Sign Conventions

- Convex surface:
  - $s_o$ is positive for objects on the incident-light side
  - $s_i$ is positive for images on the refracted-light side
  - $R$ is positive if $C$ is on the refracted-light side

\[ \frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \]
Sign Conventions

• Concave surface:
  – $s_o$ is positive for objects on the incident-light side
  – $s_i$ is negative for images on the incident-light side
  – $R$ is negative if $C$ is on the incident-light side

\[
\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}
\]

(same formula)
Magnification

- Using these sign conventions, the magnification is
  \[ m = - \frac{n_1 s_i}{n_2 s_o} \]
- Ratio of image height to object height
- Sign indicates whether the image is inverted
Thin Lenses

• The previous examples were for one spherical surface.
• Two spherical surfaces make a thin lens
Thin Lens Classification

- A flat surface corresponds to $R \to \infty$
- All possible combinations of two surfaces:
Thin Lens Equation

First surface:
\[
\frac{n_m}{s_{o1}} + \frac{n_l}{s_{i1}} = \frac{n_l - n_m}{R_1}
\]

Second surface:
\[
\frac{n_l}{-s_{i1} + d} + \frac{n_m}{s_{i2}} = \frac{n_m - n_l}{R_2}
\]

Add these equations and simplify using \( n_m = 1 \) and \( d \rightarrow 0 \):
\[
\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]
(Thin lens equation)
Gaussian Lens Formula

• Recall that the focal point was the place to which parallel rays were made to converge

• Parallel rays from the object correspond to $s_o \to \infty$ and $s_i \to f$: \[
\frac{1}{f} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)
\]

• This lens equation: \[
\frac{1}{s_i} + \frac{1}{s_o} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}
\]
Gaussian Lens Formula

- Gaussian lens formula:
  \[ \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \]

- Newtonian form:
  \[ x_o x_i = f^2 \]
  (follows from the Gaussian formula after about 5 lines of algebra)

- All you need to know about a lens is its focal length
Example

• What is the focal length of this lens?
  - Let $s_o \rightarrow \infty$, then $s_i \rightarrow f$
    \[
    \frac{1}{f} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
    \]
  - The flat surface has $R_1 \rightarrow \infty$ and we know that $R_2 = 50 \text{ mm}$
    \[
    \frac{1}{f} = (1.5 - 1) \left( \frac{1}{\infty} - \frac{1}{-50 \text{ mm}} \right) = \frac{1}{100 \text{ mm}}
    \]
    \[
    f = 100 \text{ mm}
    \]
Example

• Objects are placed at
  $s_i = 600 \text{ mm}, 200 \text{ mm}, 150 \text{ mm}, 100 \text{ mm}, 80 \text{ mm}$

• Where are their images?
  $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

  $s_i = \frac{s_of}{s_o - f}$

  $s_i = 120 \text{ mm}, 200 \text{ mm}, 300 \text{ mm}, \infty, -400 \text{ mm}$
Focal Plane

Thin lens + paraxial approximation:
- All rays that pass through the center, \( O \), do not bend
- All rays converge to points in the focal plane (back focal plane)
- \( F_o \) lies in the front focal plane
Imaging with a Thin Lens

For each point on the object we can draw three rays:

1. A ray straight through the center of the lens
2. A ray parallel to the central axis, then through the image focal point
3. A ray through the object focal point, then parallel to the central axis.
Converging Lens: Principal Rays

1) Rays parallel to principal axis pass through focal point $F_i$.
2) Rays through center of lens are not refracted.
3) Rays through $F_o$ emerge parallel to principal axis.

In this case image is real, inverted and enlarged.

Assumptions:
- Monochromatic light
- Thin lens
- Paraxial rays (near the optical axis)

Since $n$ is a function of $\lambda$, in reality each color has a different focal point: **chromatic aberration**. Contrast to mirrors: angle of incidence/reflection not a function of $\lambda$.
Diverging Lens: Forming Image

1) Rays parallel to principal axis appear to come from focal point $F_i$.
2) Rays through center of lens are not refracted.
3) Rays toward $F_o$ emerge parallel to principal axis.

Assumptions:
- paraxial monochromatic rays
- thin lens

Principal rays:

1) Rays parallel to principal axis appear to come from focal point $F_i$.
2) Rays through center of lens are not refracted.
3) Rays toward $F_o$ emerge parallel to principal axis.

Image is virtual, upright and reduced.
Converging Lens: Examples

This could be used in a camera. Big object on small film

This could be used as a projector. Small slide(object) on big screen (image)

This is a magnifying glass
Lens Magnification

\[ \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \]

Green and blue triangles are similar:

\[ \frac{y_i}{y_o} = \frac{s_i}{s_o} \]

Magnification equation:

\[ M_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o} \]

Example: \( f=10 \, \text{cm}, \, s_o=15 \, \text{cm} \)

\[ \frac{1}{15 \, \text{cm}} + \frac{1}{s_i} = \frac{1}{10 \, \text{cm}} \quad \Rightarrow \quad s_i = 30 \, \text{cm} \]

\[ M_T = -\frac{30 \, \text{cm}}{15 \, \text{cm}} = -2 \]
Longitudinal Magnification

\[ \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \]

The 3D image of the horse is distorted:
- transverse magnification changes along optical axis
- longitudinal magnification is not linear

Longitudinal magnification:

\[ M_L \equiv \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2 \]

Negative: a horse looking towards the lens forms an image that looks away from the lens

\[ x_o x_i = f^2 \rightarrow x_i = f^2 / x_o \rightarrow \frac{dx_i}{dx_o} = \frac{d}{dx_o} \left( \frac{f^2}{x_o} \right) = -\left( \frac{f^2}{x_o^2} \right) \]
Two Lens Systems

- Calculate $s_{i1}$ using $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}}$
- Ignore the first lens, treat $s_{i1}$ as the object distance for the second lens. Calculate $s_{i2}$ using $\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$
- Overall magnification: $M = m_1m_2 = \left(-\frac{s_{i1}}{s_{o1}}\right)\left(-\frac{s_{i2}}{s_{o2}}\right)$
Example: Two Lens System

An object is placed in front of two thin symmetrical coaxial lenses (lens 1 & lens 2) with focal lengths $f_1=+24\,\text{cm}$ & $f_2=+9.0\,\text{cm}$, with a lens separation of $L=10.0\,\text{cm}$. The object is 6.0 cm from lens 1. Where is the image of the object?
Example: Two Lens System

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(not really to scale...)
Example: Two Lens System

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Lens 1:  
\[
\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \quad \rightarrow \quad s_{i1} = -8 \text{ cm}
\]

Image 1 is virtual.

Lens 2: Treat image 1 as $O_2$ for lens 2. $O_2$ is outside the focal point of lens 2. So, image 2 will be real & inverted on the other side of lens 2.

\[
s_{o2} = L + |s_{i1}| \quad \rightarrow \quad s_{i2} = 18.0 \text{ cm}
\]

Image 2 is real.

Magnification: 
\[
M_T = \left( -\frac{8 \text{ cm}}{6 \text{ cm}} \right) \left( -\frac{18 \text{ cm}}{18 \text{ cm}} \right) = -1.33
\]