PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

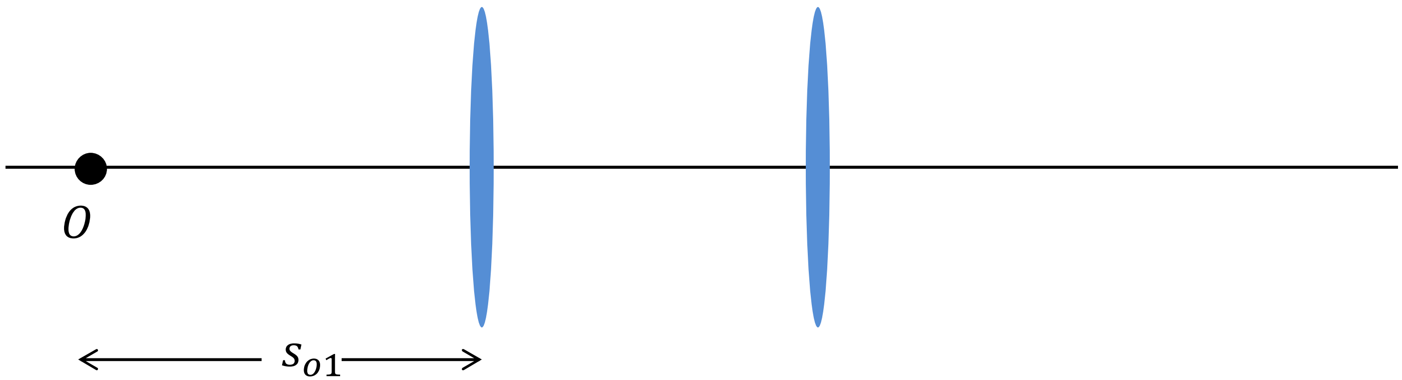
Waves & Oscillations

Lecture 31 – Geometric Optics

Spring 2013 Semester

Matthew Jones

Two Lens Systems



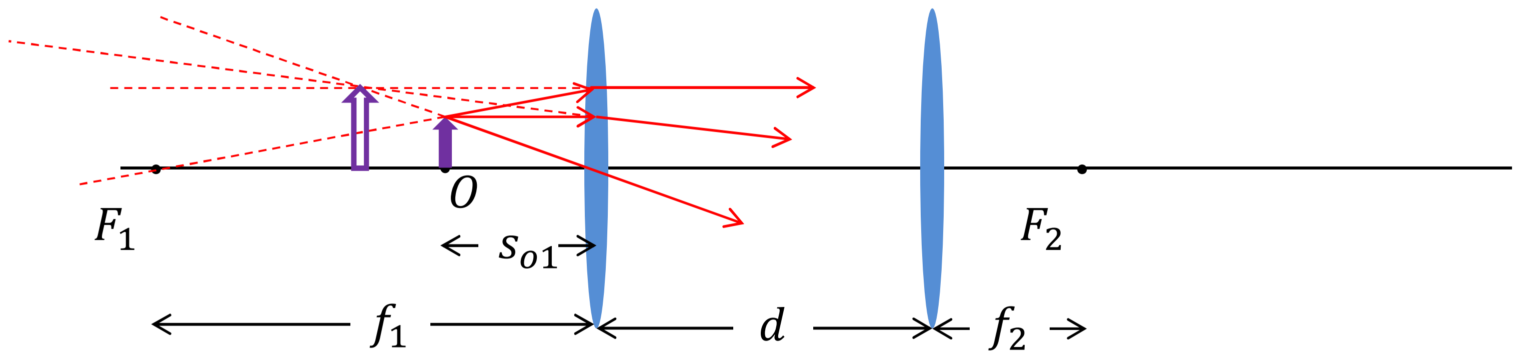
- Calculate s_{i1} using $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}}$
- Ignore the first lens, treat s_{i1} as the object distance for the second lens. Calculate s_{i2} using

$$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$$

- Overall magnification: $M = m_1 m_2 = \left(-\frac{s_{i1}}{s_{o1}}\right)\left(-\frac{s_{i2}}{s_{o2}}\right)$

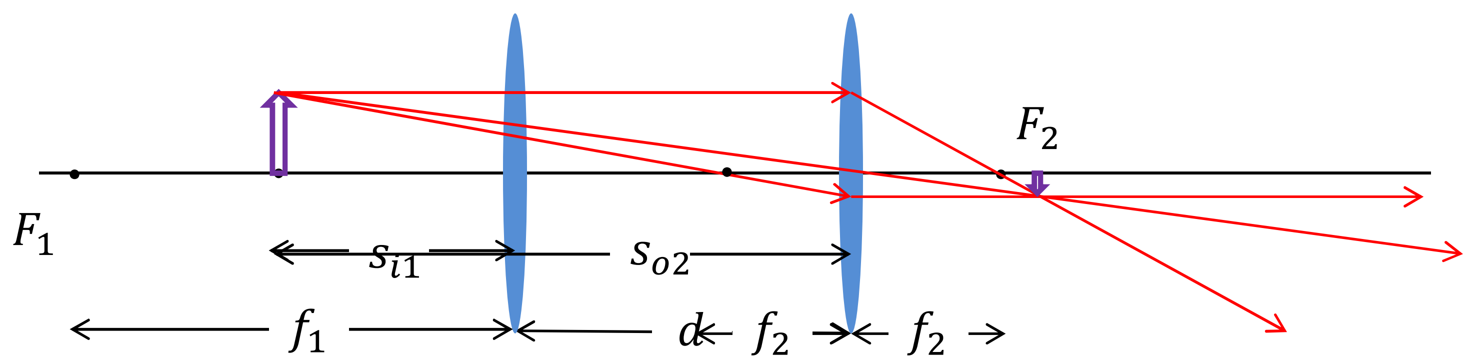
Example: Two Lens System

An object is placed in front of two thin symmetrical coaxial lenses (lens 1 & lens 2) with focal lengths f_1=+24 cm & f_2=+9.0 cm, with a lens separation of L=10.0 cm. The object is 6.0 cm from lens 1. Where is the image of the object?



Example: Two Lens System

An object is placed in front of two thin symmetrical coaxial lenses (lens 1 & lens 2) with focal lengths f_1=+24 cm & f_2=+9.0 cm, with a lens separation of L=10.0 cm. The object is 6.0 cm from lens 1. Where is the image of the object?



(not really to scale…)

Example: Two Lens System

An object is placed in front of two thin symmetrical coaxial lenses (lens 1 & lens 2) with focal lengths $f_1 = +24\ cm$ and $f_2 = +9.0\ cm$, with a lens separation of $d = 10.0\ cm$. The object is 6.0 cm from lens 1. Where is the image of the object?

Lens 1:

$$\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \quad \Rightarrow \quad s_{i1} = -8\ cm$$

Image 1 is virtual.

Lens 2: Treat image 1 as O_2 for lens 2. O_2 is outside the focal point of lens 2. So, image 2 will be real & inverted on the other side of lens 2.

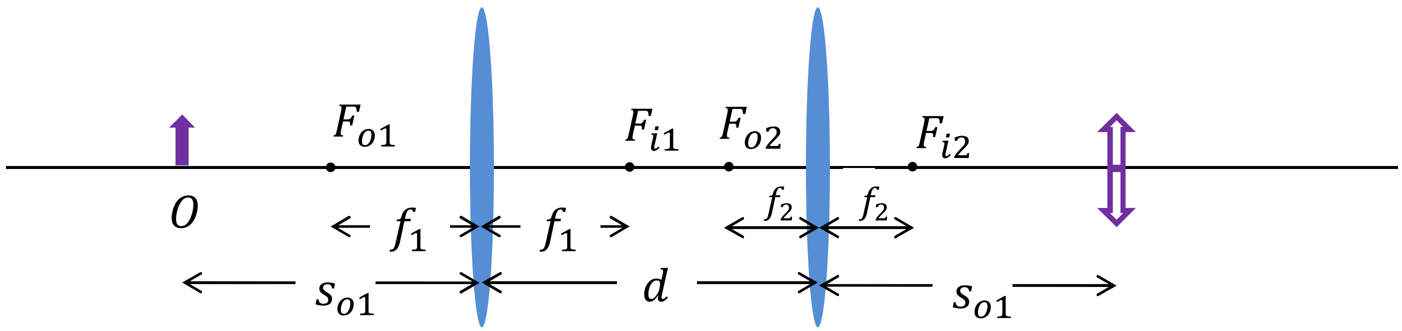
$$s_{o2} = L + |s_{i1}| \quad \Rightarrow \quad s_{i2} = 18.0\ cm$$

$$\frac{1}{f_2} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}}$$

Image 2 is real.

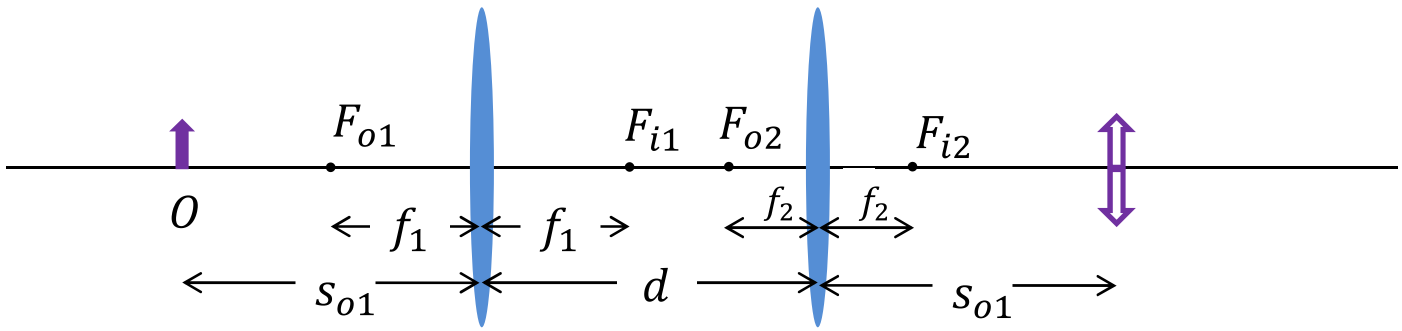
Magnification: $M_T = \left(-\frac{-8\ cm}{6\ cm}\right)\left(-\frac{18cm}{18cm}\right) = -1.33$

Another Example



- Parameters:
  - Focal length: $f_1 = 10\ cm, f_2 = 5\ cm$
  - Lens separation: $d = 42\ cm$
  - Object distance: $s_{o1} = 15\ cm$
- Problem:
  - Calculate the image distance, s_{i2}
  - Calculate the transverse magnification

Another Example

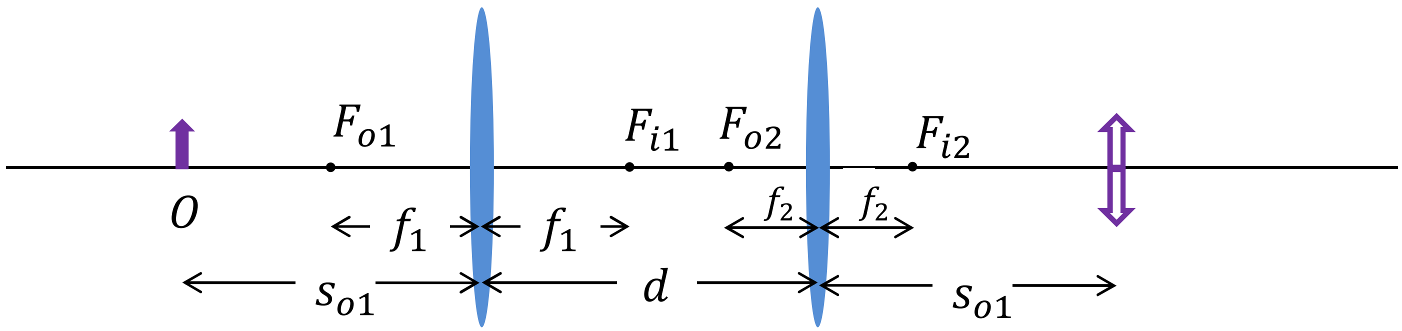


- Step 1: calculate position of first image
  - Thin lens equation: $\frac{1}{s_{o1}} + \frac{1}{s_{i1}} = \frac{1}{f_1}$
  - Solve for s_{i1}:

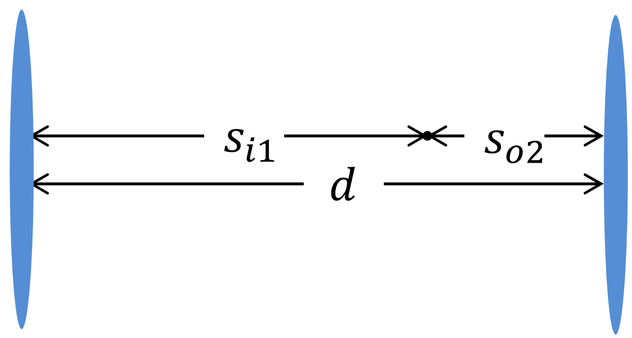
$$s_{i1} = \left(\frac{1}{f_1} - \frac{1}{s_{o1}}\right)^{-1} = \left(\frac{1}{10\ cm} - \frac{1}{15\ cm}\right)^{-1} = 30\ cm$$

  - This is a positive number, so the image is to the right of the first lens.

Another Example

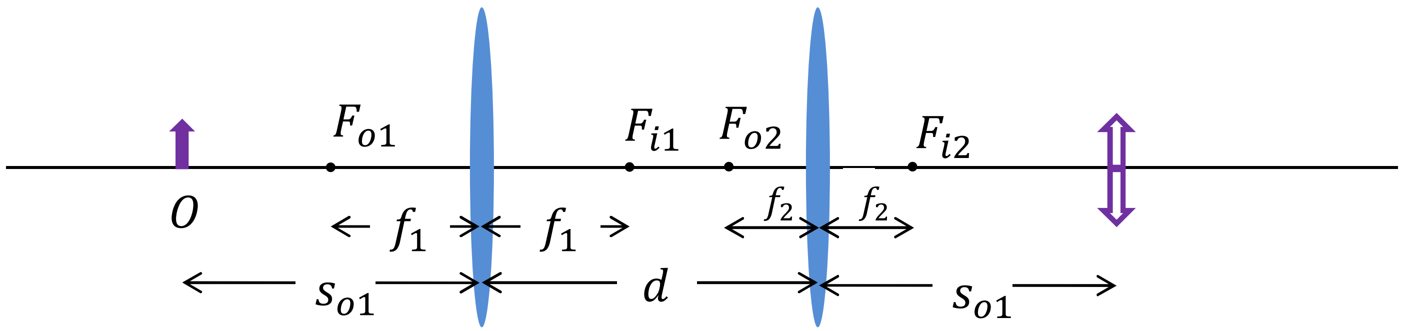


- Step 2: Calculate the second object distance, s_{o2}



$$s_{o2} = d - s_{i1} = 42\ cm\ - 10\ cm$$
$$s_{o2} = 12\ cm$$

Another Example

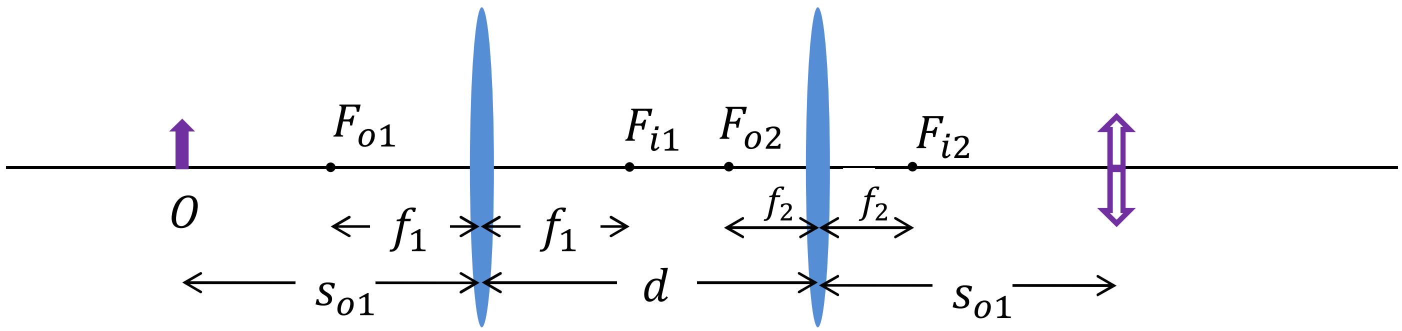


- Step 3: calculate position of second image
  - Thin lens equation: $\frac{1}{s_{o2}} + \frac{1}{s_{i2}} = \frac{1}{f_2}$
  - Solve for s_{i2}:

$$s_{i2} = \left(\frac{1}{f_2} - \frac{1}{s_{o2}}\right)^{-1} = \left(\frac{1}{5\ cm} - \frac{1}{12\ cm}\right)^{-1} = 8.6\ cm$$

  - This is a positive number, so the image is to the right of the second lens.

Another Example



- Step 4: calculate the transverse magnification
  - Magnification of the first lens:

$$m_{T1} = -\frac{s_{i1}}{s_{o1}} = -\frac{30\ cm}{15\ cm} = -2$$

  - Magnification of the second lens:

$$m_{T2} = -\frac{s_{i2}}{s_{o2}} = -\frac{8.6\ cm}{12\ cm} = -0.72$$

  - Total magnification:

$$m_T = m_{T1} m_{T2} = (-2)(-0.72) = +1.44$$

Two Lens Equation

- Can we derive an equation that expresses s_{i2} in terms of s_{o1}, f_1, f_2 and d?
- Thin lens equation:

$$\frac{1}{s_{o2}} + \frac{1}{s_{i2}} = \frac{1}{f_2} \qquad s_{i2} = \left(\frac{1}{f_2} - \frac{1}{s_{o2}}\right)^{-1} = \frac{f_2 s_{o2}}{s_{o2} - f_2}$$

- Relation between s_{o2} and s_{i1}:

$$s_{o2} = d - s_{i1} = d - \frac{f_1 s_{o1}}{s_{o1} - f_1}$$

- Two lens equation:

$$s_{i2} = \frac{f_2 d - f_2 f_1 s_{o1}/(s_{o1} - f_1)}{d - f_2 - f_1 s_{o1}/(s_{o1} - f_1)}$$

Two Lens Equation

$$s_{i2} = \frac{f_2 d - f_2 f_1 s_{o1}/(s_{o1} - f_1)}{d - f_2 - f_1 s_{o1}/(s_{o1} - f_1)}$$

- Can we describe the two-lens system in terms of a focal length?
  - Yes, but the front and back focal lengths are different.
- Front focal length: $s_{i2} \to \infty$ or $1/s_{i2} \to 0$
  - Outgoing rays parallel to optical axis

$$\frac{d - f_2 - f_1 s_{o1}/(s_{o1} - f_1)}{f_2 d - f_2 f_1 s_{o1}/(s_{o1} - f_1)} = 0$$

$$s_{o1} = \mathrm{f.f.l.} = \frac{f_1(d - f_2)}{d - (f_1 + f_2)}$$

Two Lens Equation

$$s_{i2} = \frac{f_2 d - f_2 f_1 s_{o1}/(s_{o1} - f_1)}{d - f_2 - f_1 s_{o1}/(s_{o1} - f_1)}$$

- Front focal length: $s_{i2} \to \infty$ or $1/s_{i2} \to 0$

$$s_{o1} = \mathrm{f.f.l.} = \frac{f_1(d - f_2)}{d - (f_1 + f_2)}$$

- Back focal length: $s_{o1} \to \infty$ or $1/s_{o1} \to 0$
  - Incoming rays parallel to optical axis

$$s_{i2} = \mathrm{b.f.l.} = \frac{f_2(d - f_1)}{d - (f_1 + f_2)}$$

Two Lens Equation

- Front focal length:

$$\text{f.f.l.} = \frac{f_1(d-f_2)}{d-(f_1+f_2)}$$

- Back focal length:

$$\text{b.f.l.} = \frac{f_2(d-f_1)}{d-(f_1+f_2)}$$

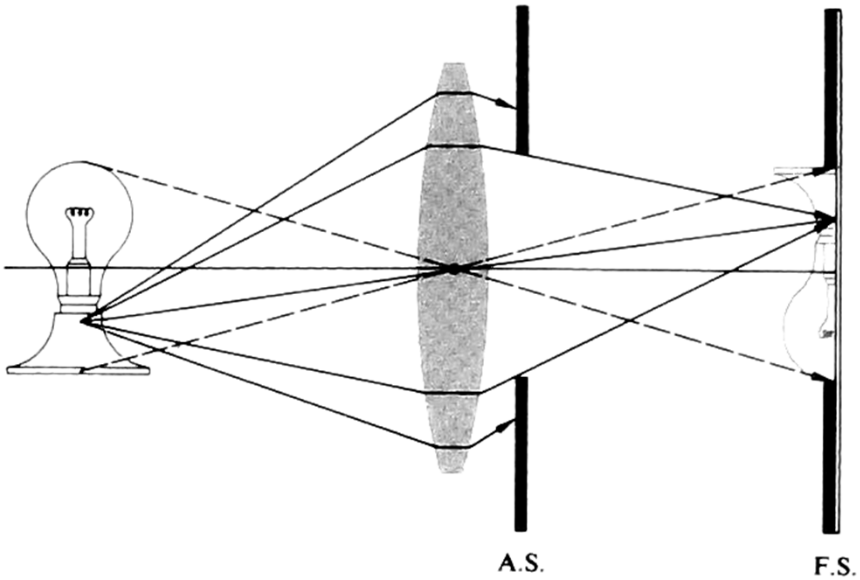
- If the lenses are close together, $d \to 0$ and we have

$$\text{f.f.l.} = \text{b.f.l.} = f = \frac{f_1 f_2}{f_1+f_2} = \left(\frac{1}{f_1}+\frac{1}{f_2}\right)^{-1}$$

- In general, for several closely spaced thin lenses

$$\frac{1}{f} = \frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\cdots$$

Apertures and Stops



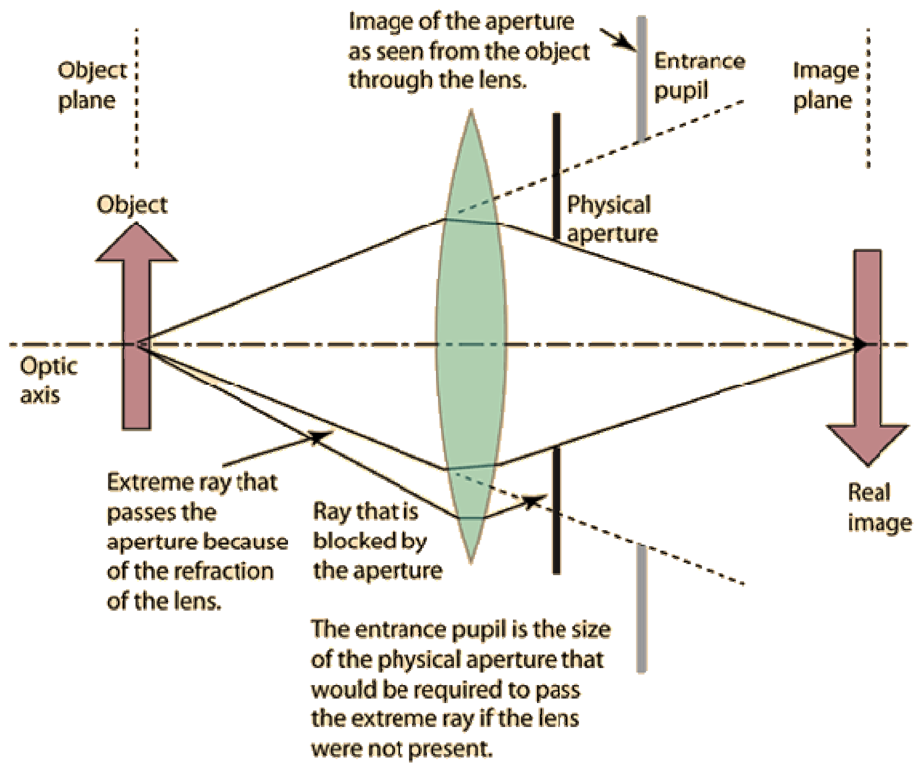
Field stop - an element limiting the size, or angular breadth of the image (for example film edge in camera)

Aperture stop - an element that determines the amount of light reaching the image

- Field stop determines the field of view and limits the size of objects that can be imaged.
- Aperture determines amount of light only

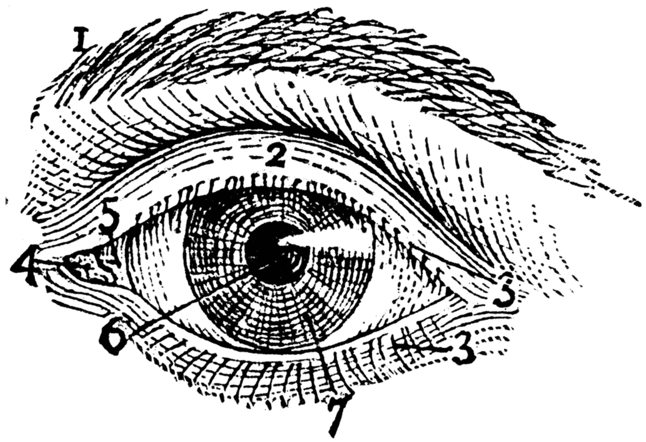
Entrance Pupil

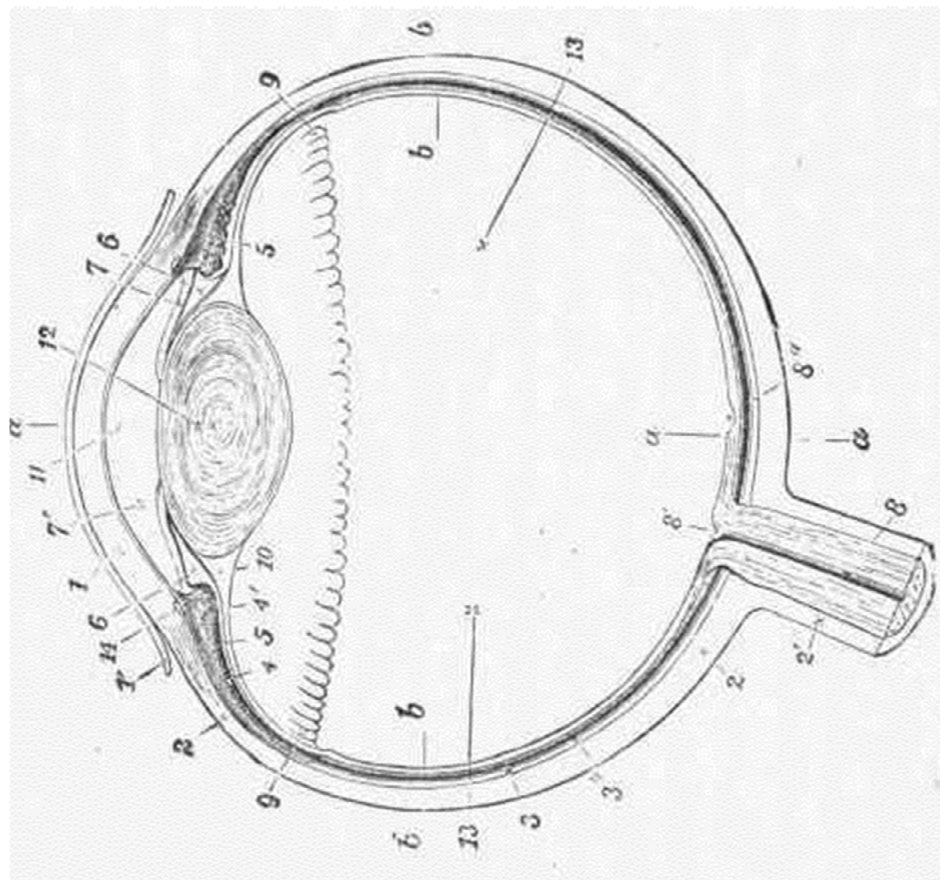
- How big does the aperture stop appear when viewed from the position of the object?



(Aperture stop is behind the lens)

Entrance Pupil

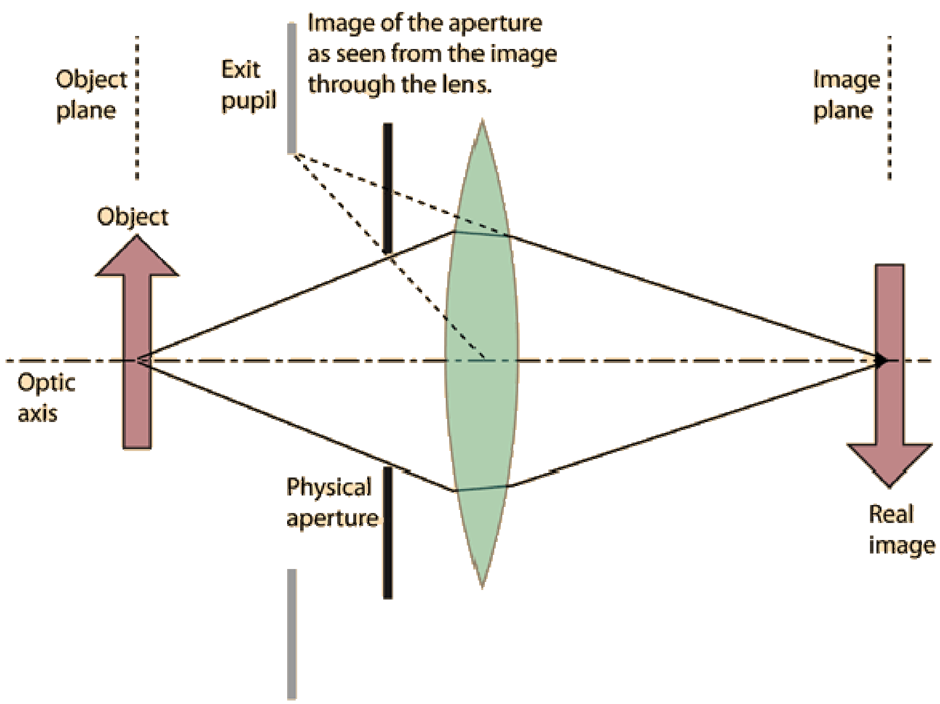




- If the cornea were removed, the pupil would appear smaller
- The cornea magnifies the image of the pupil

Exit Pupil

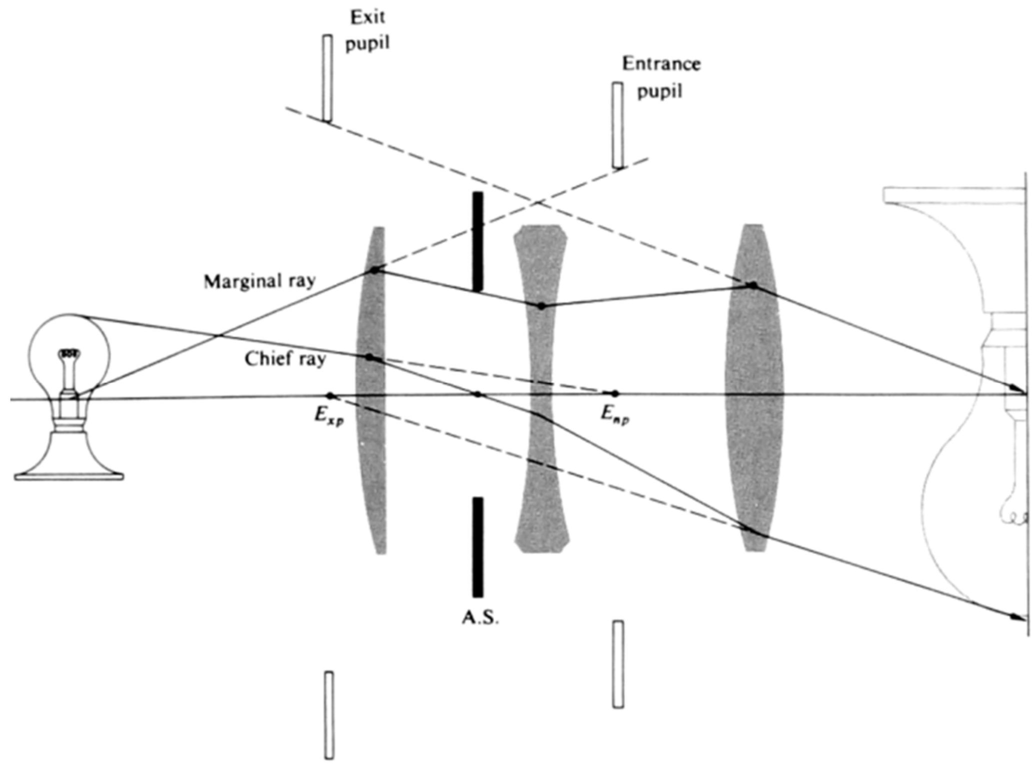
- How big does the aperture stop appear when viewed from the image plane?



(Aperture stop is in front of the lens)

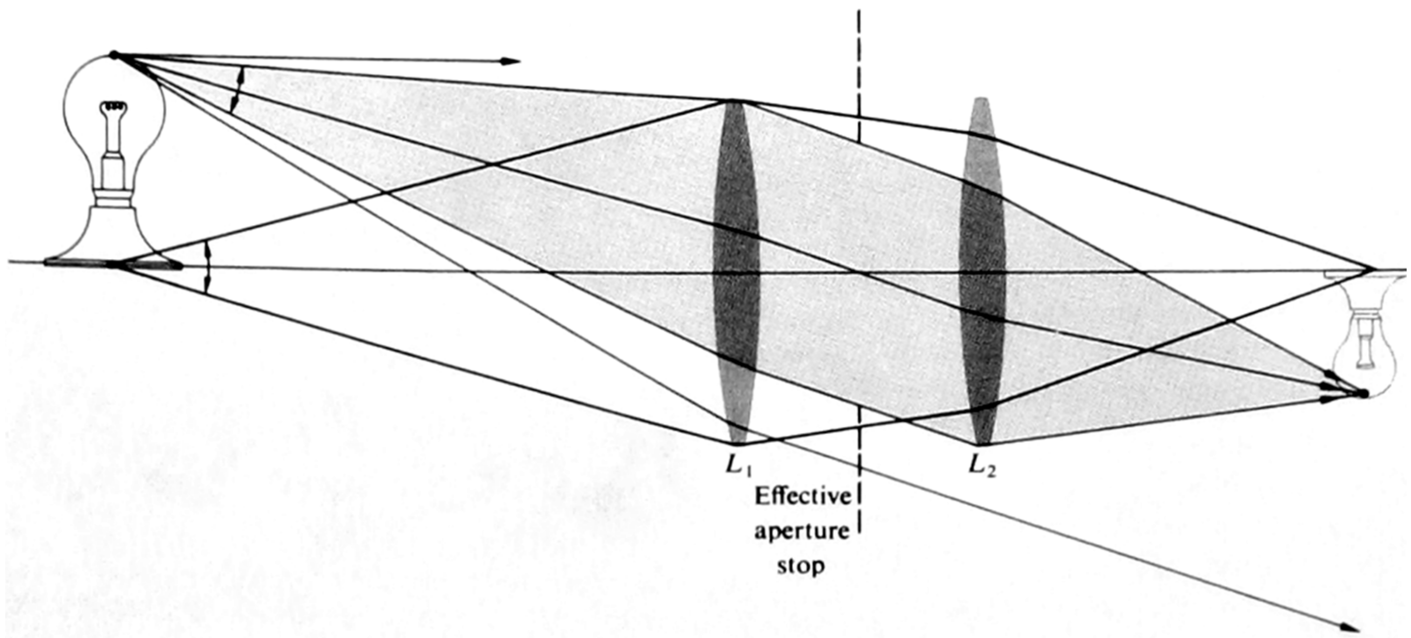
Chief and marginal rays

Marginal ray: the ray that comes from point on object and marginally passes the aperture stop



Chief ray: any ray from an object point that passes through the middle of the aperture stop
It is effectively the central ray of the bundle emerging from a point on an object that can get through the aperture.
Importance: aberrations in optical systems

Vignetting

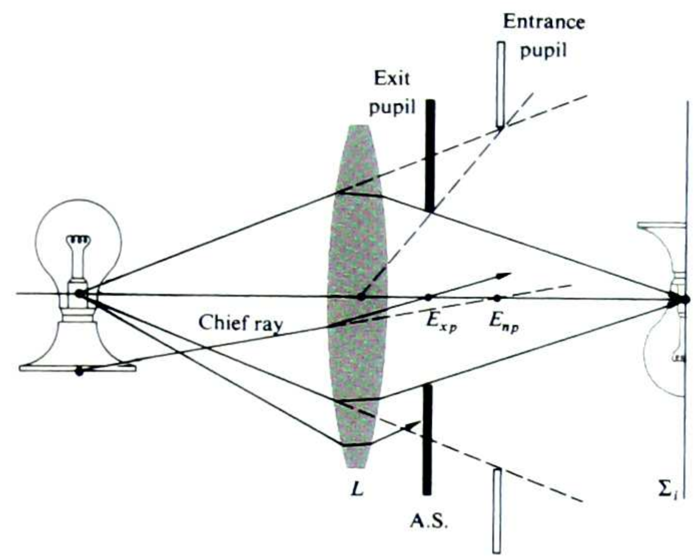


The cone of rays that reaches image plane from the top of the object is smaller than that from the middle. There will be less light on the periphery of the image - a process called **vignetting**

Example: entrance pupil of the eye can be as big as 8 mm. Telescopes are designed to have exit pupil of 8 mm for maximum brightness of the image

Relative Aperture

- The area of the entrance pupil determines how much light will reach the image plane.
- Pupils are typically circular: the area varies as the square of the diameter, D.



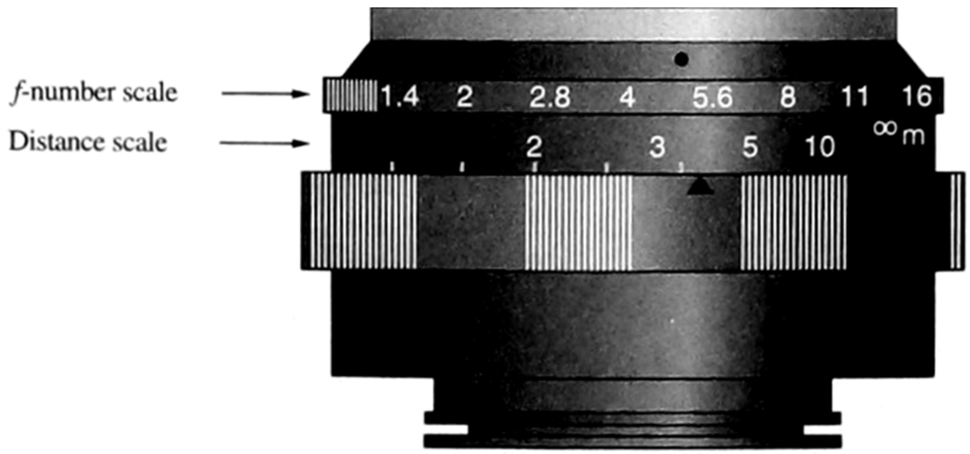
- The image area varies as the square of the lateral dimension, $A \sim f^2$
- Light intensity at the image plane varies as $(D/f)^2$
- (D/f) is called the *relative aperture*

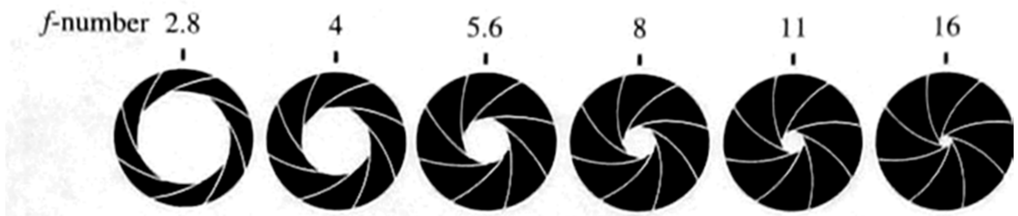
Relative Aperture

- Relative aperture: $f/D =$ (focal length/diameter)
- For optical equipment (camera lenses) this is usually labeled as $f/\#$
- Example:
  - $f = 50\, mm$
  - $D = 25\, mm$

  $\Big\}$ $f/D = 2$ denoted "$f/2$"
- This provides a standard way to reference the intensity of light shining on film or other photosensitive material.

f-number of a camera lens

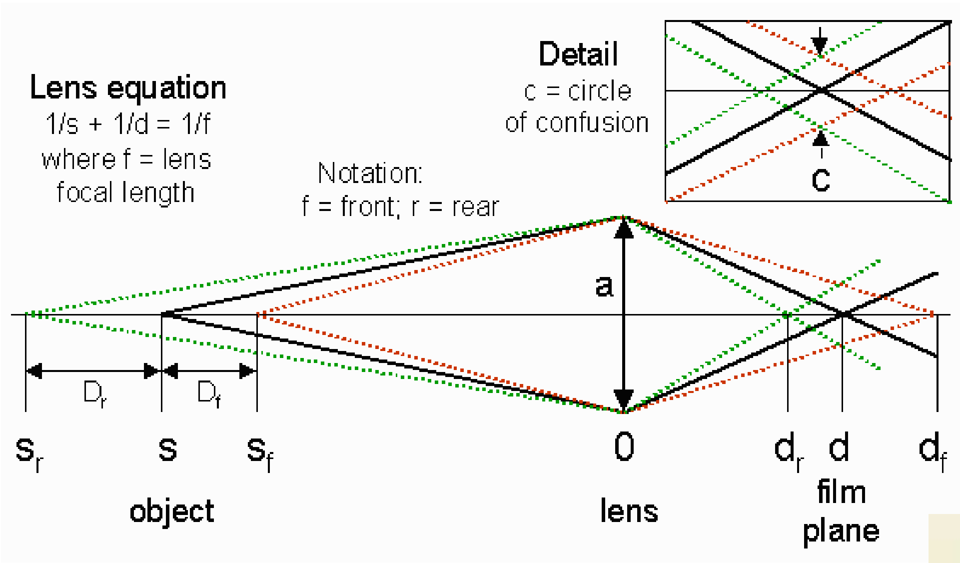




Change in neighboring numbers is $\sqrt{2}$

Intensity is ~1/(*f/#*)^2: changing diaphragm from one label to another changes light intensity on film 2 times

Depth of Field



Depth of Field

- Extreme case is the pinhole camera