linear polarized light incident on a quarter-wave plate with \( \theta_i = 0 \).
2. Light that is linearly polarized along the $x$-axis passes through a quarter-wave plate which has its fast axis aligned at an angle $\theta$ with respect to the $x$-axis.

(a) What is the Jones vector that describes the resulting light?
(b) What are the Stokes’ parameters for the resulting light?

3. Coherent light is emitted from a source located a distance $a/2$ above an interface between oil on top ($n = 3/2$) and water below ($n = 4/3$) as shown:

(a) At what height $y$ will bright interference fringes be observed on the screen, located a distance $d$ from the light source?

(b) At what height $y$ will bright interference fringes be observed on the screen if the light source were located a distance $a/2$ below the interface?
4. The optical elements of a Michelson interferometer are shown below:

If the beam splitter, B, and the corrector plate, G, have index of refraction \( n \), by how much would the optical path length difference between the two beams change if the corrector plate was removed?

5. Consider a pair of thin lenses which have focal lengths \( f_1 \) and \( f_2 \) for light with wavelength \( \lambda \), but which have different focal lengths \( f'_1 \) and \( f'_2 \) for light with a different wavelength \( \lambda' \). Derive an equation that determines the distance \( L \) between the lenses that will produce an image that does not suffer from chromatic aberration, at least for these two particular wavelengths. You don’t have to solve the equation.

6. The equation of motion for an electron bound to an atomic nucleus can be modelled as a simple harmonic oscillator:

\[
m \ddot{x} + b \dot{x} + kx = 0
\]

Suppose that light passed through a material containing many such electrons. Find an expression that shows how the intensity of light that passed through this material varies with the frequency \( \omega \) of the light.
7. Find an expression for the focal length of a thin lens composed of a material with index of refraction \( n_\ell \), with spherical surfaces of radius \( R_1 \) and \( R_2 \), which has air on one side \( n = 1 \) but a liquid material with index of refraction \( n' \) on the other side. (The air interface has radius of curvature \( R_1 \) while the liquid interface has radius of curvature \( R_2 \).)

8. Consider a Fabre-Perot interferrometer constructed using a glass plate with index of refraction \( n \) and thickness \( d \).

(a) Which angles of incidence produce constructive interference in the reflected light?

(b) Which angles of incident light would produce constructive interference if the glass plate were immersed in a liquid with index of refraction \( n' > n \)?