PURDUE UNIVERSITY | DEPARTMENT OF PHYSICS

Physics 42200
Waves & Oscillations

Lecture 37 – Interference

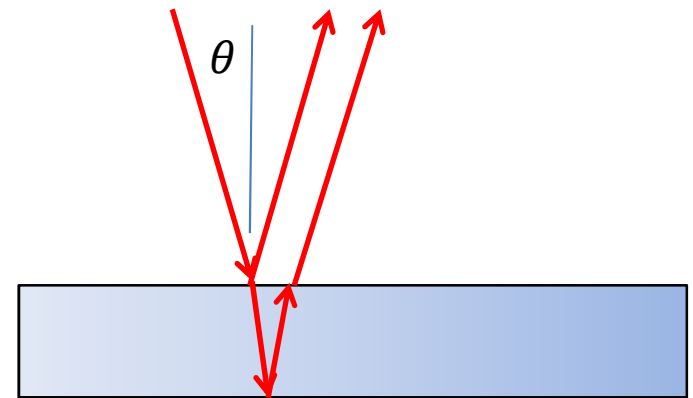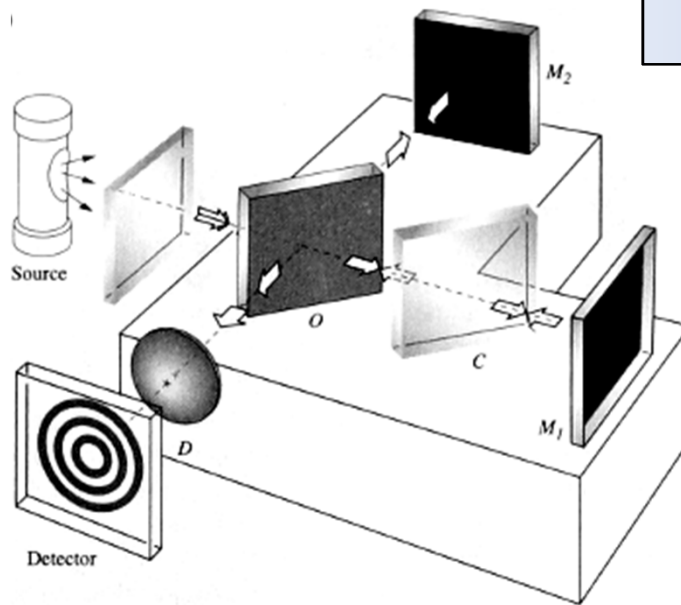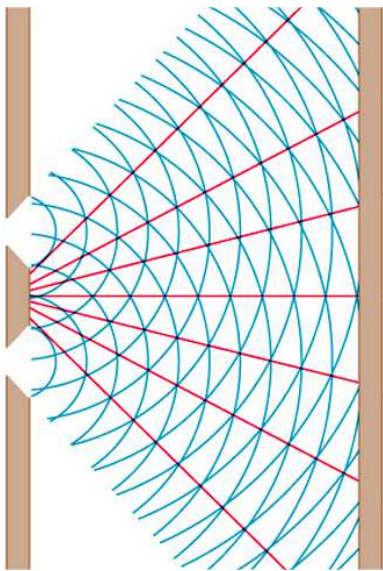## Spring 2013 Semester

Matthew Jones

- Multiple beam interferometers
  - Thin films
 - Fabre-Perot interferometer

Multiple Beam Interference

- Previously we considered only two interfering beams:

Multiple Beam Interference

- In many situations, a coherent beam can interfere with itself multiple times

- Consider a beam incident on a thin film
  - Some component of the light will be reflected at each surface and some will be transmitted
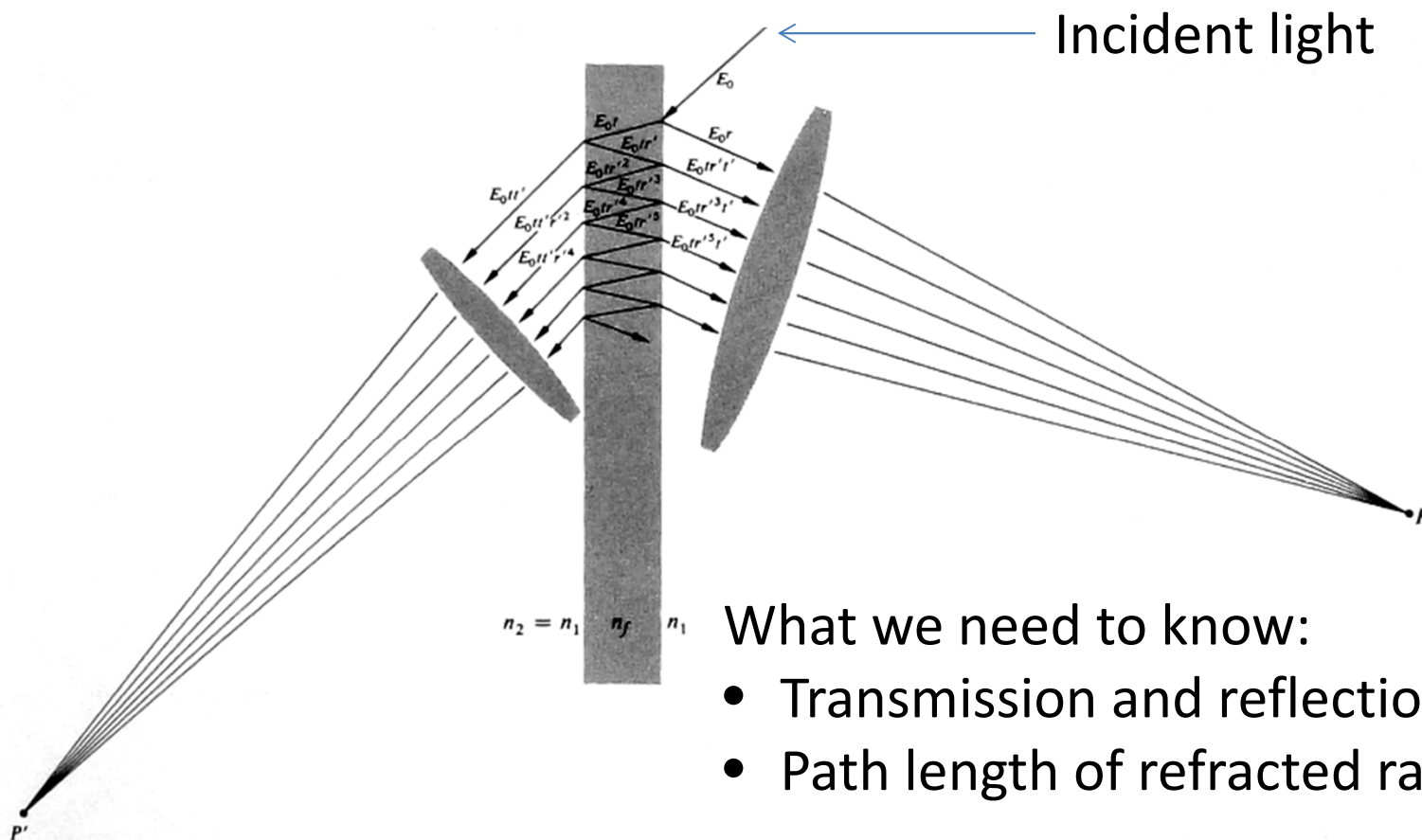
Incident light
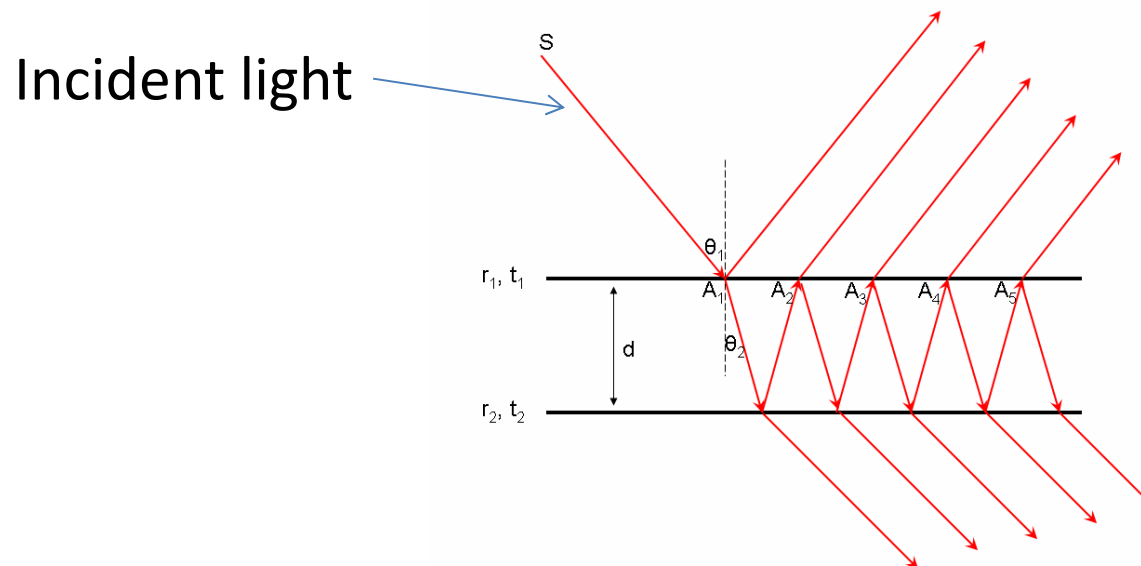
Each transmitted beam will have a different phase relative to the adjacent beams.

What is the total intensity of the reflected light?

Multiple Beam Interference

- All transmitted and reflected rays will be parallel
- They can be focused onto points P and P' by lenses:

Incident light

E_0

$E_0 t$

$E_0 tr'$

$E_0 tr'^2$

$E_0 tr'^3$

$E_0 tr'^4$

$E_0 tr'^5$

$E_0 tt'$

$E_0 tt'r'^2$

$E_0 tt'r'^4$

$E_0 r$

$E_0 tr't'$

$E_0 tr'^3 t'$

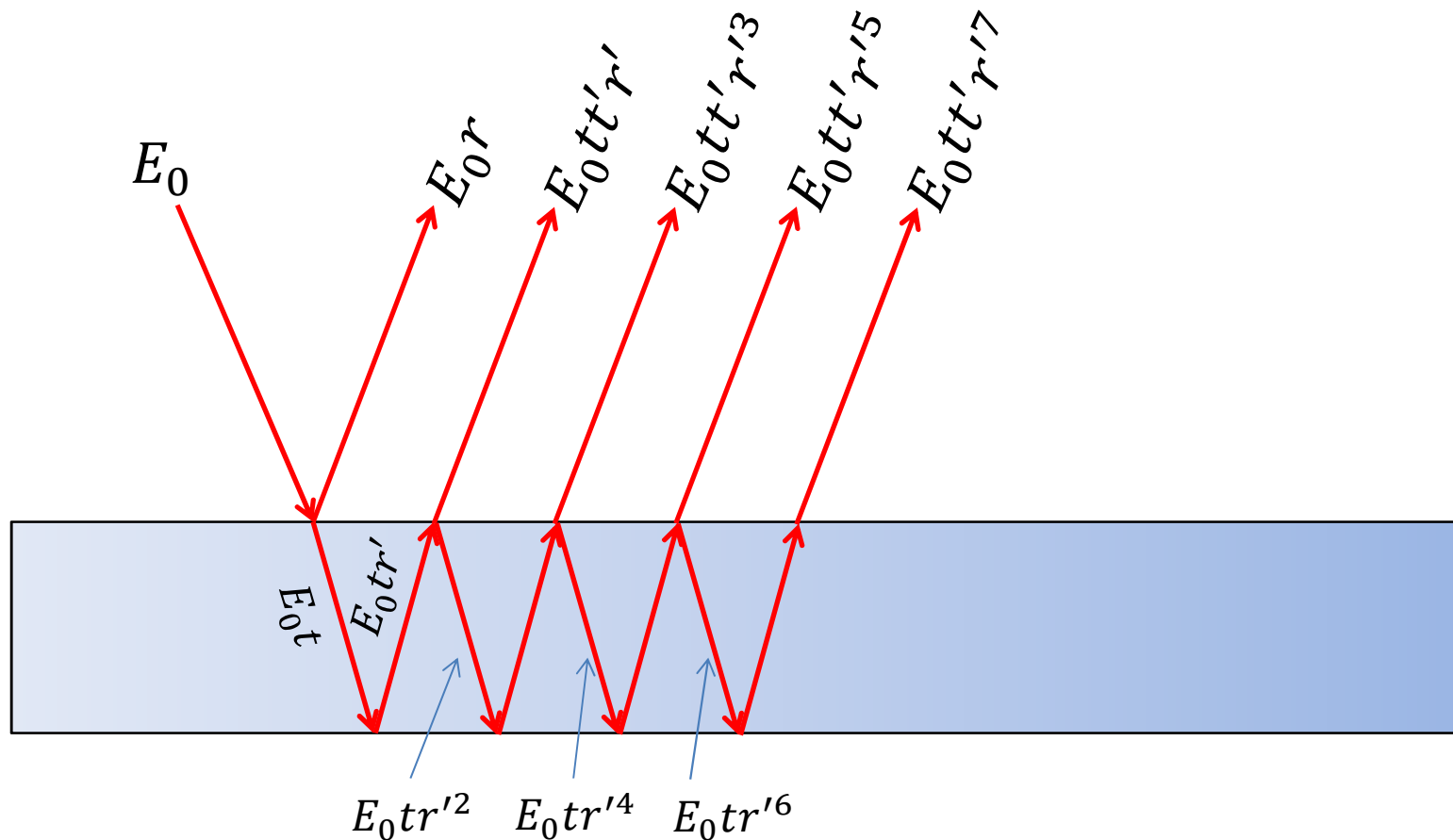$E_0 tr'^5 t'$

$n_2 = n_1$ n_f n_1

P

P'

What we need to know:
- Transmission and reflection coefficients
- Path length of refracted rays in the film

Multiple Beam Interference

- Reflection coefficients: r and r'

- Transmission coefficients: t and t'

E_0

$E_0 r$

$E_0 tt'r'$

$E_0 tt'r'^3$

$E_0 tt'r'^5$

$E_0 tt'r'^7$

$E_0 t$
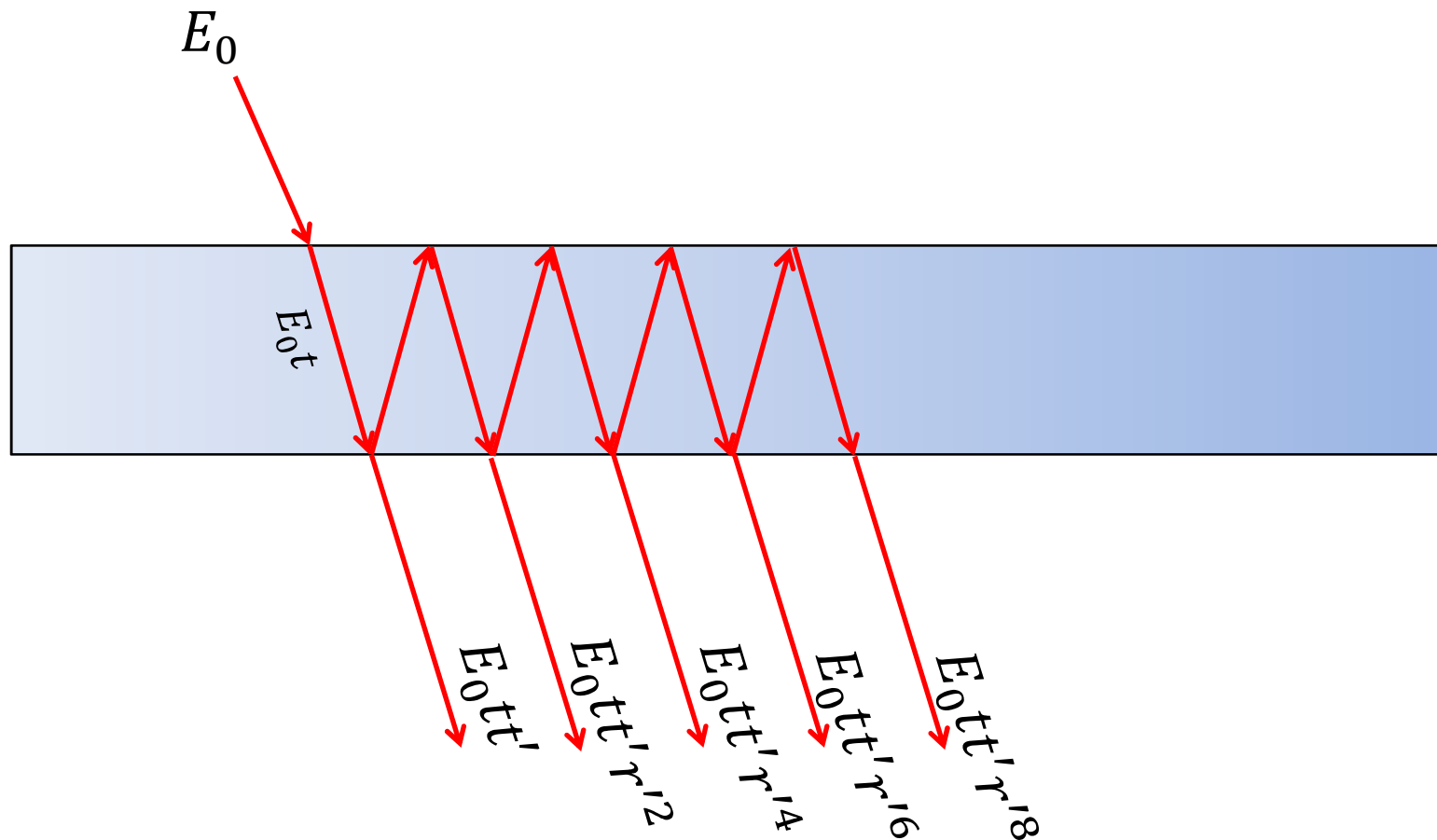
$E_0 tr'$

$E_0 tr'^2 \quad E_0 tr'^4 \quad E_0 tr'^6$

Multiple Beam Interference

- Reflection coefficients: r and r'
- Transmission coefficients: t and t'

Multiple Beam Interference

- The additional phase in the film is always the same:

$$\delta = \frac{2nkd}{\cos\theta_t}$$

- If the initial phase is zero, then

$$E_{1r} = E_0 r e^{i\omega t}$$
$$E_{2r} = E_0 tt' r' e^{i(\omega t - \delta)}$$
$$E_{3r} = E_0 tt' r'^3 e^{i(\omega t - 2\delta)}$$
$$E_{4r} = E_0 tt' r'^5 e^{i(\omega t - 3\delta)}$$
$$\cdots$$

- In general:

$$E_{Nr} = E_0 e^{i\omega t} tt' \; r'^{2N-3} e^{-i(N-1)\delta}$$
$$= E_0 e^{-i\omega t} tt' r' e^{-i\delta} \left(r'^2 e^{-i\delta} \right)^{N-2}$$

Multiple Beam Interference

- The total electric field on one side of the film:

$$E_r = E_0 e^{i\omega t} r + E_0 e^{i\omega t} t t' r' e^{-i\delta} \times$$

$$\left[1 + r'^2 e^{-i\delta} + \left(r'^2 e^{-i\delta} \right)^2 + \left(r'^2 e^{-i\delta} \right)^3 + \cdots \right]$$

- This is in infinite sum of the form:

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z} \quad \text{(when } |z| < 1\text{)}$$

- Total electric field:

$$E_r = E_0 e^{i\omega t} \left[r + \frac{t t' r' e^{-i\delta}}{1 - r'^2 e^{-i\delta}} \right]$$

Multiple Beam Interferometry

- Simplifications:

$$r' = -r$$
$$tt' = 1 - r^2$$

- Total electric field:

$$E_r = E_0 e^{i\omega t} r \left[1 - \frac{(1-r^2)e^{-i\delta}}{1 - r^2 e^{-i\delta}} \right]$$

$$= E_0 e^{i\omega t} r \left[\frac{1 - r^2 e^{-i\delta} - e^{-i\delta} + r^2 e^{-i\delta}}{1 - r^2 e^{-i\delta}} \right]$$

$$= E_0 e^{i\omega t} r \left[\frac{1 - e^{-i\delta}}{1 - r^2 e^{-i\delta}} \right]$$

Multiple Beam Interferometry

- The intensity of the light is $I_r \propto |E_r|^2$

$$I_r = I_0 \left\{ r \left[\frac{1 - e^{-i\delta}}{1 - r^2 e^{-i\delta}} \right] \right\}^* \left\{ r \left[\frac{1 - e^{-i\delta}}{1 - r^2 e^{-i\delta}} \right] \right\}$$

$$= I_0 r^2 \frac{(1 - e^{i\delta})(1 - e^{-i\delta})}{(1 - r^2 e^{i\delta})(1 - r^2 e^{-i\delta})}$$

$$= I_0 \frac{2r^2(1 - \cos\delta)}{(1 + r^4) - 2r^2 \cos\delta}$$

- The intensity of the transmitted light is $I_t \propto |E_t|^2$

$$I_t = I_0 \frac{1 - r^2}{(1 + r^4) - 2r^2 \cos\delta}$$

Multiple Beam Interferometry

- One more identity will clean this up a bit:

$$\cos \delta = 1 - 2 \sin^2(\delta/2)$$

- Reflected intensity:

$$I_r = I_0 \frac{F \sin^2(\delta/2)}{1 + F \sin^2(\delta/2)}$$

- Transmitted intensity:

$$I_t = I_0 \frac{1}{1 + F \sin^2(\delta/2)}$$

- The parameter $F = \left(\frac{2r}{1-r^2}\right)^2$ is called the ***coefficient of finesse***

- Notice that $I_0 = I_r + I_t$
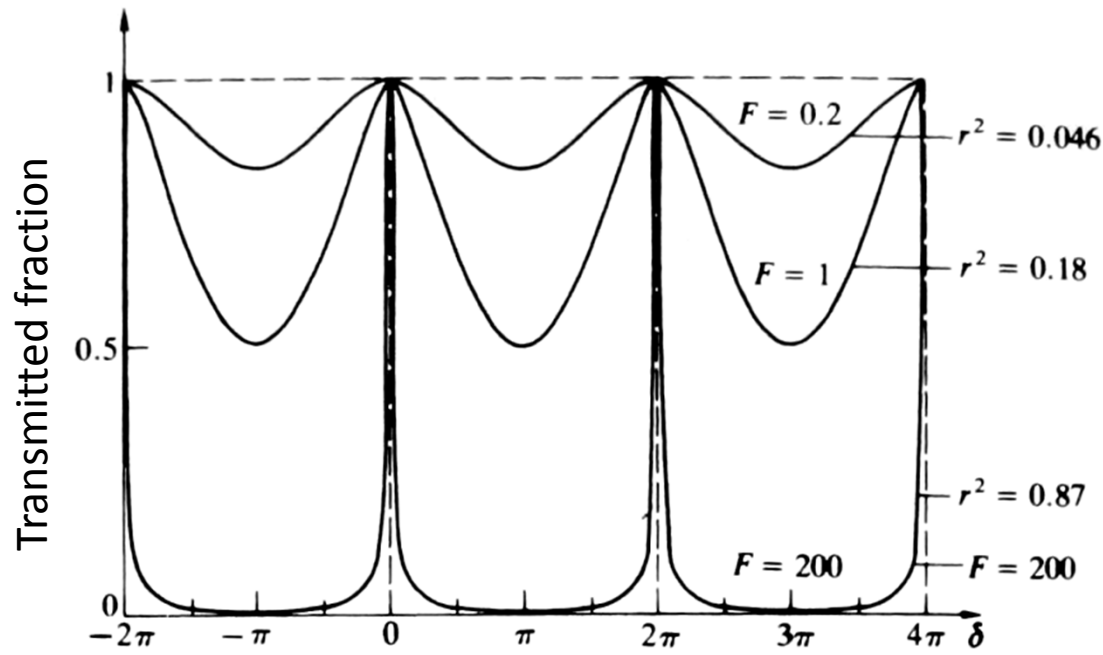
  - We assumed that no energy was lost in the film

Multiple Beam Interferometry

The function

$$\mathcal{A}(\theta) = \frac{1}{1 + F \sin^2(\delta/2)}$$

is called the Airy function.

Remember that δ is a function of the angle of incidence, θ.

Multiple Beam Interferometry

- In practice, some fraction of the light will be absorbed
- Absorptance, A, is defined by:
$$T + R + A = 1$$
- This modifies the transmitted intensity:
$$I_t = I_0 \left[1 - \frac{A}{1 - R} \right]^2 \mathcal{A}(\theta)$$
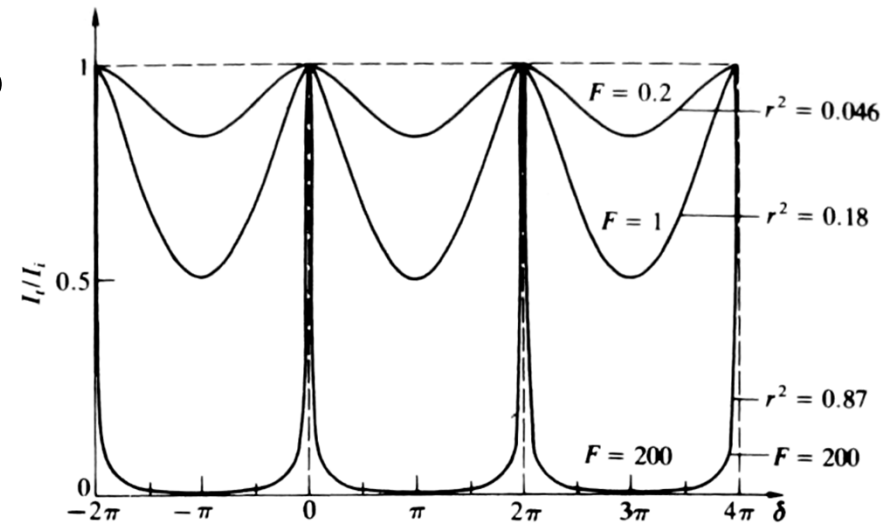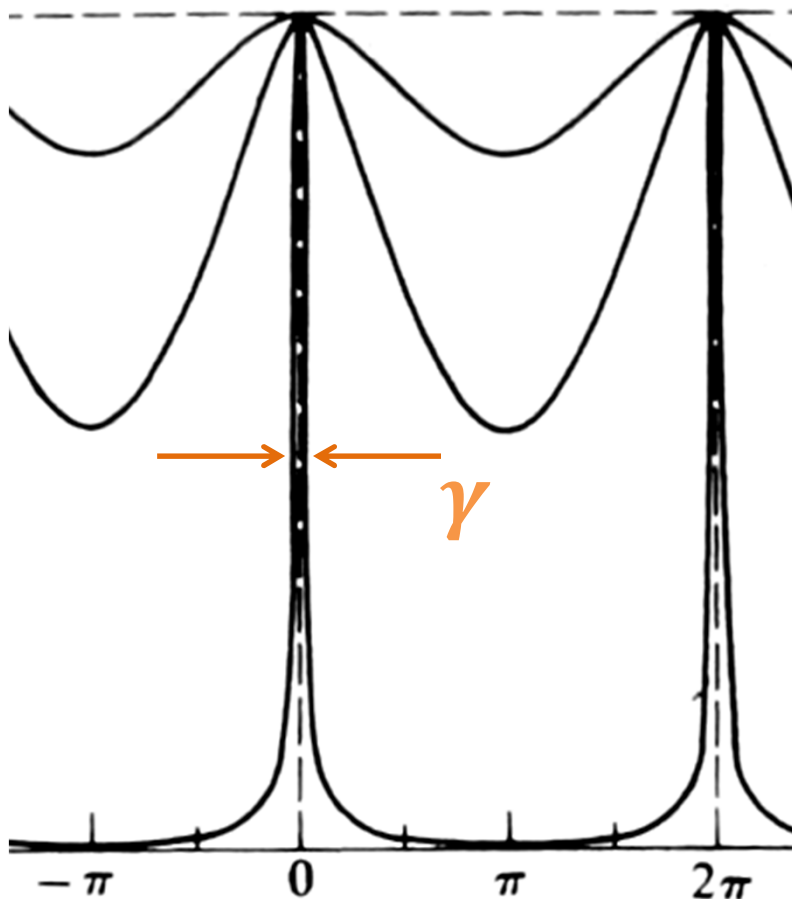- Example: silver film, 50 nm thick, deposited on glass
$$R = 0.94, T = 0.01, A = 0.05$$
$$\left[1 - \frac{A}{1 - R} \right]^2 = 0.0278$$
$$F = 1044$$

Multiple Beam Interference

- How sharp are the peaks?

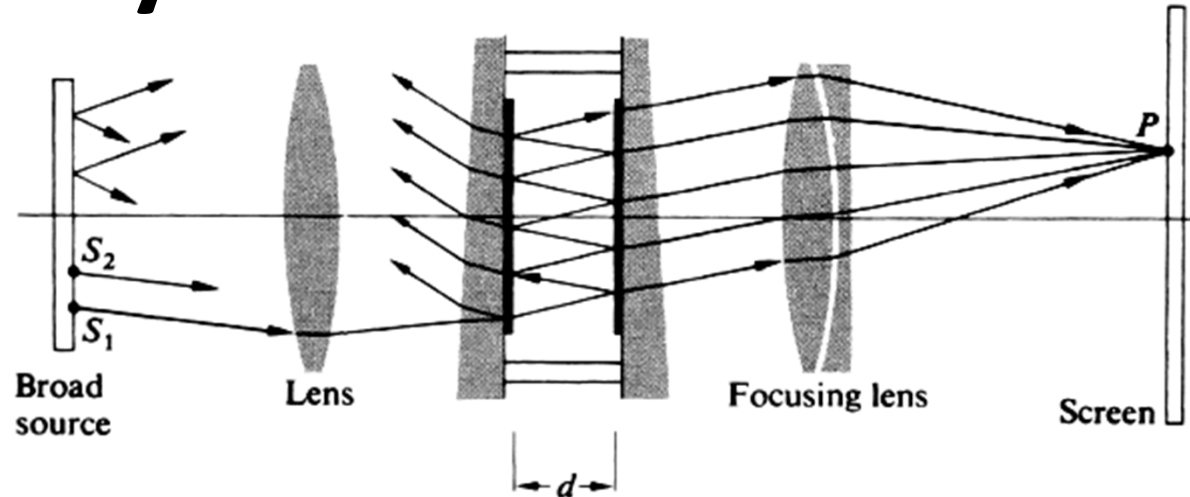

Width of one line:
$$\gamma = 4/\sqrt{F}$$
Ratio of line spacing to the width:
$$\mathcal{F} = \frac{2\pi}{\gamma} = \frac{\pi\sqrt{F}}{2}$$
"Finesse" $\mathcal{F}$, not to be confused with the "coefficient of finesse" F.

Previous example: $\mathcal{F} \approx 50$

Fabry-Perot Interferometer

- Phase difference:

$$\delta = \frac{4\pi n_f}{\lambda_0} d \cos \theta_i = 2\pi m$$

$$m\lambda_0 = 2n_f d \cos \theta_i$$

- Differentiate: $m \, \Delta\lambda_0 + \Delta m \, \lambda_0 = 0$

$$\frac{\Delta m}{m} = -\frac{\Delta\lambda_0}{\lambda}$$

Fabry-Perot Interferometer

$$\frac{\lambda_0}{\Delta\lambda_0} = \frac{2\pi m}{\Delta\delta}$$

- Smallest resolvable wavelength difference:

$$(\Delta\lambda_0)_{min} = \frac{\lambda_0(\Delta\delta)_{min}}{2\pi m}$$

- Minimum resolvable phase shift:

$$(\Delta\delta)_{min} \sim \gamma = 4/\sqrt{F}$$

- Chromatic resolving power:

$$\mathcal{R} = \frac{\lambda_0}{(\Delta\lambda_0)_{min}} \approx \mathcal{F}m \approx \mathcal{F}\,\frac{2n_f d}{\lambda_0}$$
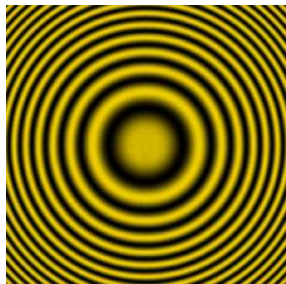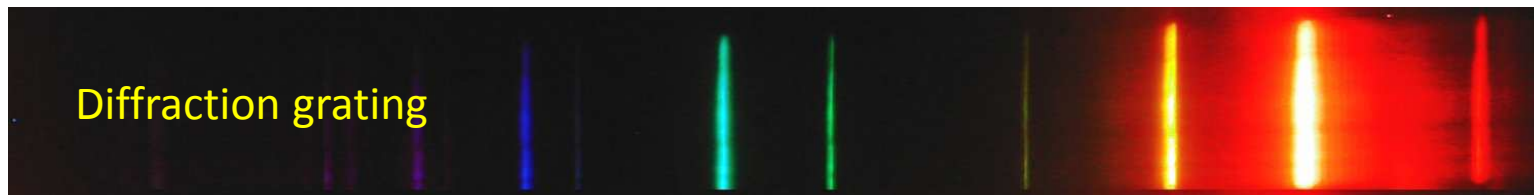
Fabry-Perot Interferometer
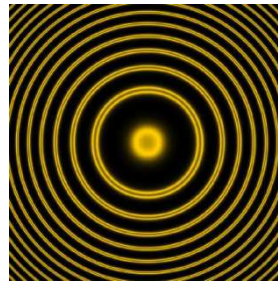
- Typical values:

$$\mathcal{F} = 50$$

$$n_f d = 1 \; cm$$

$$\lambda_0 = 500 \; nm$$

$$\mathcal{R} = \frac{2 \times 50 \times 1 \; cm}{500 \; nm} = 2 \times 10^6$$

Diffraction grating

Michelson interferometer

Fabry-Perot interferometer

Fabry-Perot Interferometer

- The effective gap between the surfaces can be adjusted by changing the pressure of a gas, or by means of piezoelectric actuators