PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 33 – Geometric Optics

Spring 2013 Semester

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Aberrations

- We have continued to make approximations:
  - Paraxial rays
  - Spherical lenses
  - Index of refraction independent of wavelength
- How do these approximations affect images?
  - There are several ways…
  - Sometimes one particular effect dominates the performance of an optical system
  - Useful to understand their source in order to introduce the most appropriate corrective optics
- How can these problems be reduced or corrected?

Aberrations

- Limitations of paraxial rays:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$$

- Paraxial approximation:

$$\sin\theta \approx \theta$$

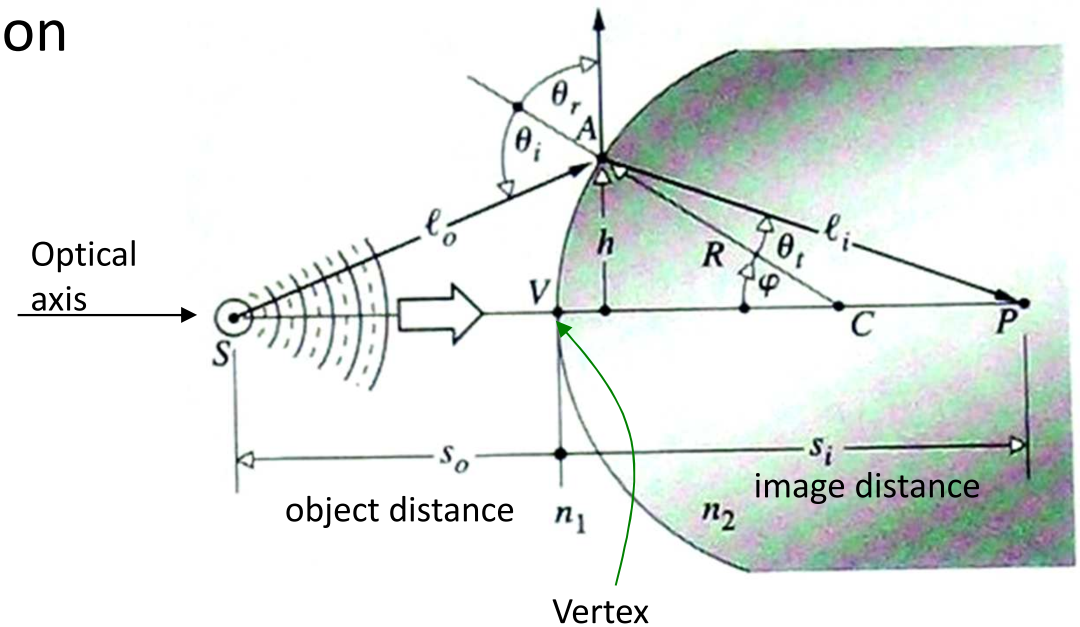
- Third-order approximation:

$$\sin\theta \approx \theta - \frac{\theta^3}{3!}$$

- The optical equations are now non-linear
  - The lens equations are only approximations
  - Perfect images might not even be possible!
  - Deviations from perfect images are called aberrations
  - Several different types are classified and their origins identified.

Aberrations

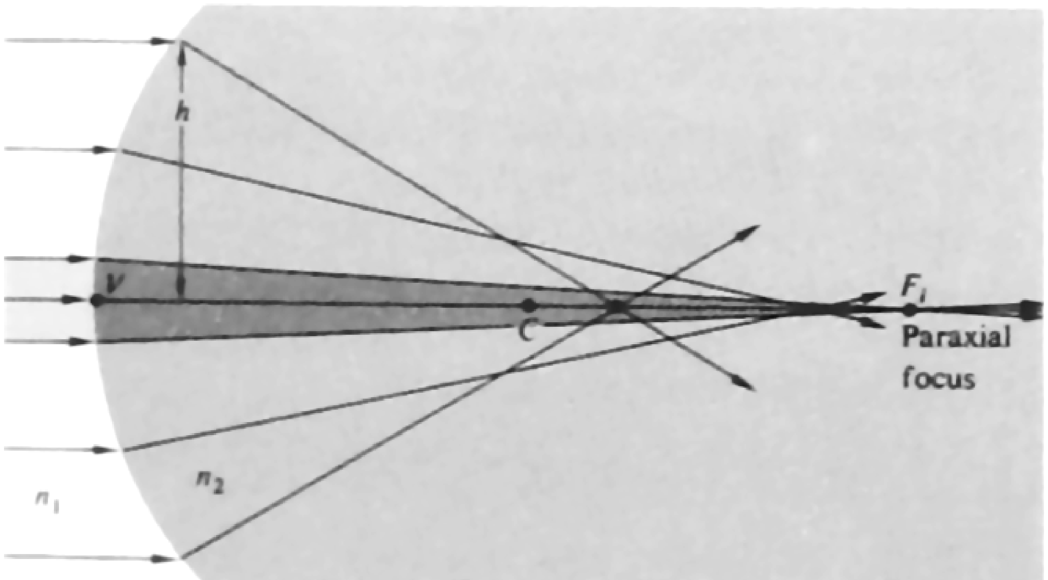
- Departure from the linear theory at third-order were classified into five types of ***primary aberrations*** by Phillip Ludwig Seidel (1821-1896):
  - Spherical aberration
  - Coma
  - Astigmatism
  - Field curvature
  - Distortion



Spherical Aberration

- We first derived the shape of a surface that changes spherical waves into plane waves
  - It was either a parabola, ellipse or hyperbola
- But this only worked for light sources that were on the optical axis
- To form an image, we need to bring rays into focus from points that lie off the optical axis
- A sphere looks the same from all directions so there are no “off-axis” points
- It is still not perfect – there are aberrations

Spherical Aberration



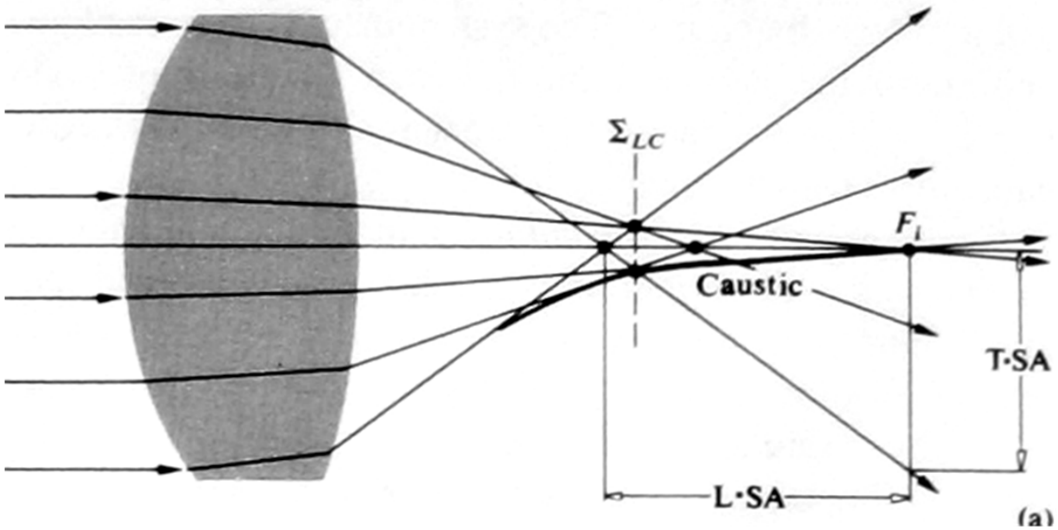
Paraxial approximation:

$$\frac{n_1}{s_o}+\frac{n_2}{s_i}=\frac{n_2-n_1}{R}$$

Third order approximation:

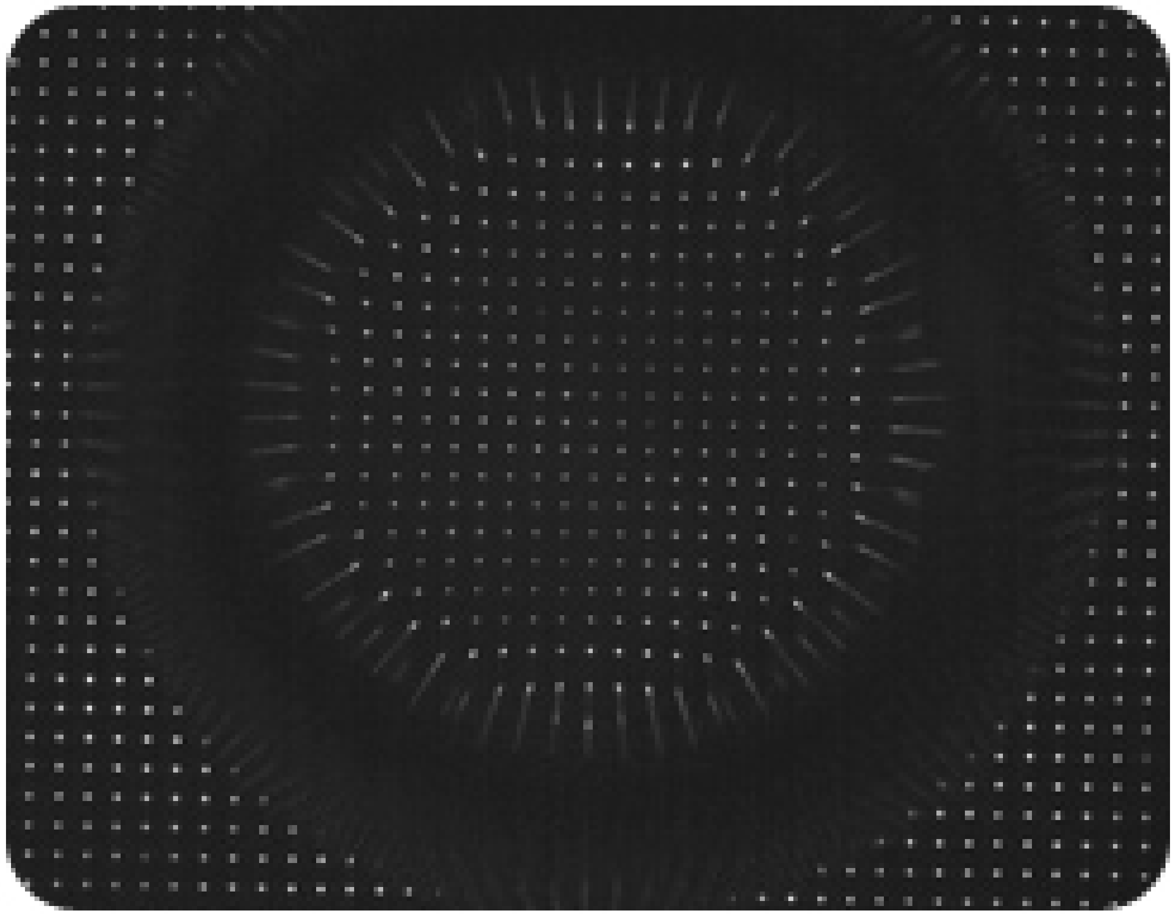
$$\frac{n_1}{s_o}+\frac{n_2}{s_i}=\frac{n_2-n_1}{R}+\underbrace{h^2\left[\frac{n_1}{2s_o}\left(\frac{1}{s_o}+\frac{1}{R}\right)^2+\frac{n_2}{2s_i}\left(\frac{1}{R}-\frac{1}{s_i}\right)^2\right]}_{\text{Deviation from first-order theory}}$$

Spherical Aberrations



- Longitudinal Spherical Aberration: $\mathrm{L} \cdot SA$
  - Image of an on-axis object is longitudinally stretched
  - Positive $\mathrm{L} \cdot SA$ means that marginal rays intersect the optical axis in front of F_i (paraxial focal point).
- Transverse Spherical Aberration: $\mathrm{T} \cdot SA$
  - Image of an on-axis object is blurred in the image plane
- Circle of least confusion: Σ_{LC}
  - Smallest image blur

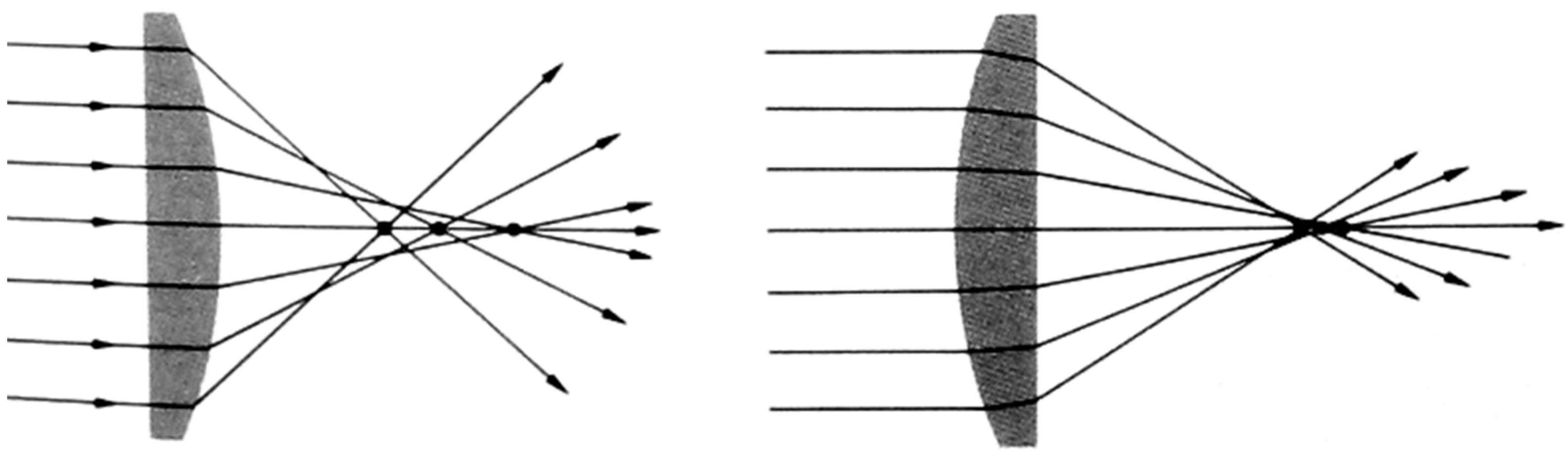
Spherical Aberration



Example from http://www.spot-optics.com/index.htm

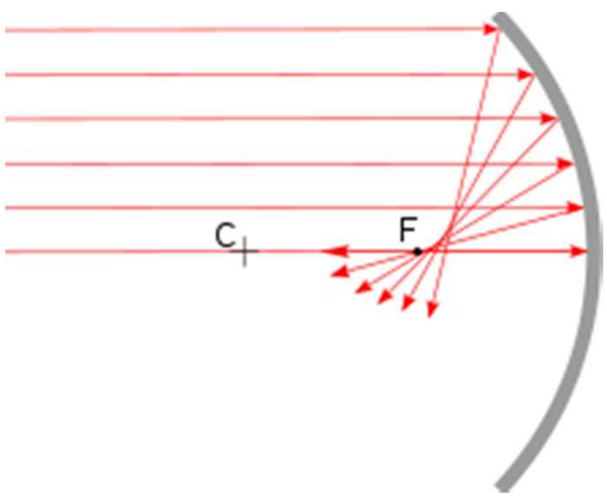
Spherical Aberration

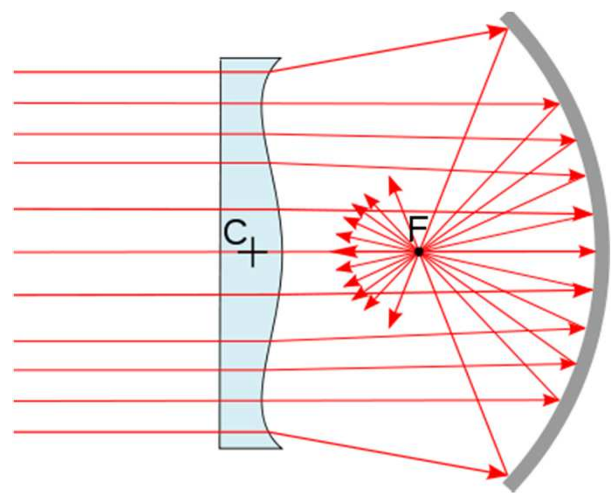
- In third-order optics, the orientation of the lenses does matter
- Spherical aberration depends on the lens arrangement:



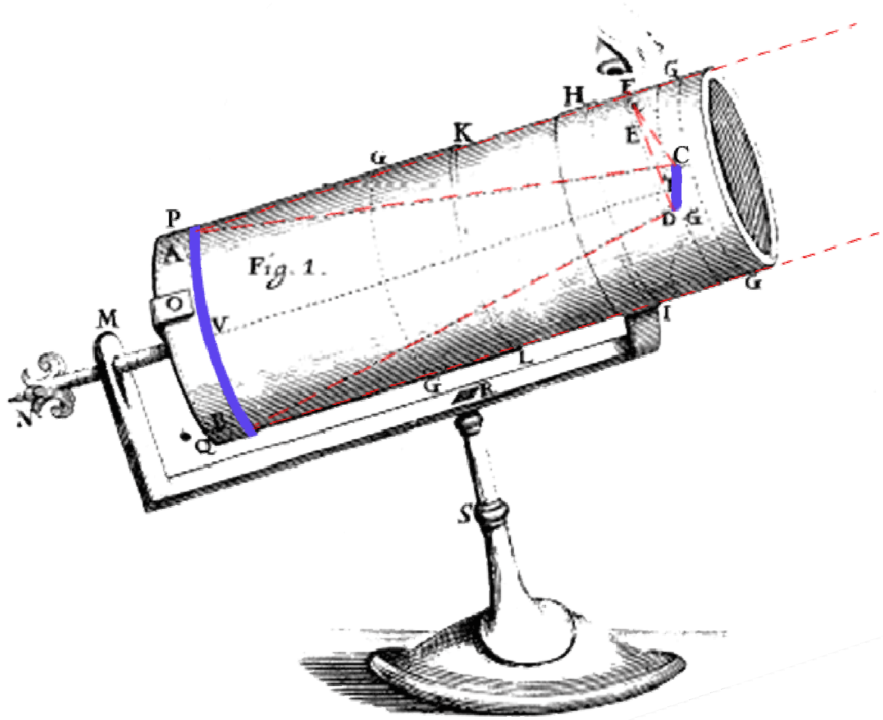
Spherical Aberration of Mirrors

- Spherical mirrors also suffer from spherical aberration
  - Parabolic mirrors do not suffer from spherical aberration, but they distort images from points that do not lie on the optical axis
- ***Schmidt corrector plate*** removes spherical aberration without introducing other optical defects.

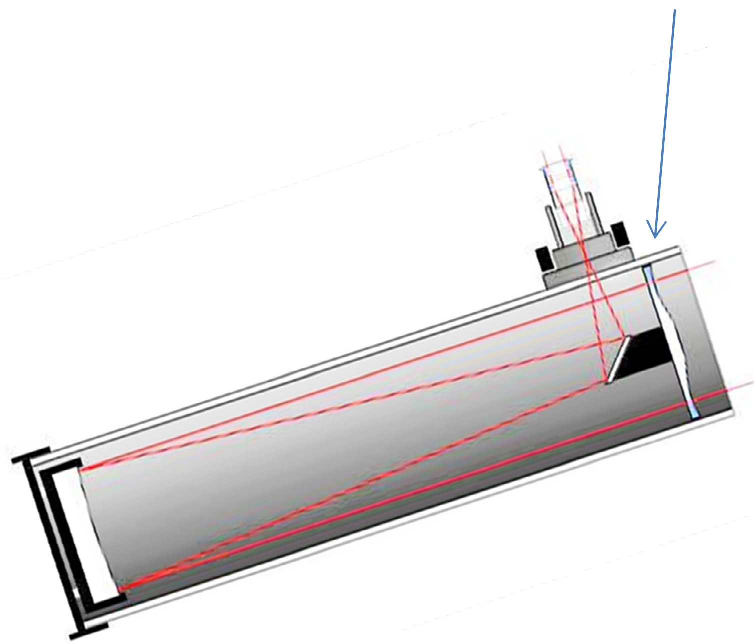




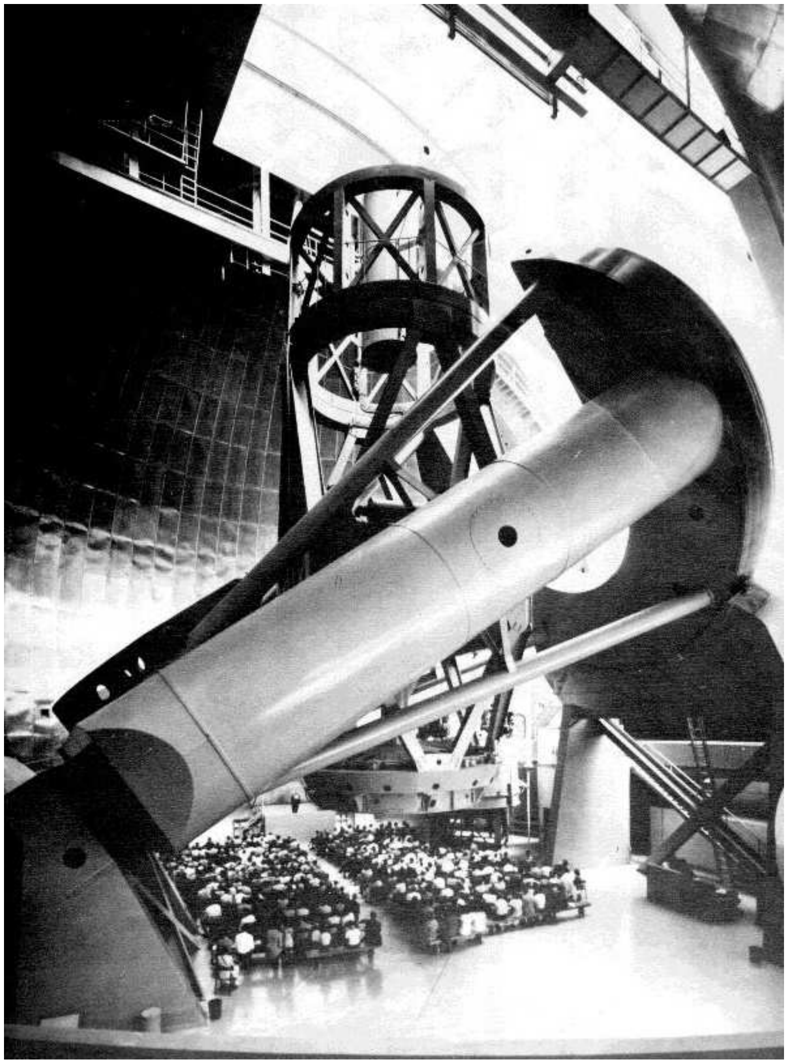
Newtonian Telescope



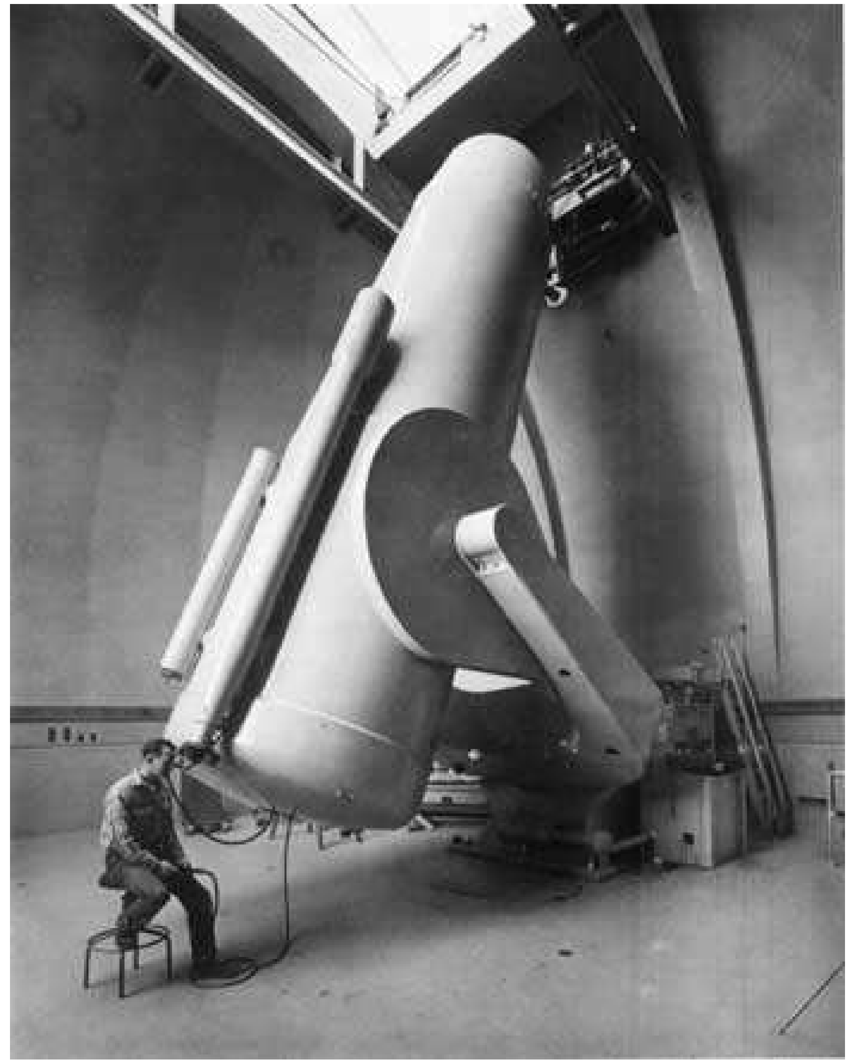
Schmidt corrector plate



Schmidt 48-inch Telescope



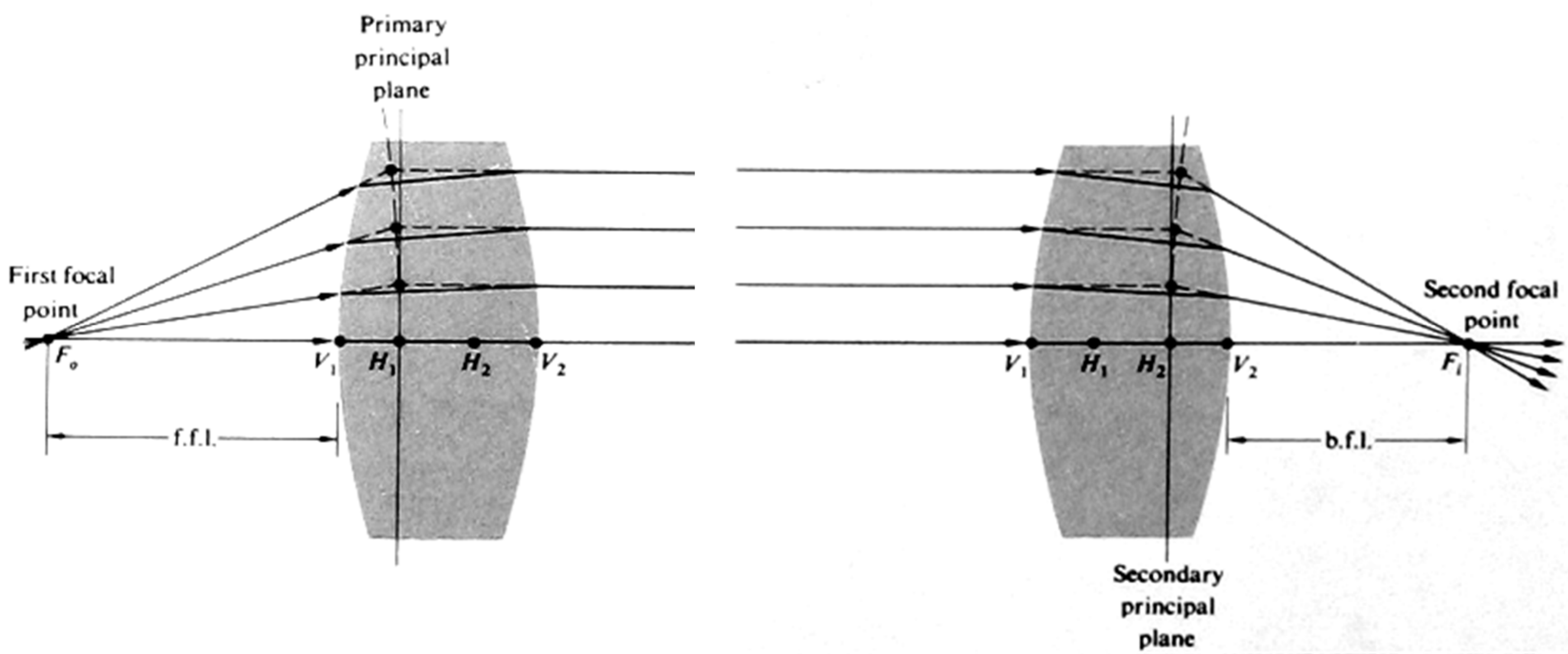
200 inch Hale telescope



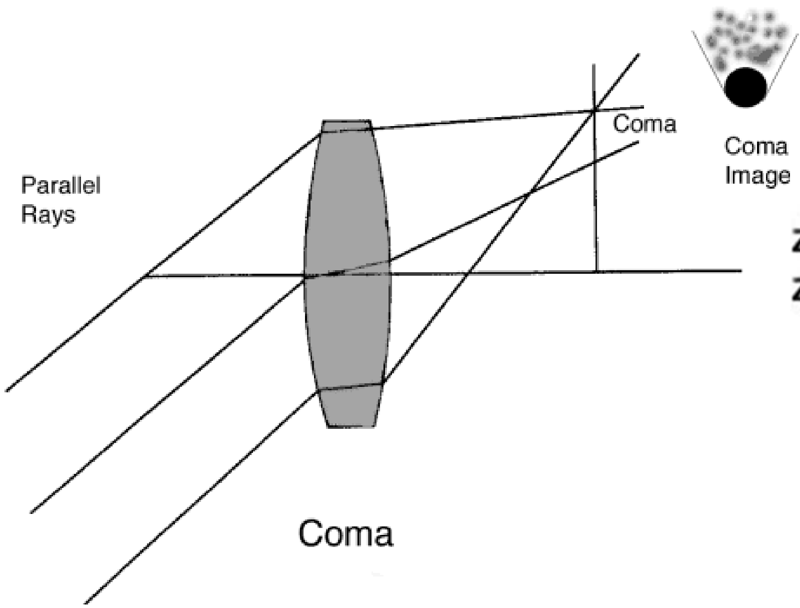
48-inch Schmidt telescope

Coma (comatic aberration)

- Principle planes are not flat – they are actually curved surfaces.
- Focal length is different for off-axis rays



Coma



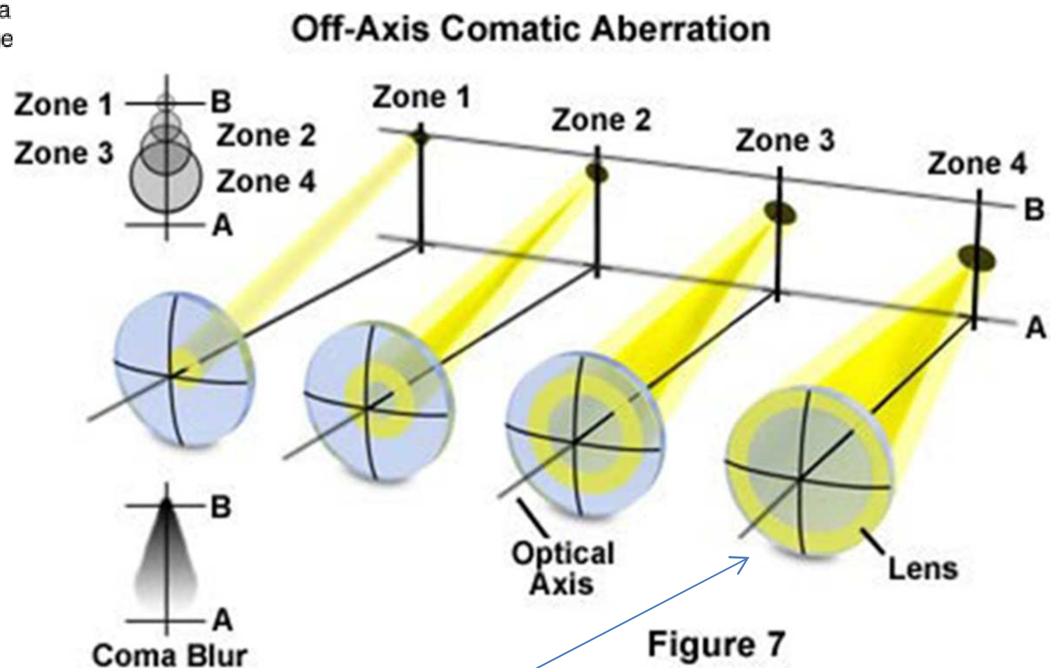
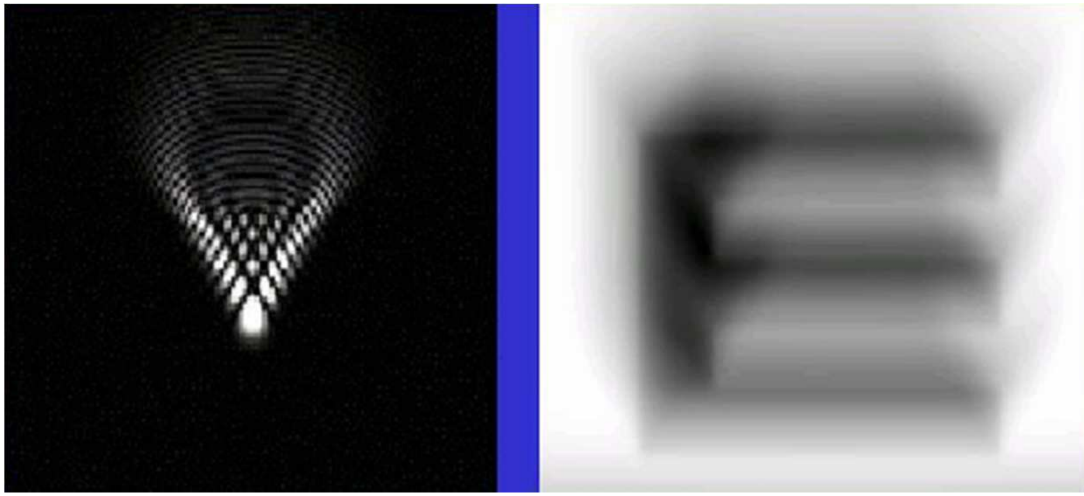


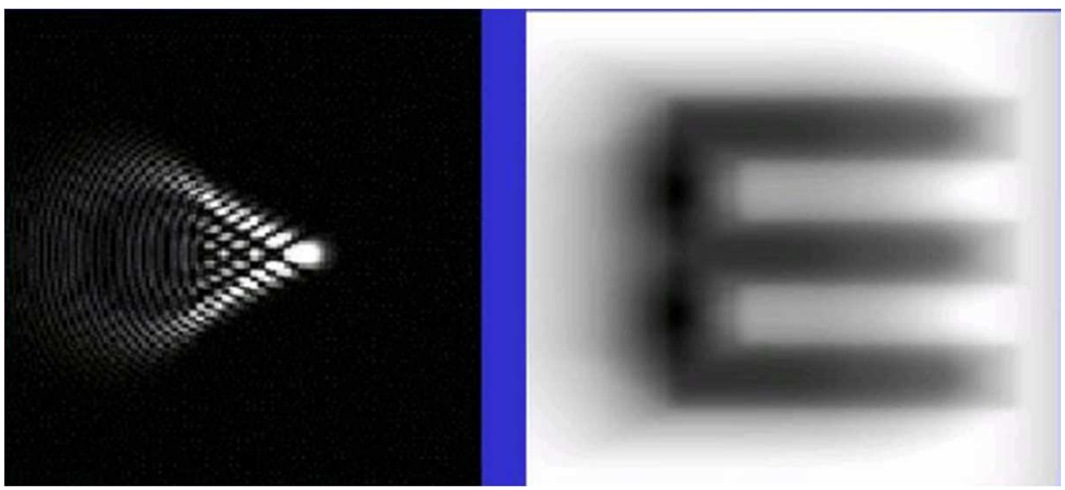
Figure 7

- Negative coma: meridional rays focus closer to the principal axis

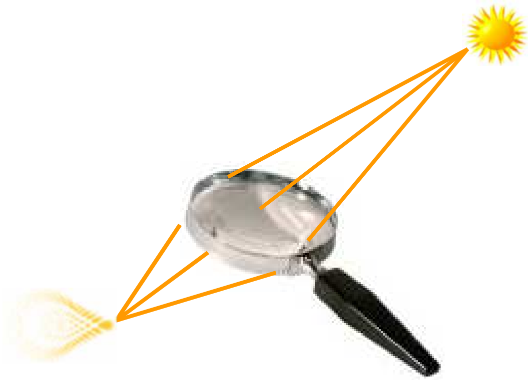
Coma



Vertical coma



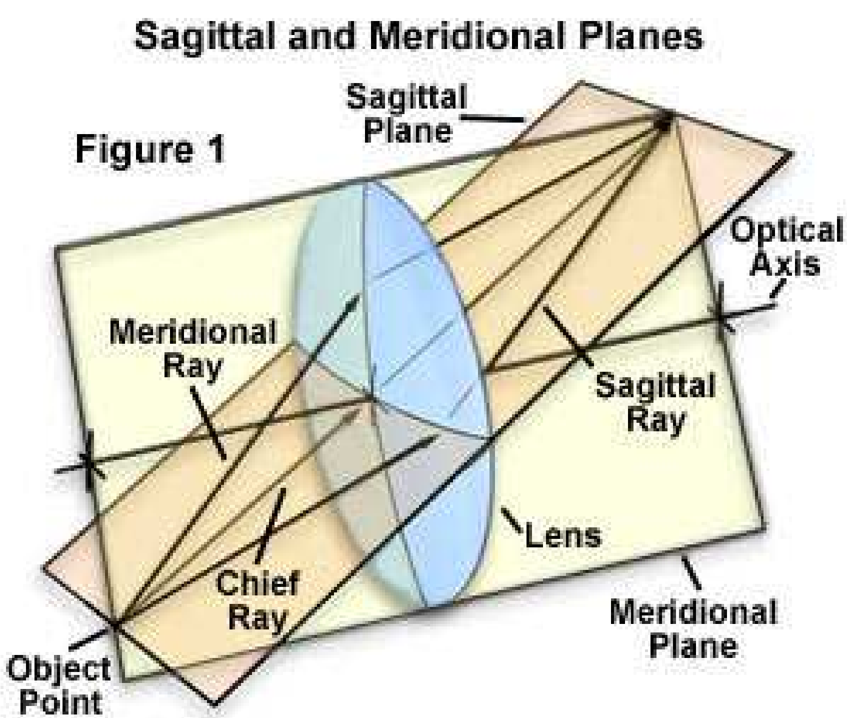
Horizontal coma

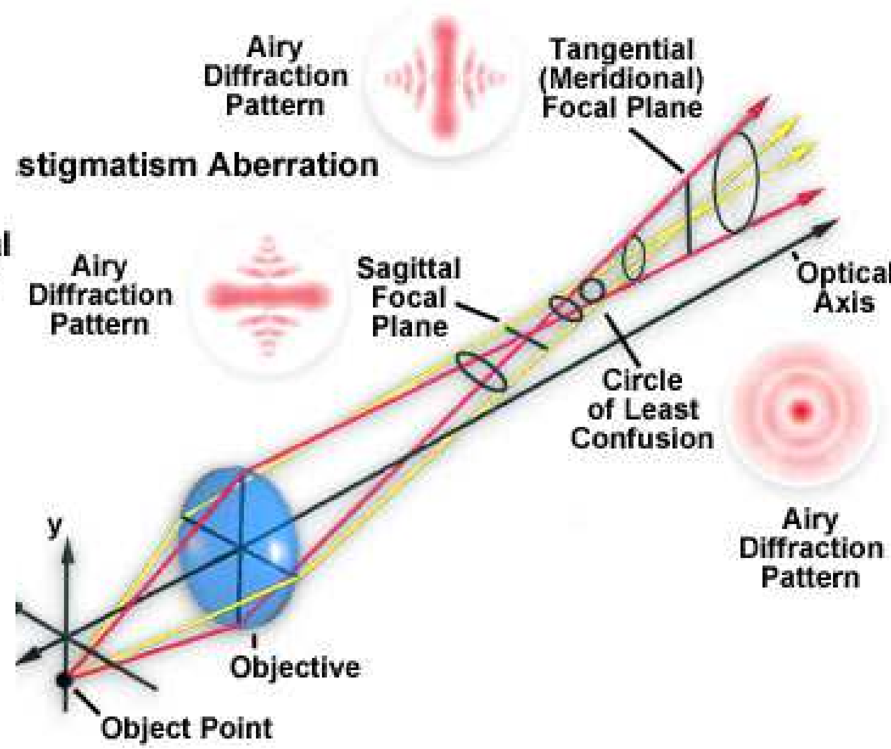


Coma can be reduced by introducing a stop positioned at an appropriate point along the optical axis, so as to remove the appropriate off-axis rays.

Astigmatism

- Parallel rays from an off-axis object arrive in the plane of the lens in one direction, but not in a perpendicular direction:

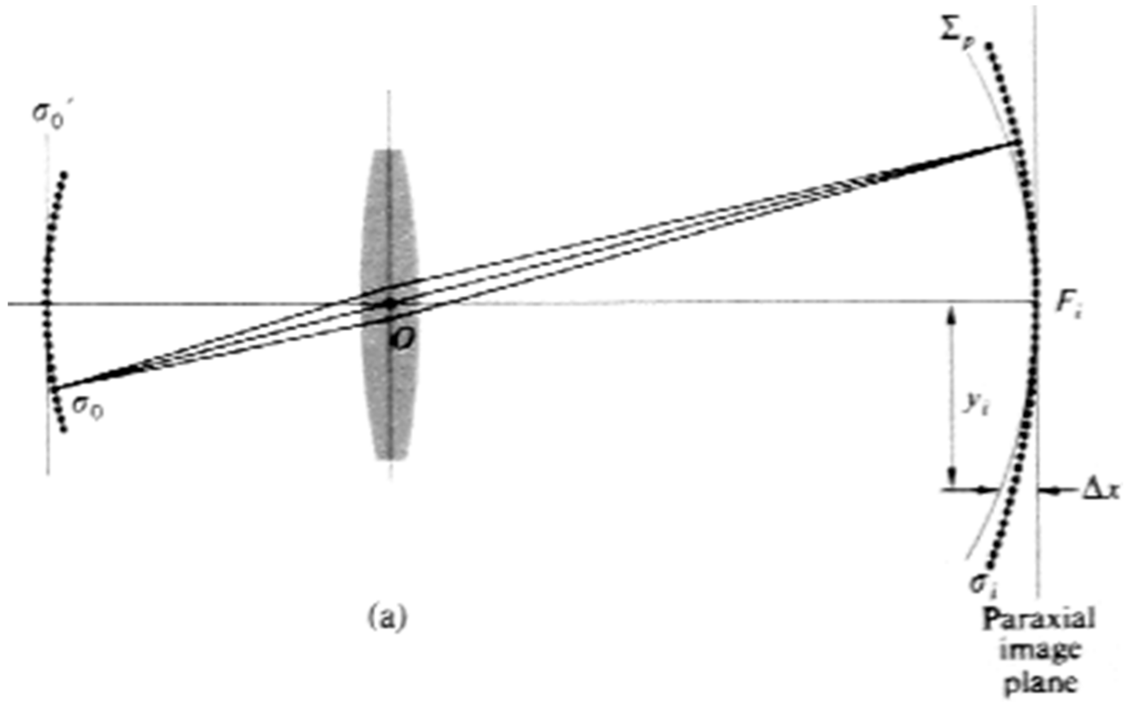




Astigmatism

- This formal definition is different from the one used in ophthalmology which is caused by non-spherical curvature of the surface and lens of the eye.

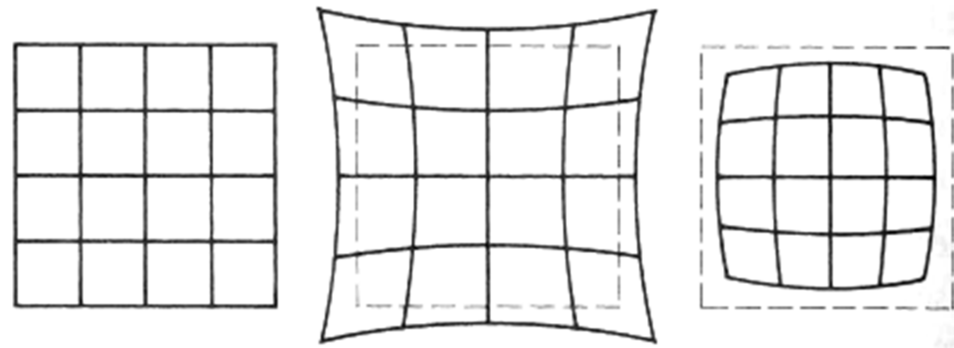
Field Curvature



- The focal plane is actually a curved surface
- A negative lens has a field plane that curves away from the image plane
- A combination of positive and negative lenses can cancel the effect

Field Curvature

- Transverse magnification, m_T, can be a function of the off-axis distance:

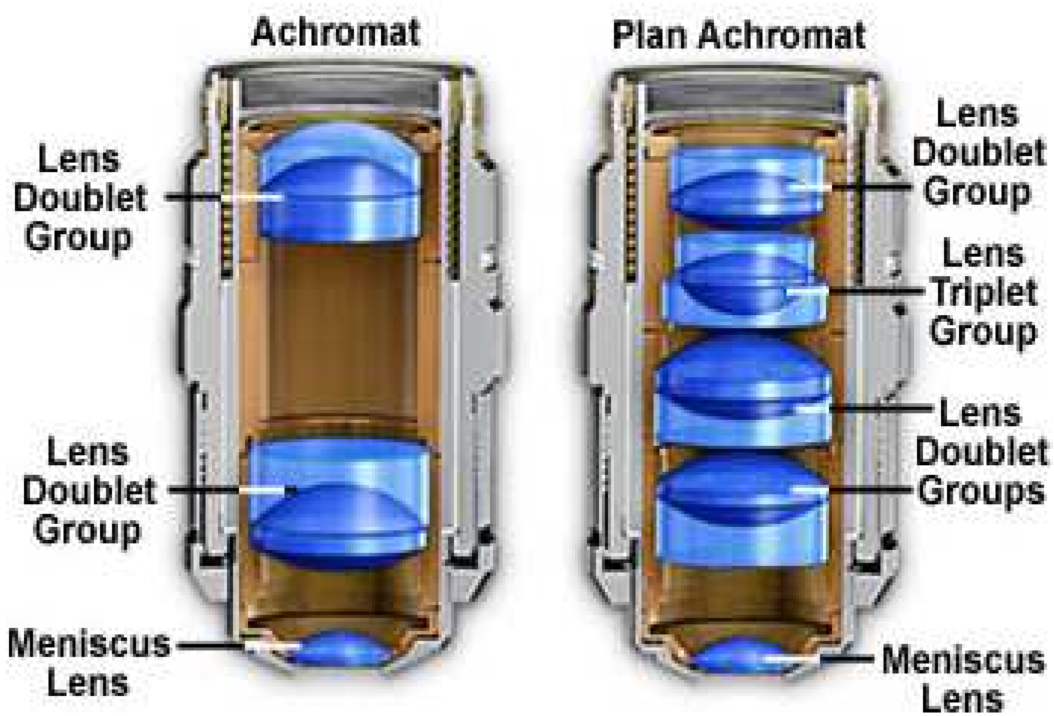


Positive (pincushion) distortion

Negative (barrel) distortion

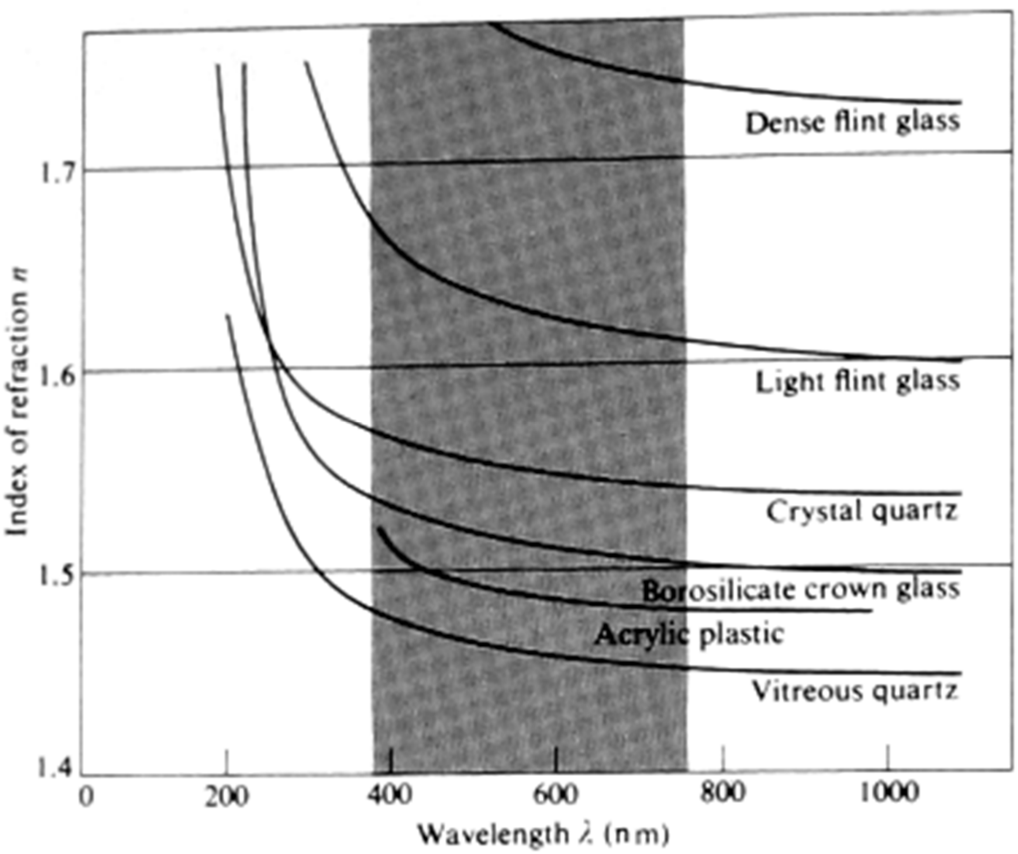
Correcting Monochromatic Aberrations

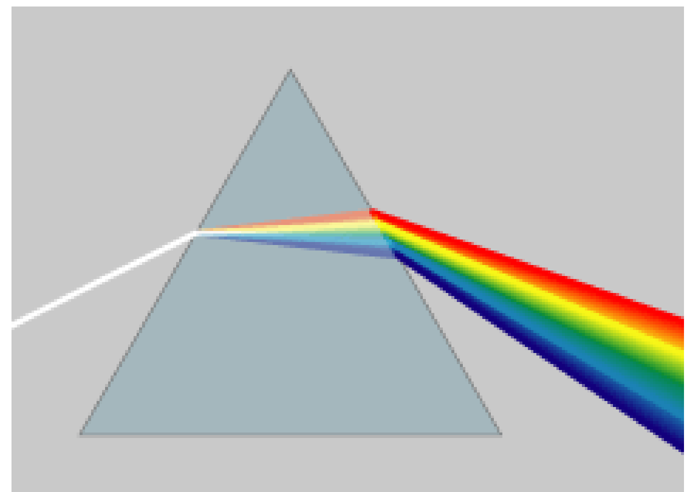
- Combinations of lenses with mutually cancelling aberration effects
- Apertures
- Aspherical correction elements.



Chromatic Aberrations

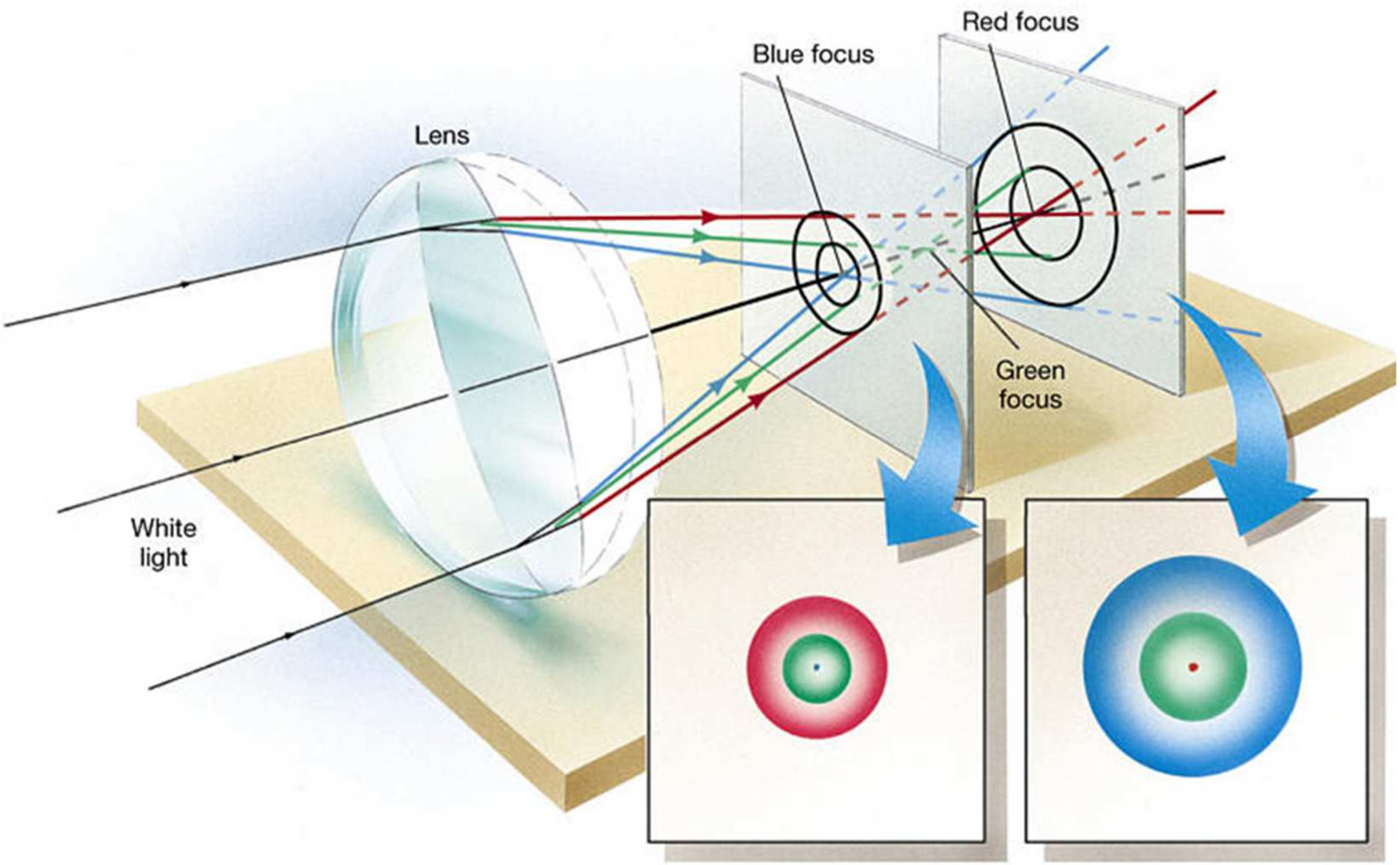
- Index of refraction depends on wavelength





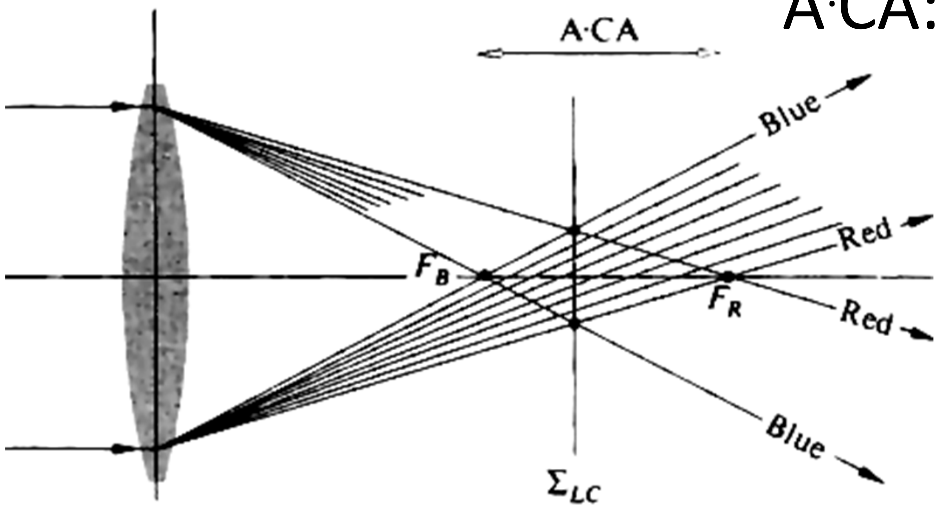
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Chromatic Aberrations

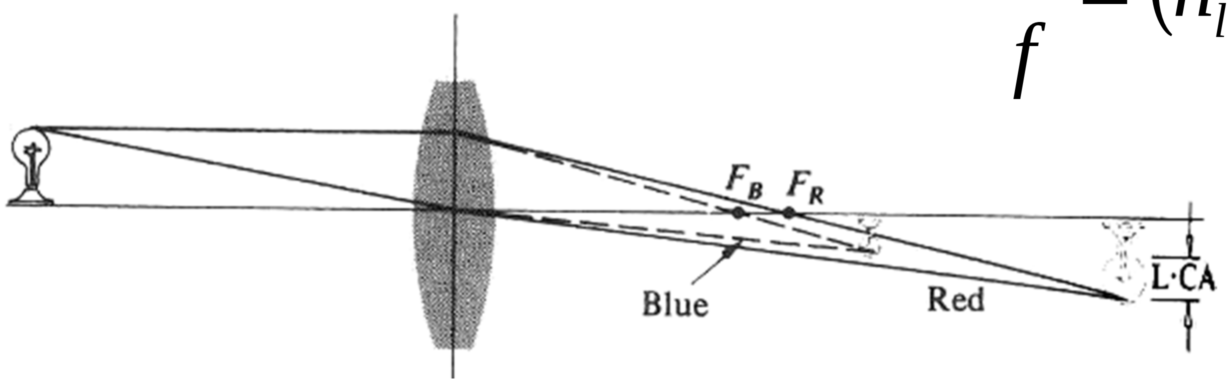


Chromatic Aberrations

A·CA: axial chromatic aberration



$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

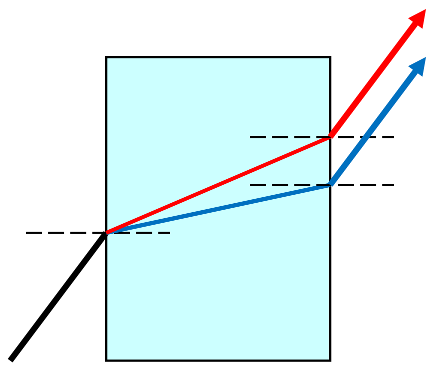


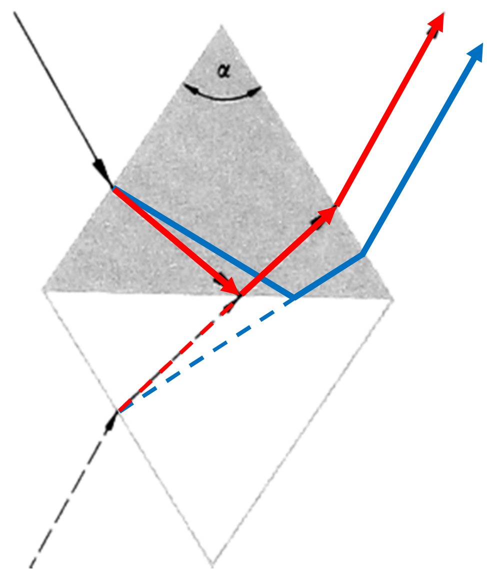
L·CA: lateral chromatic aberration

Chromatic Aberration

Correcting for Chromatic Aberration

- It is possible to have refraction without chromatic aberration even when n is a function of λ:
  - Rays emerge displaced but *parallel*
  - If the thickness is small, then there is no distortion of an image
  - Possible even for non-parallel surfaces:
  - Aberration at one interface is compensated by an opposite aberration at the other surface.





Chromatic Aberration

- Focal length:

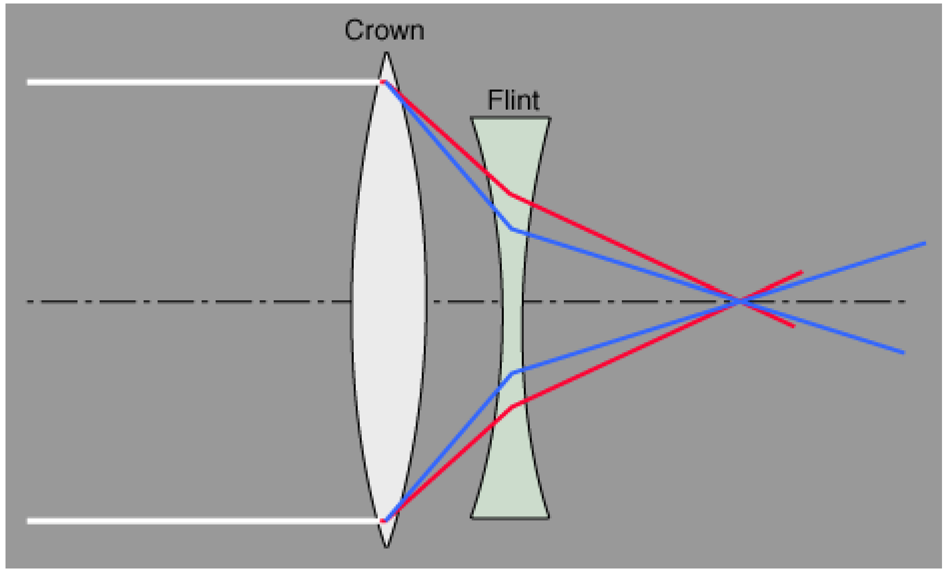
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- Thin lens equation:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

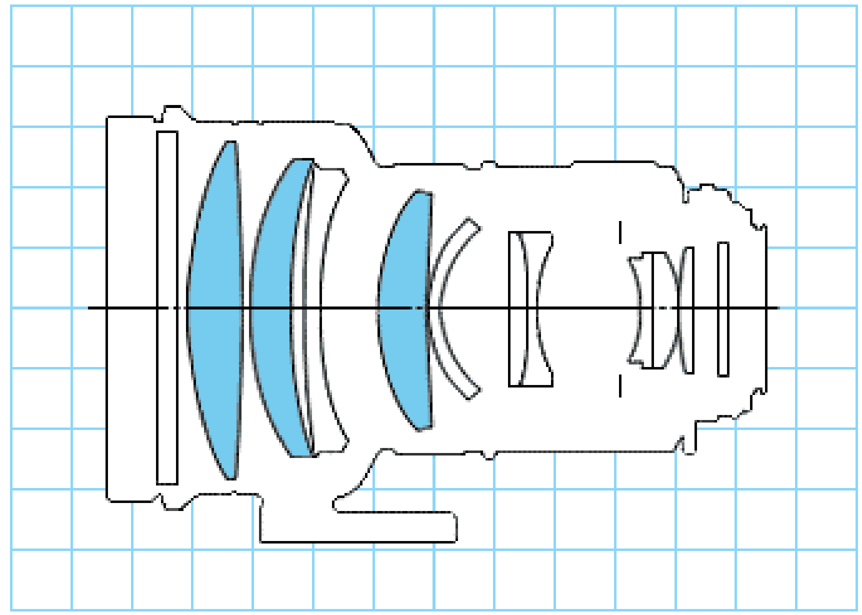
- Cancel chromatic aberration using a combination of concave and convex lenses with different index of refraction

Chromatic Aberration



- This design does not eliminate chromatic aberration completely – only two wavelengths are compensated.

Commercial Lens Assemblies



- Some lens components are made with ultra-low dispersion glass, eg. calcium fluoride