PURDUE UNIVERSITY
DEPARTMENT OF PHYSICS

Physics 42200
Waves & Oscillations

Lecture 33 – Geometric Optics

Spring 2013 Semester

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Optical Elements and Configurations

- The mathematics is important for designing and analyzing optical systems

- Equally important is a good knowledge of how to solve specific problems in optics

- Many different possible configurations of optical elements

- This lecture is intended to provide an overview of various optical systems and how their optical elements satisfy their requirements

- Very little quantitative discussion, but…

Optical Elements and Configurations

- Example exam questions:

"Describe the similarities and differences of the images that are formed when rays of light pass through a right-angle prism and a penta-prism."
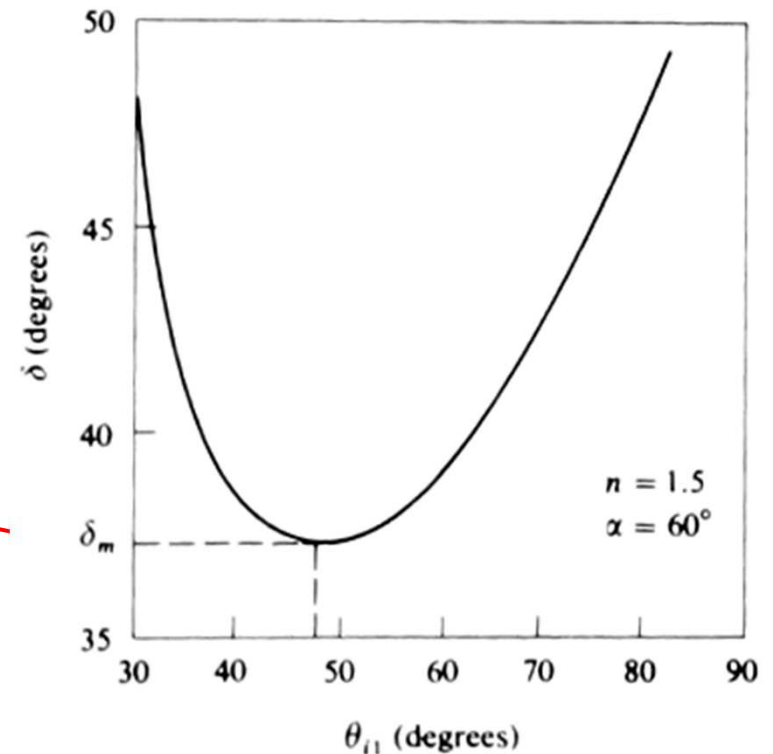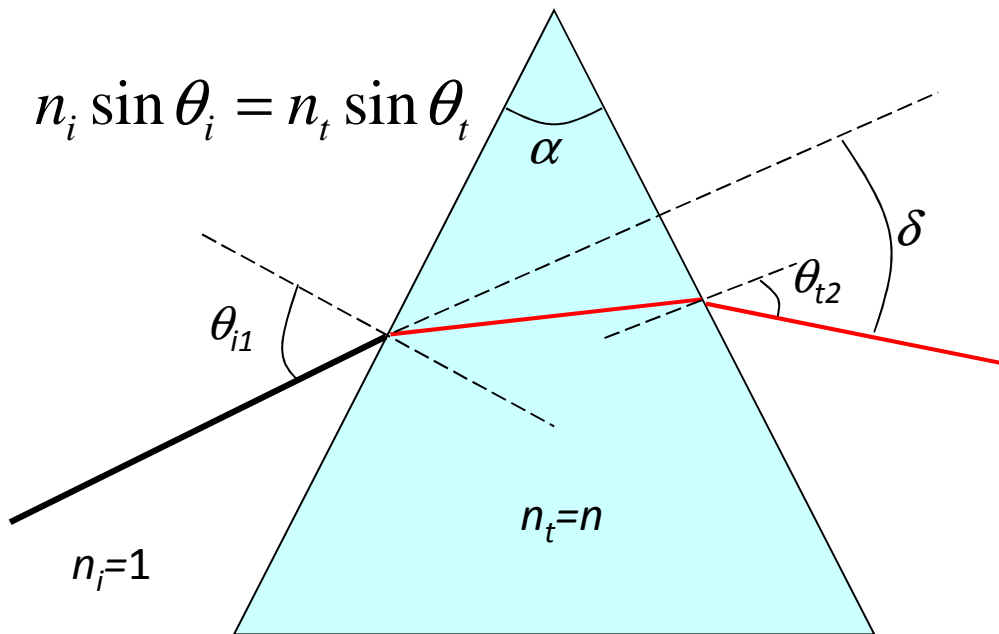
"Describe an optical system that provides no magnification, but inverts the orientation an image that would be formed on the optical axis.

Optical Elements

- Wave plate (modifies polarization)
- Prisms: dispersing, reflecting
- Beam splitters
- Fiber optics
- Nonlinear optics

Prisms

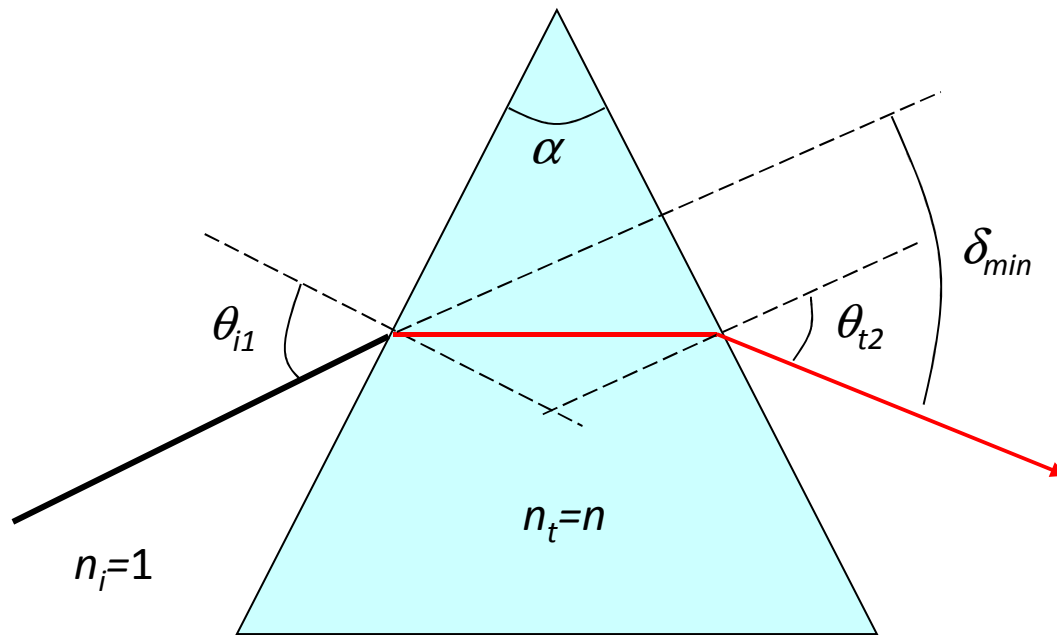- Dispersing prism:

$$n_i \sin \theta_i = n_t \sin \theta_t$$

- Total deviation:

$$\delta = \theta_{i1} + \sin^{-1}\left[(\sin \alpha)\sqrt{n^2 - \sin^2 \theta_{i1}} - \sin \theta_{i1} \cos \alpha\right] - \alpha$$

Prisms

- The minimum deflection, δ_{min}, occurs when $\theta_{i1} = \theta_{t2}$:
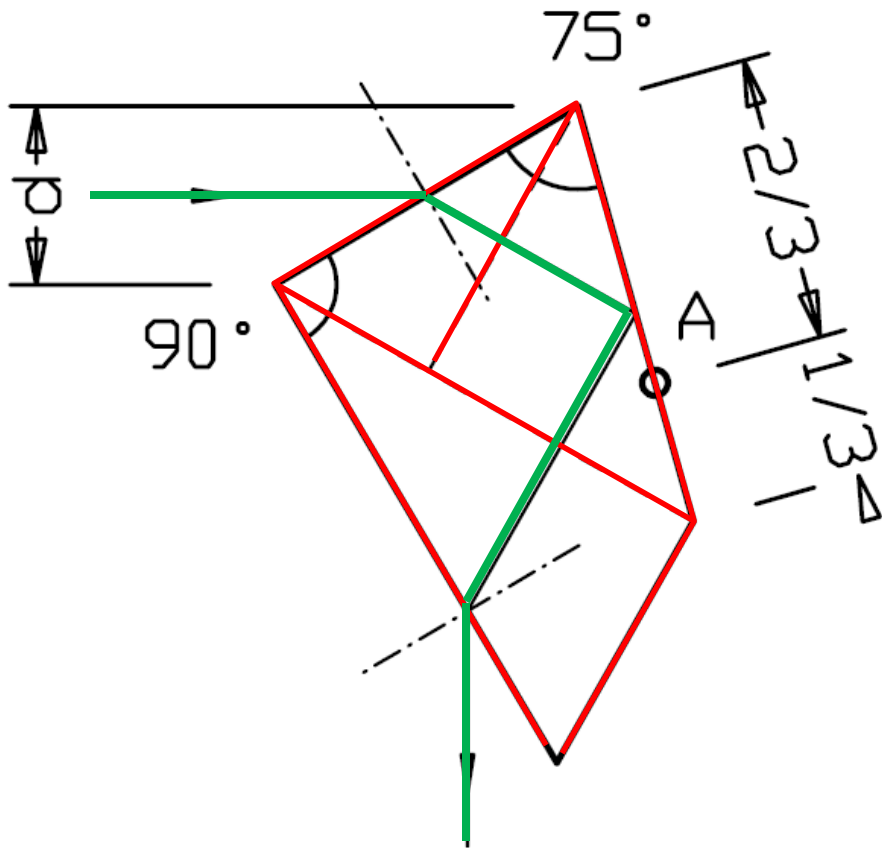

$$n = \frac{\sin[(\delta_{min} + \alpha)/2]}{\sin(\alpha/2)}$$

This can be used as an effective method to measure n

A disadvantage for analyzing colors is the variation of δ_{min} with θ_{i1} - the angle of incidence must be known precisely to

Pellin-Broca Prism

One color is refracted through exactly 90°.

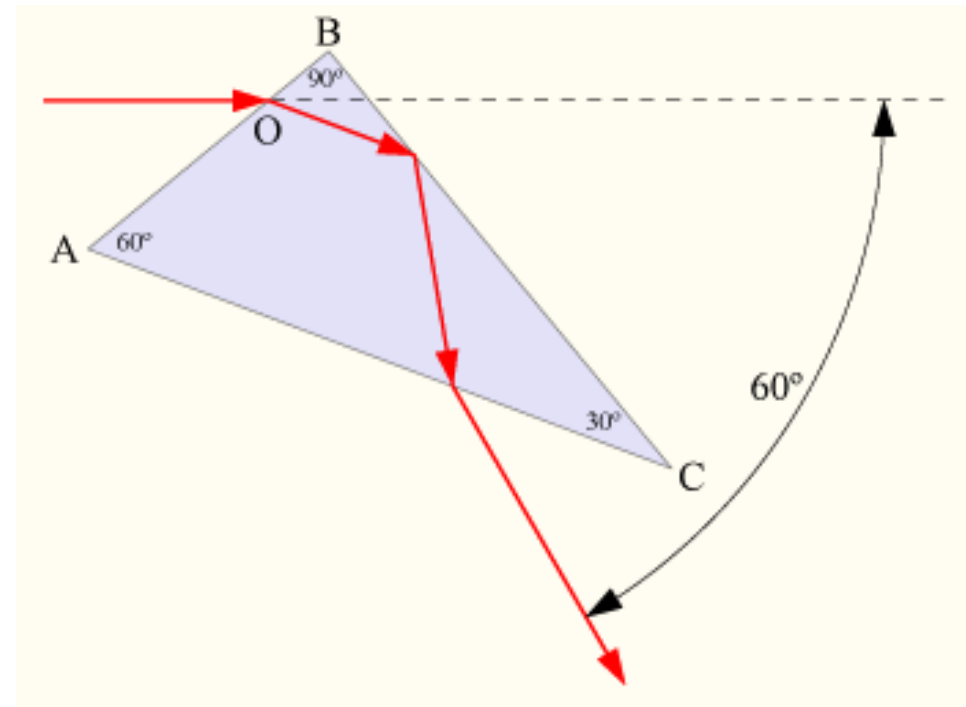Rotating the prism about point A selects different colors.

Ideal for selecting a particular wavelength with minimal change to an optical system.

Abbe Prism

- A particular wavelength is refracted through 60°
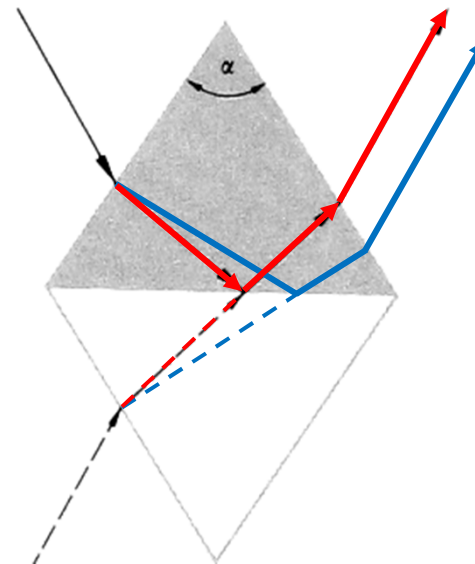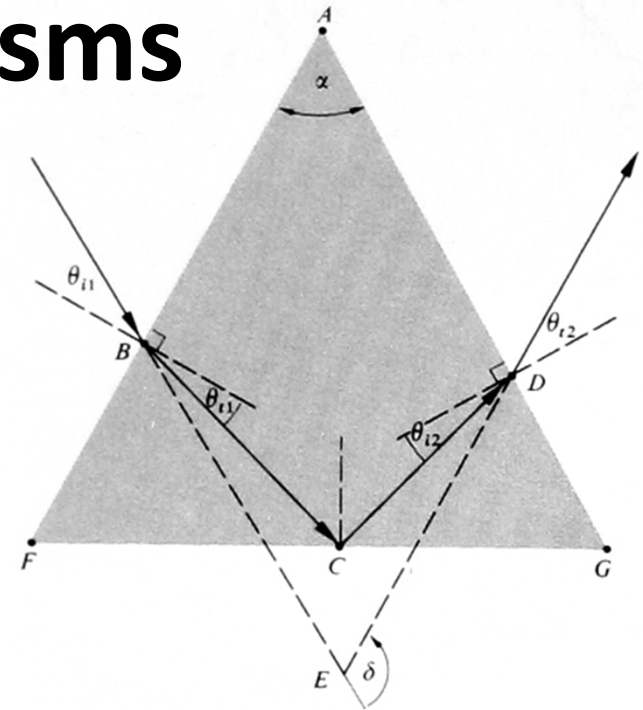- Rotating the prism about point O selects different colors.


Ernst Abbe
1840-1905

Reflective Prisms

- Total internal reflection on one surface

- Equal and opposite refraction at the other surfaces

- Deflection angle:
$$\delta = 2\theta_{i1} + \alpha$$

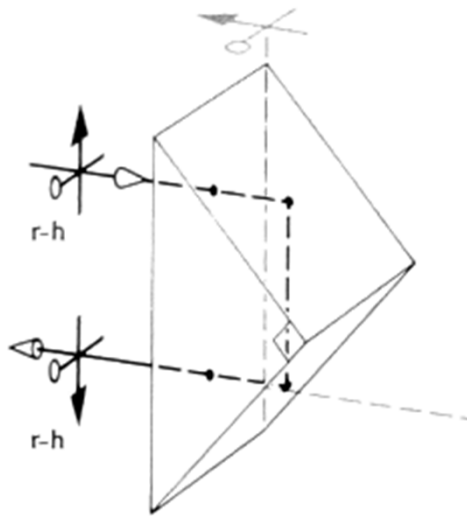- Independent of wavelength (non-dispersive or achromatic prism)

Reflecting Prisms

- Why not just use a mirror?
  - Mirrors produce a reflected image
- Prisms can provide ways to change the direction of light while simultaneously transforming the orientation of an image.
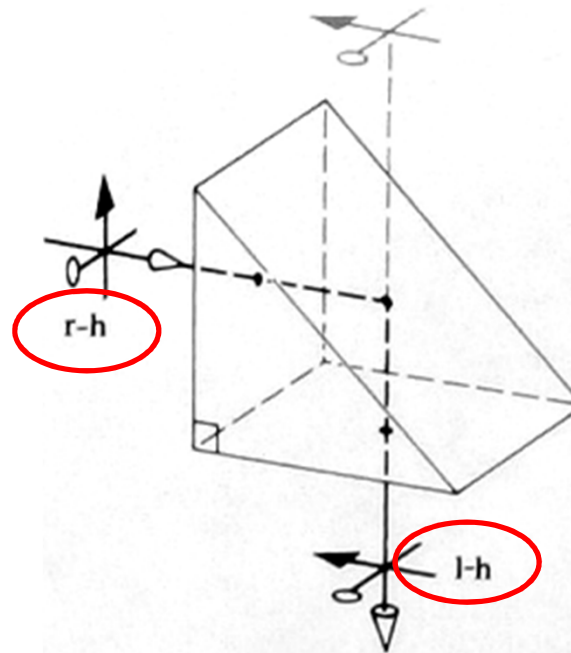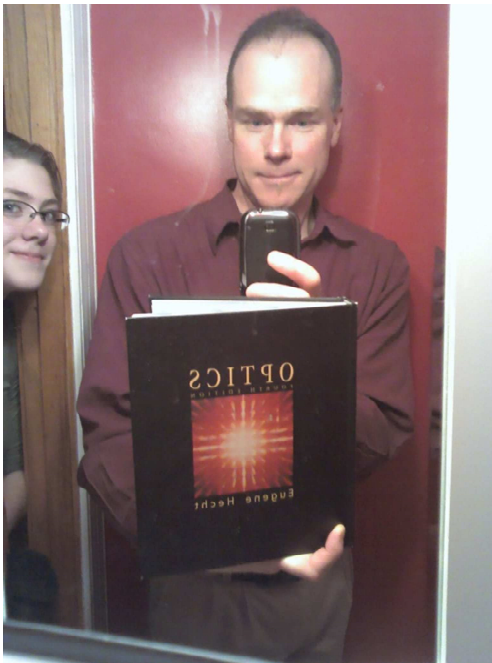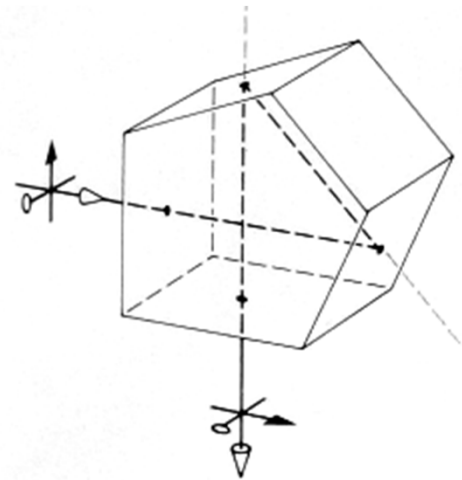
Reflecting Prisms

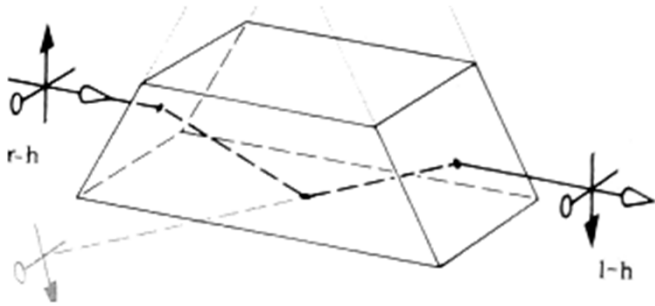- Two internal reflections restores the orientation of the original image.

The Porro prism
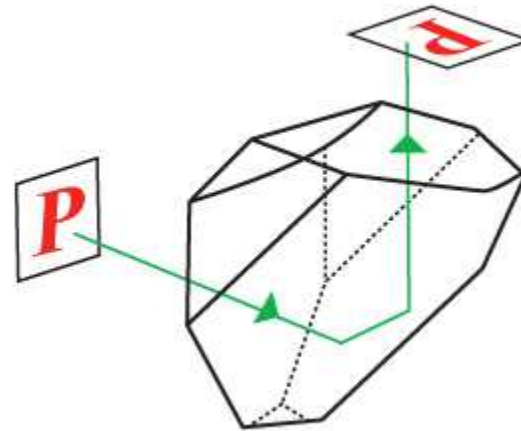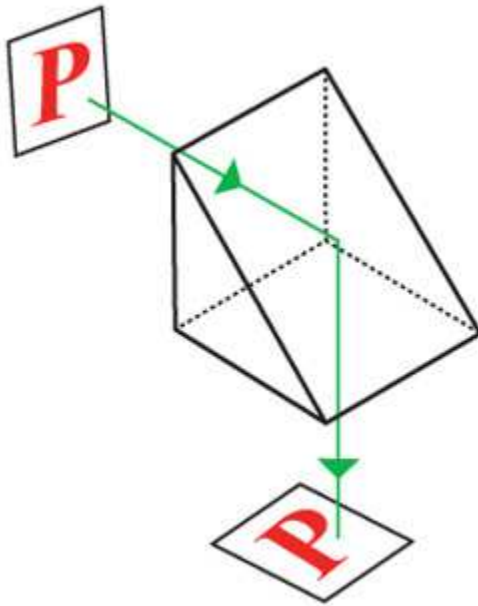

The penta prism

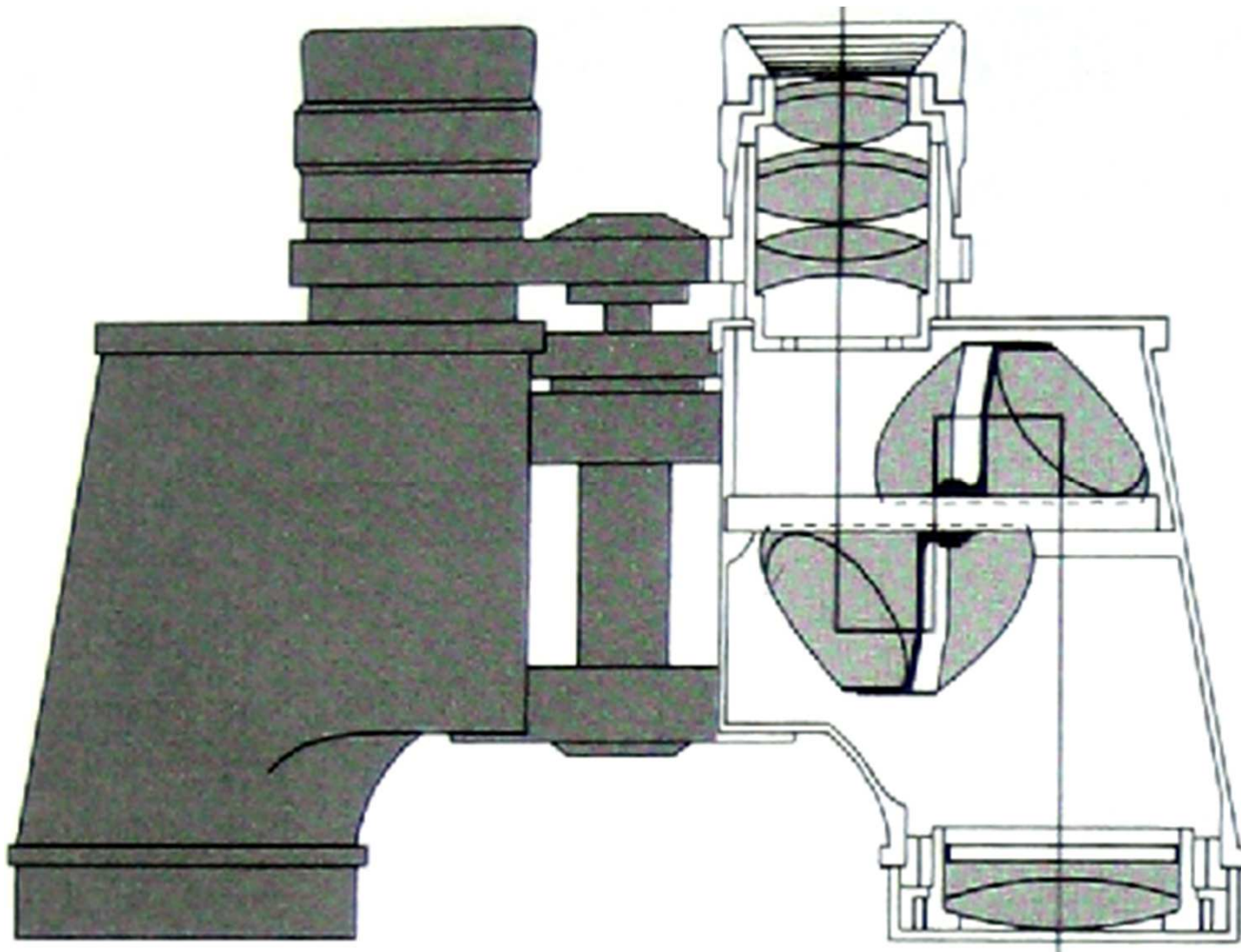# Dove Prism/Image Rotator

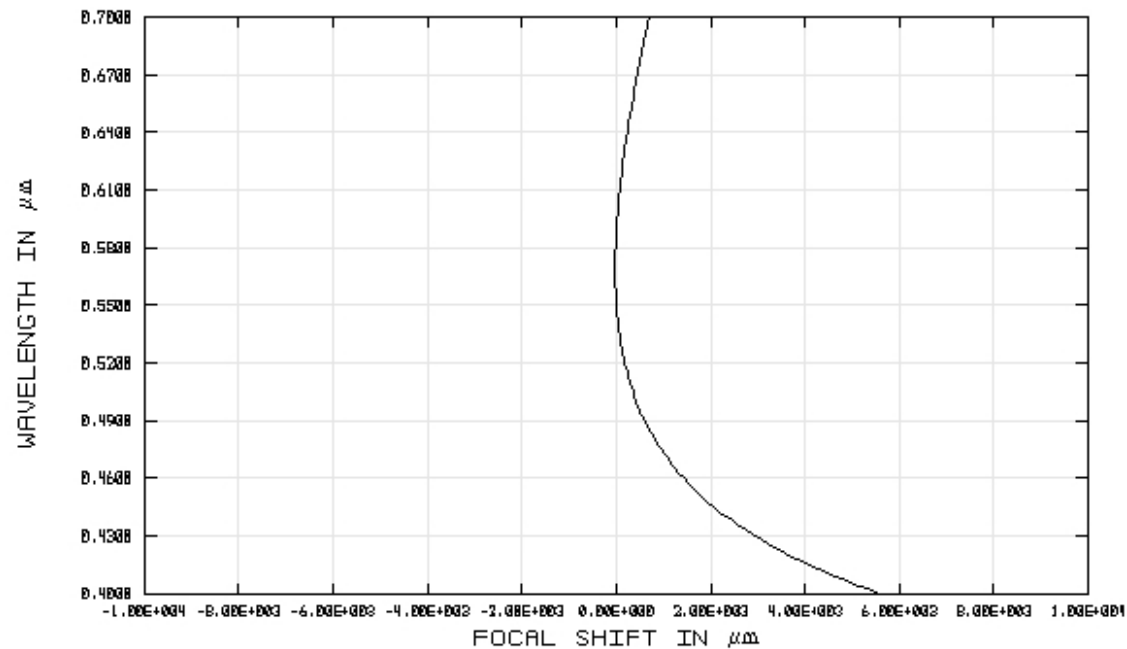The Dove prism

Roof Prism

- Right-angle reflection without image reversal (image rotation)
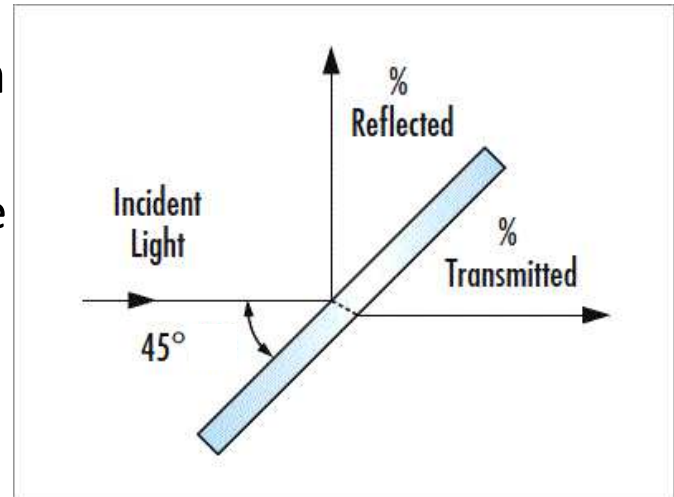
Binoculars

Achromatic Doublets
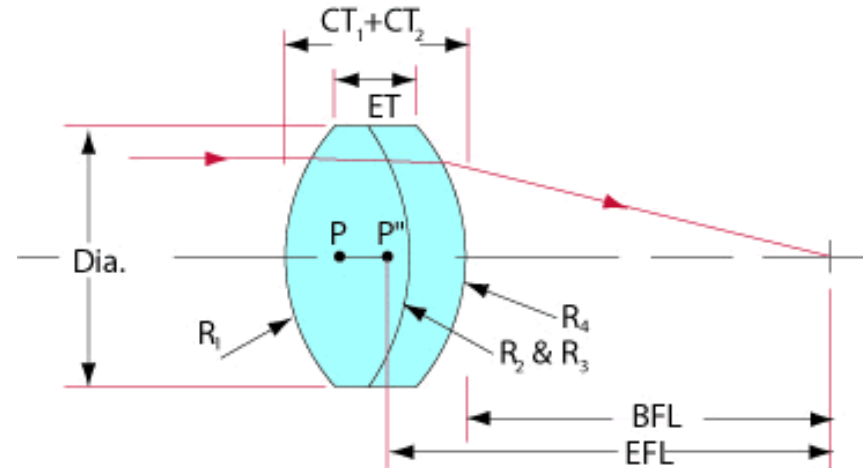
Beam Splitters

- Reflect half the light in a different direction
- Important application: interferometry
  - Transmitted and reflected beams are phase coherent.
- Beam splitter plate
  - Partially reflective surfaces

- Beam splitter cube:
  - Right angle prisms cemented together
  - Match transmission of both polarization components

Fiber Optics

- Development of fiber optics:
  - 1854: John Tyndall demonstrated that light could be bent by a curved stream of water
  - 1888: Roth and Reuss used bent glass rods to illuminate body cavities for surgical procedures
  - 1920's: Baird and Hansell patented an array of transparent rods to transmit images
- Significant obstacles:
  - Light loss through the sides of the fibers
  - Cross-talk (transfer of light between fibers)
  - 1954: Van Heel studied fibers clad with a material that had a lower index of refraction than the core

Fiber Optics: Losses

Consider large fiber: diameter $D \gg \lambda \rightarrow$ can use geometric optics
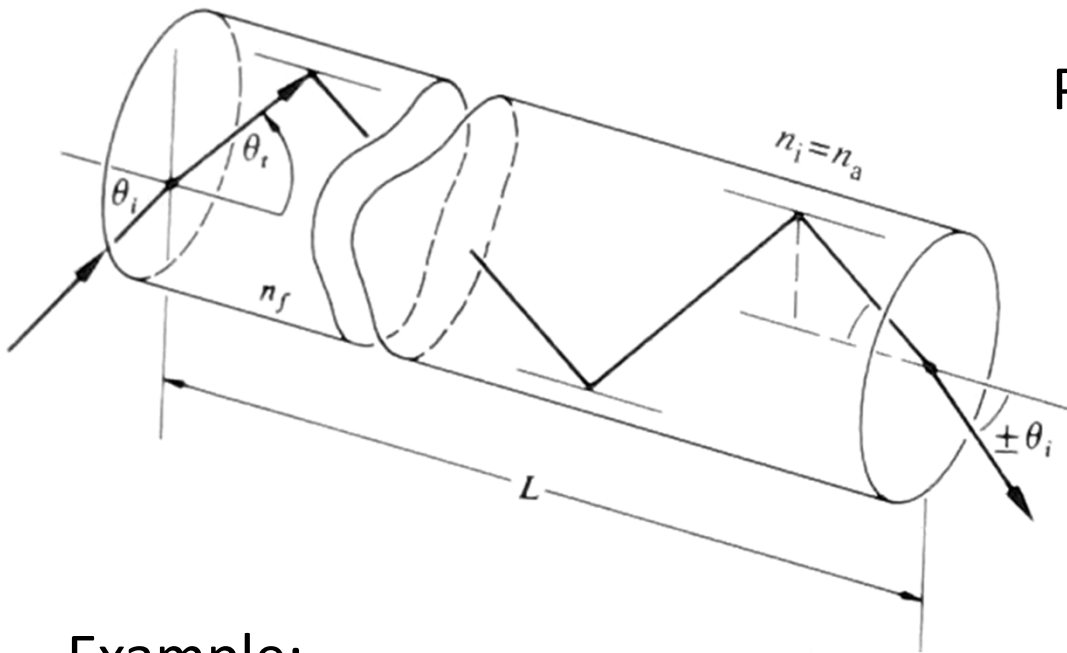
Path length traveled by ray:

$$l = L/\cos\theta_t$$

Number of reflections:

$$N = \frac{l}{D/\sin\theta_t} \pm 1$$

Example:

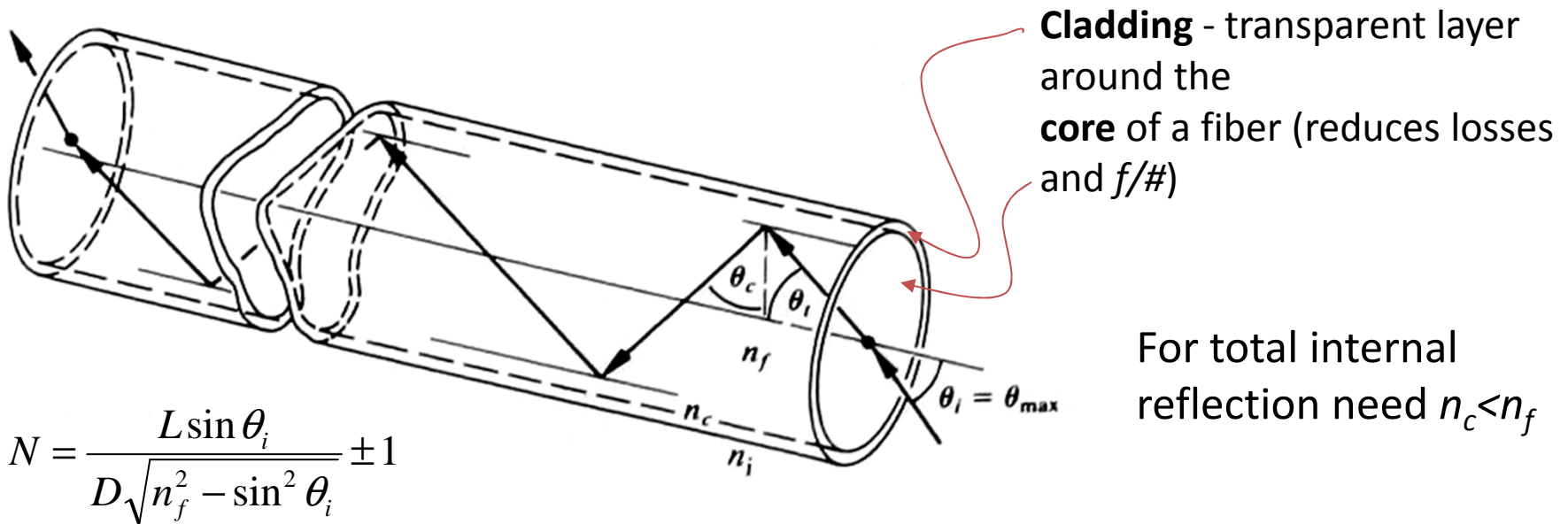L = 1 km, D = 50 μm, n_f =1.6, θ_i = 30º

$$N = 6,580,000$$

Note: frustrated internal reflection, irregularities $\rightarrow$ losses!

Using Snell's Law for θ_t:

$$N = \frac{L\sin\theta_i}{D\sqrt{n_f^2 - \sin^2\theta_i}} \pm 1$$

'Step-index' Fiber

Cladding - transparent layer around the **core** of a fiber (reduces losses and $f/\#$)

For total internal reflection need $n_c < n_f$

$$N = \frac{L\sin\theta_i}{D\sqrt{n_f^2 - \sin^2\theta_i}} \pm 1$$
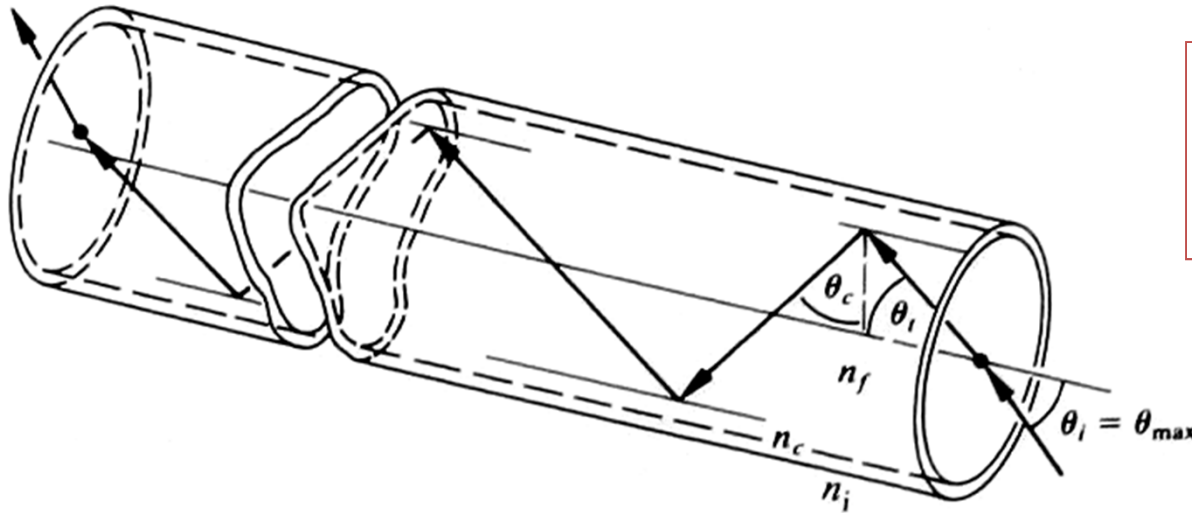
For lower losses need to reduce N, or maximal θ_i, the latter is defined by critical angle for total internal reflection:

$$\sin\theta_c = \frac{n_c}{n_f} = \sin(90° - \theta_t) = \cos(\theta_t) \implies \boxed{\sin\theta_{max} = \frac{\sqrt{n_f^2 - n_c^2}}{n_i}}$$

$n_i = 1$ for air

Fiber and *f/#*

$$\sin\theta_{max} = \frac{\sqrt{n_f^2 - n_c^2}}{n_i}$$

Angle θ_{max} defines the light gathering efficiency of the fiber, or numerical aperture NA:

$$NA \equiv n_i \sin\theta_{max} = \sqrt{n_f^2 - n_c^2}$$

And *f/#* is:

$$f/\# \equiv \frac{1}{2(NA)}$$

Largest NA=1
Typical NA = 0.2 ... 1

Data Transfer Limitations

1. Distance is limited by losses in a fiber. Losses α are measured in decibels (dB) per km of fiber (dB/km), i.e. in logarithmic scale:

$$\alpha \equiv -\frac{10}{L}\log\left(\frac{P_o}{P_i}\right) \implies \frac{P_o}{P_i} = 10^{-\alpha L/10}$$

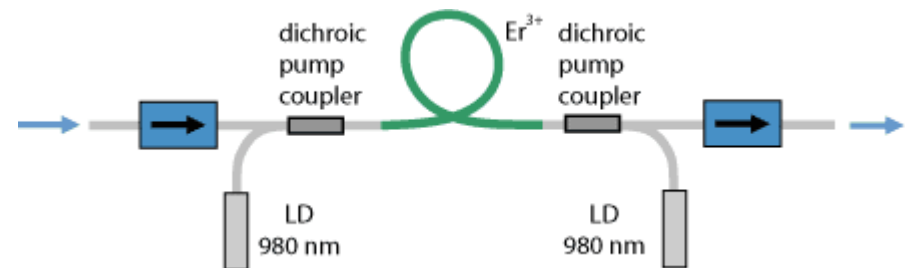P_o - output power
P_i - input power
L - fiber length

Example:	α	P_o/P_i over 1 km
	10 dB	1:10
	20 dB	1:100
	30 dB	1:1000

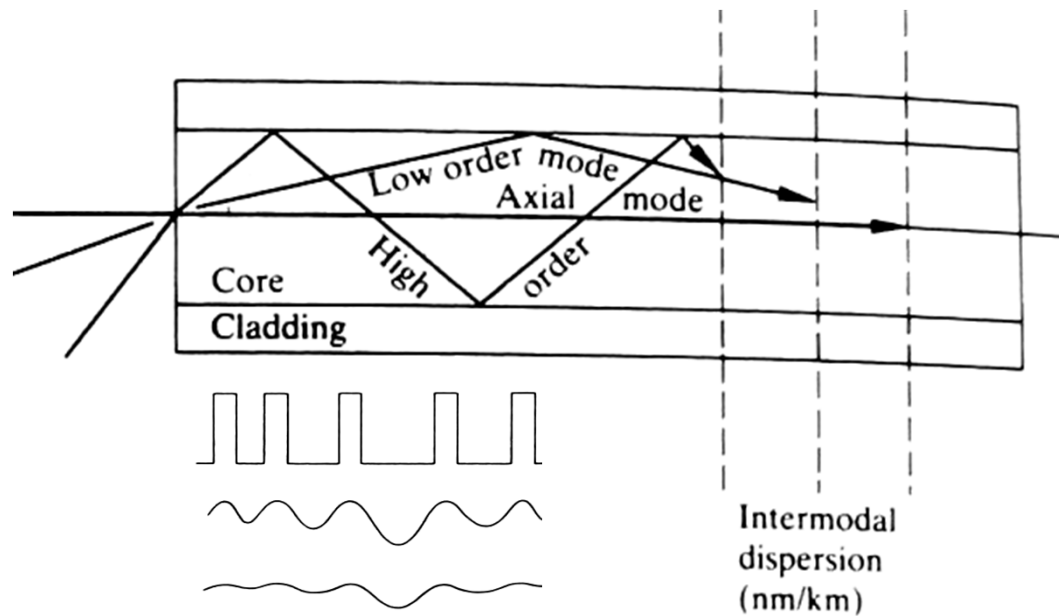Workaround: use light amplifiers to boost and relay the signal

2. Bandwidth is limited by pulse *broadening* in fiber and processing electronics

Pulse Broadening

Multimode fiber: there are many rays (modes) with different OPLs and initially short pulses will be broadened (*intermodal dispersion*)

For ray along axis:
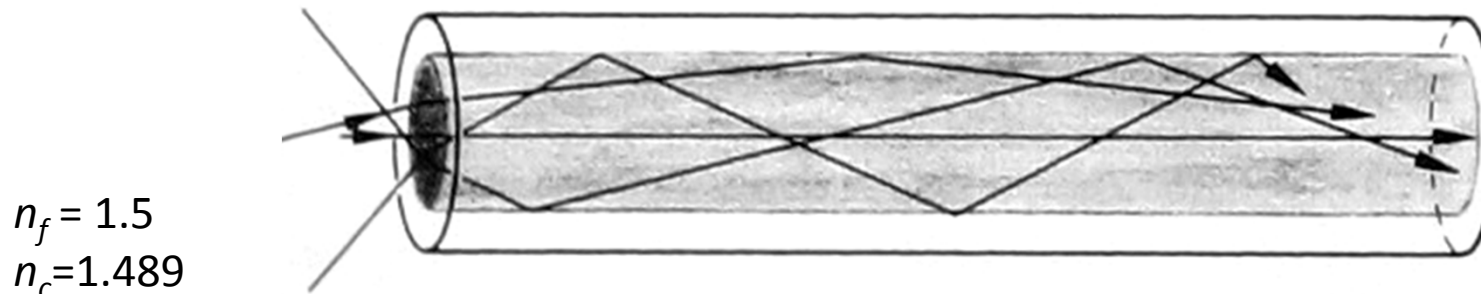
$$t_{min} = L/v_f = Ln_f/c$$

For ray entering at θ_{max}:

$$t_{max} = l/v_f = Ln_f^2/(cn_c)$$

The initially short pulse will be broadened by:

Making n_c close to n_f reduces the effect! ⟶

$$\Delta t = t_{max} - t_{min} = \frac{Ln_f}{c}\left(\frac{n_f}{n_c} - 1\right)$$

Pulse Broadening: Example

n_f = 1.5
n_c = 1.489
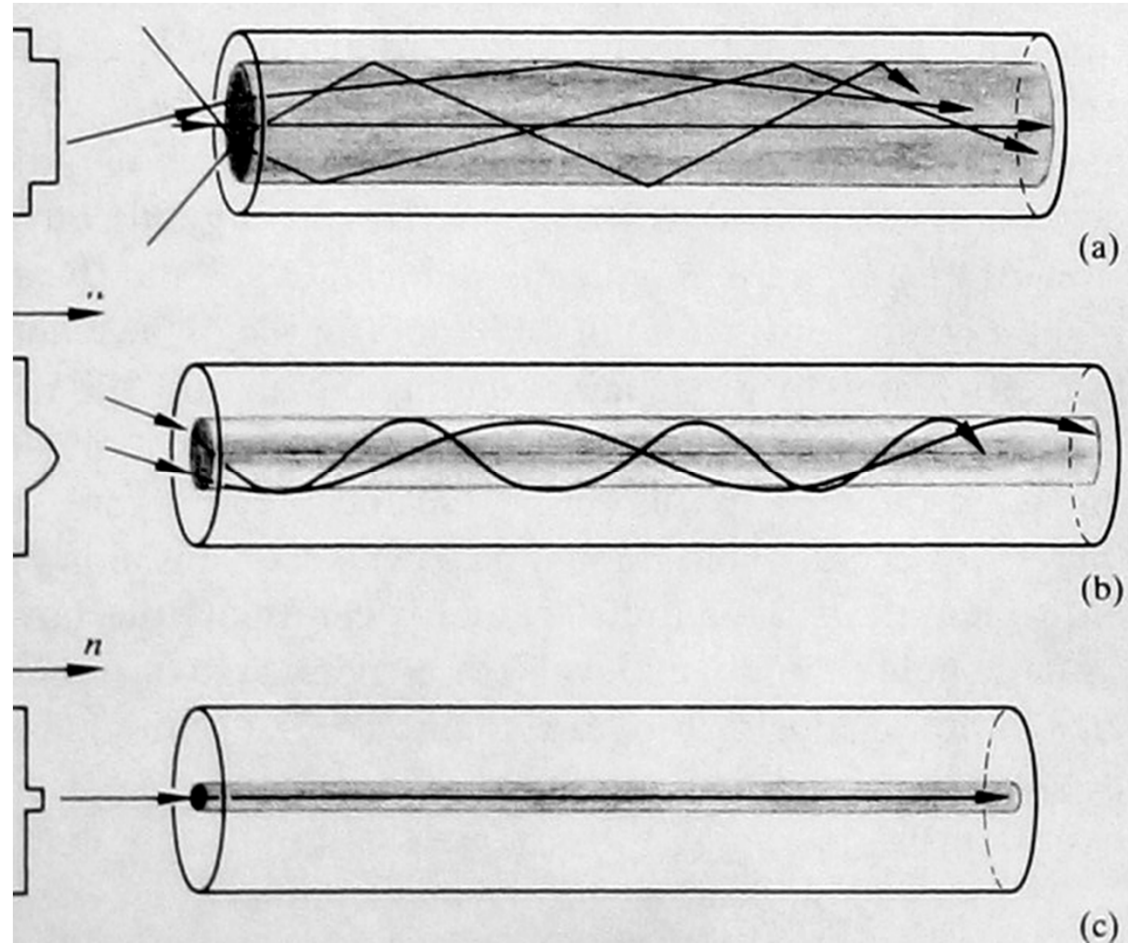Estimate the bandwidth limit for 1000 km transmission.

Solution:

$$\Delta t = \frac{Ln_f}{c}\left(\frac{n_f}{n_c}-1\right) = \frac{10^6 \cdot 1.5}{3\times10^8}\left(\frac{1.5}{1.489}-1\right)s = 3.7\times10^{-5}s = 37\mu s$$

Even the shortest pulse will become ~37 μs long

Bandwidth ~ $\dfrac{1}{3.7\times10^{-5}s} = 27$ kbps ⟵ kilobits per second
= ONLY 3.3 kbytes/s

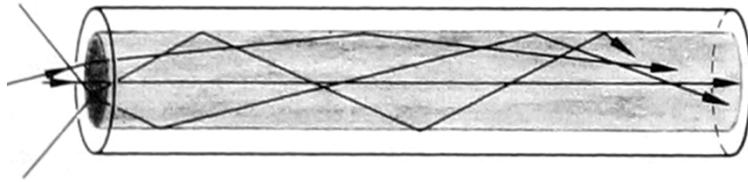## Multimode fibers are not used for communication!

Graded and Step Index Fibers

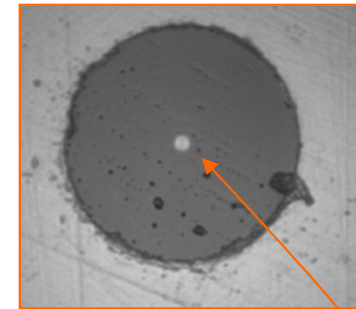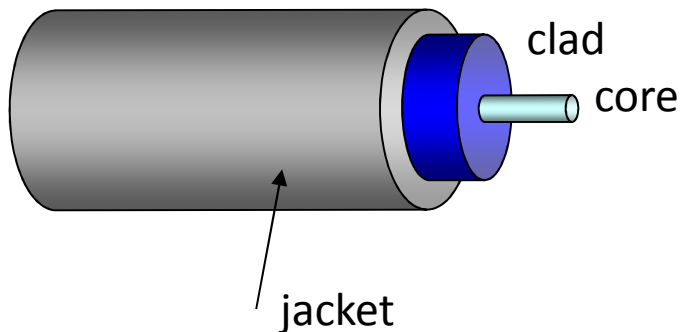Step index: the change in n is abrupt between cladding and core
Graded index: n changes smoothly from n_c to n_f

Single Mode Fiber

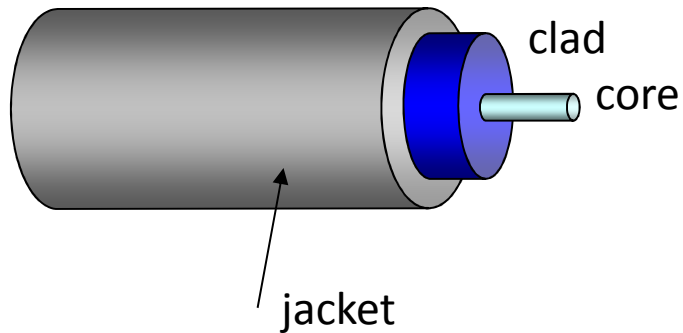

To avoid broadening need to have only one path, or mode

Single mode fiber: there is only one path, all other rays escape from the fiber

clad
core

jacket

Geometric optics does not work anymore: need wave optics. Single mode fiber core is usually only 2-7 micron in diameter

Single Mode Fiber: Broadening

clad

core

jacket

Problem: shorter the pulse, broader the spectrum.
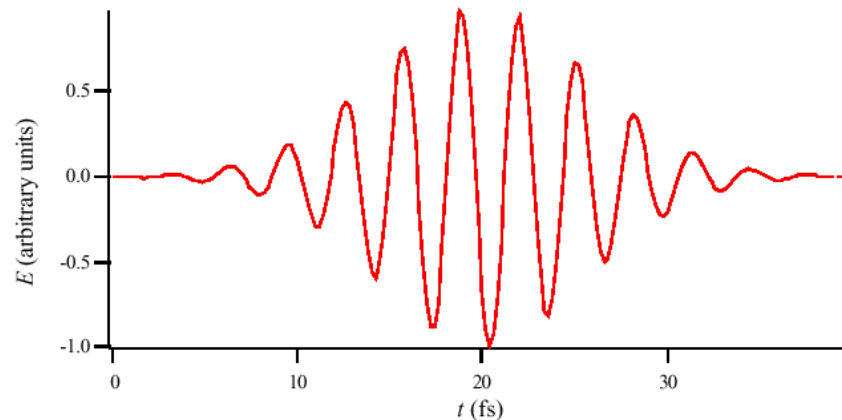refraction index depends on wavelength

'Transform' limited pulse product of spectral full width at half maximum (fwhm) by time duration fwhm:

$$\Delta f \Delta t \approx 0.2$$

A 10 fs pulse at 800 nm is ~40 nm wide spectrally
If second derivative of n is not zero this pulse will broaden in fiber rapidly

Solitons: special pulse shapes that do not change while propagating