PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 29 – Geometric Optics

Spring 2013 Semester

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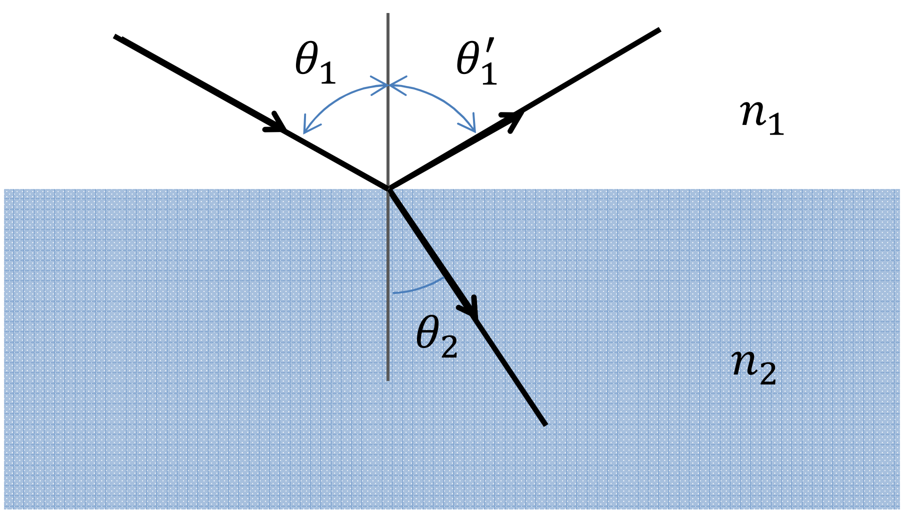
Geometric Optics

- When the wavelength of light is much smaller than the dimensions of objects it interacts with, we can ignore its wave nature.
- Multiple paths by which light can reach a given point – phases are random (incoherent).
- We are generally not concerned with polarization.
- Treat light as rays propagating in straight lines

Geometric Optics

- Under these conditions, the only physical principles we need to describe the propagation of light are:



Reflection:

$$\theta_1' = \theta_1$$

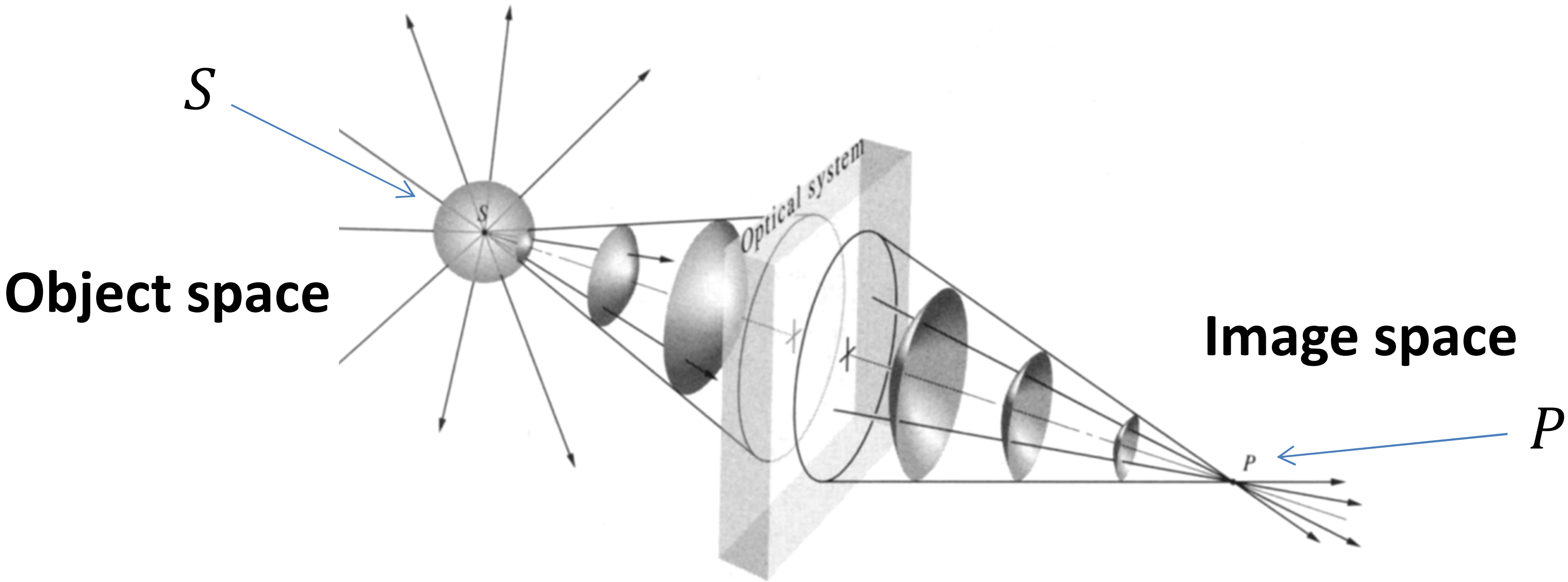
Refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Geometric Optics

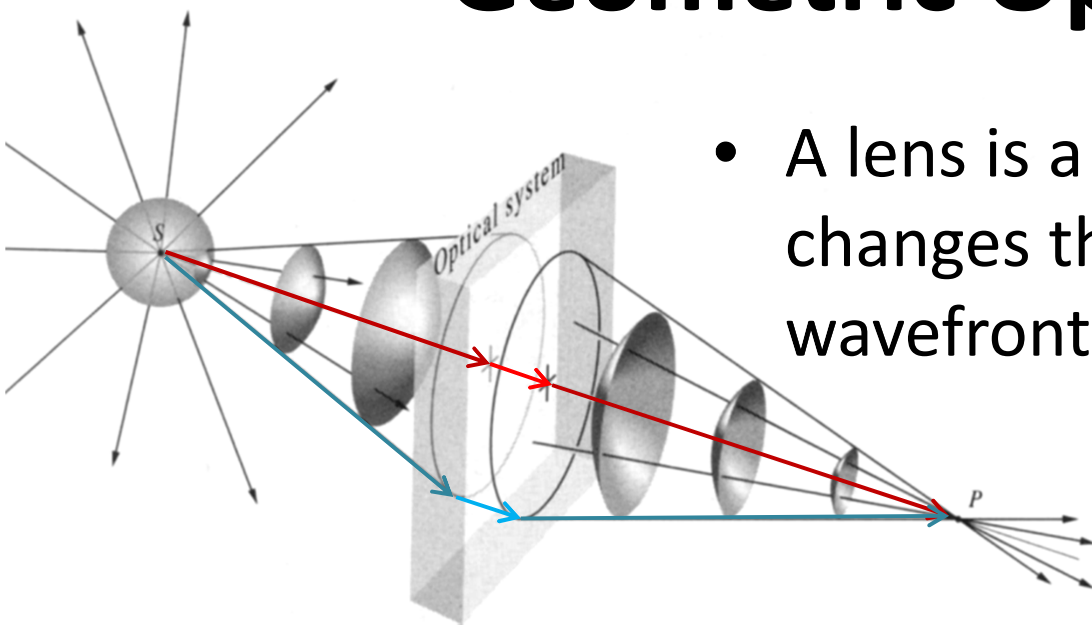
- Each point on an illuminated surface is a source of spherical waves
  - Rays diverge from that point
  - We perceive an image as the collection of points from which the rays emerge
- An optical system can cause the rays to diverge from a different point
  - We perceive this point as an image of the original object

Geometric Optics



- A point from which a portion of the spherical wave diverges is a focus of the bundle of rays
- A point to which the portion of the spherical wave converges is also a focus of the bundle of rays
- The paths are reversible
- P and S are *conjugate points*

Geometric Optics



- A lens is a refractive device that changes the curvature of the wavefront

- All points on the wavefront have the same *optical path length (OPL)*

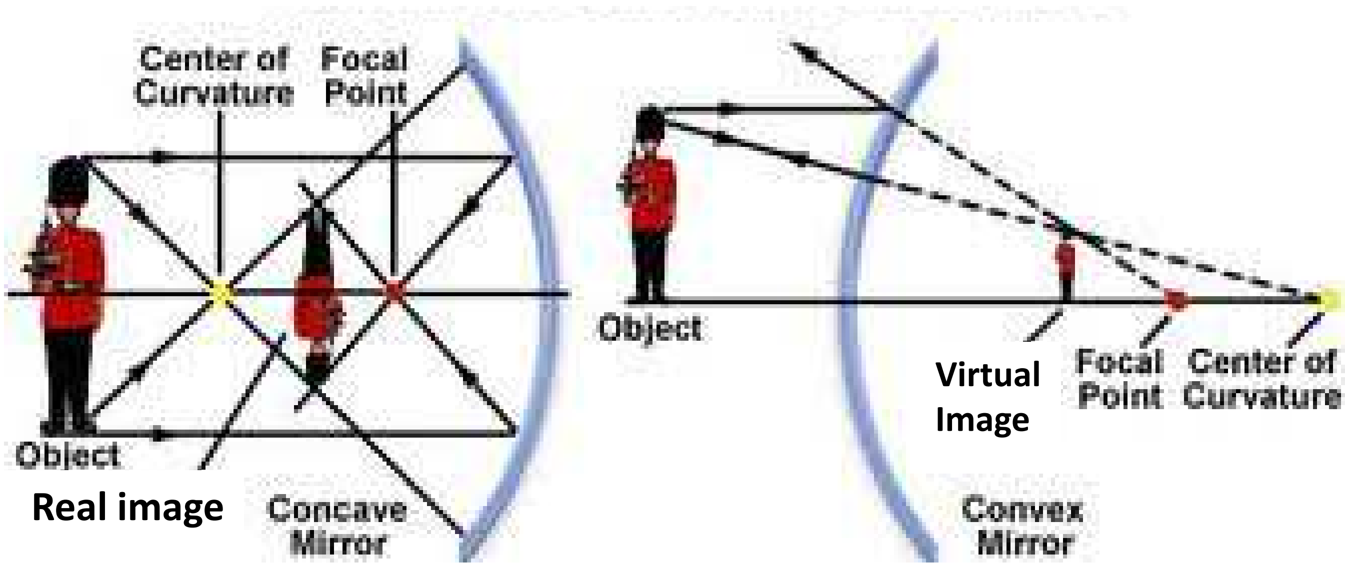
$$t = \frac{1}{c}\sum_{i=1}^{N} n_i s_i \rightarrow OPL = \int_S^P n(s)ds$$

Geometric Optics

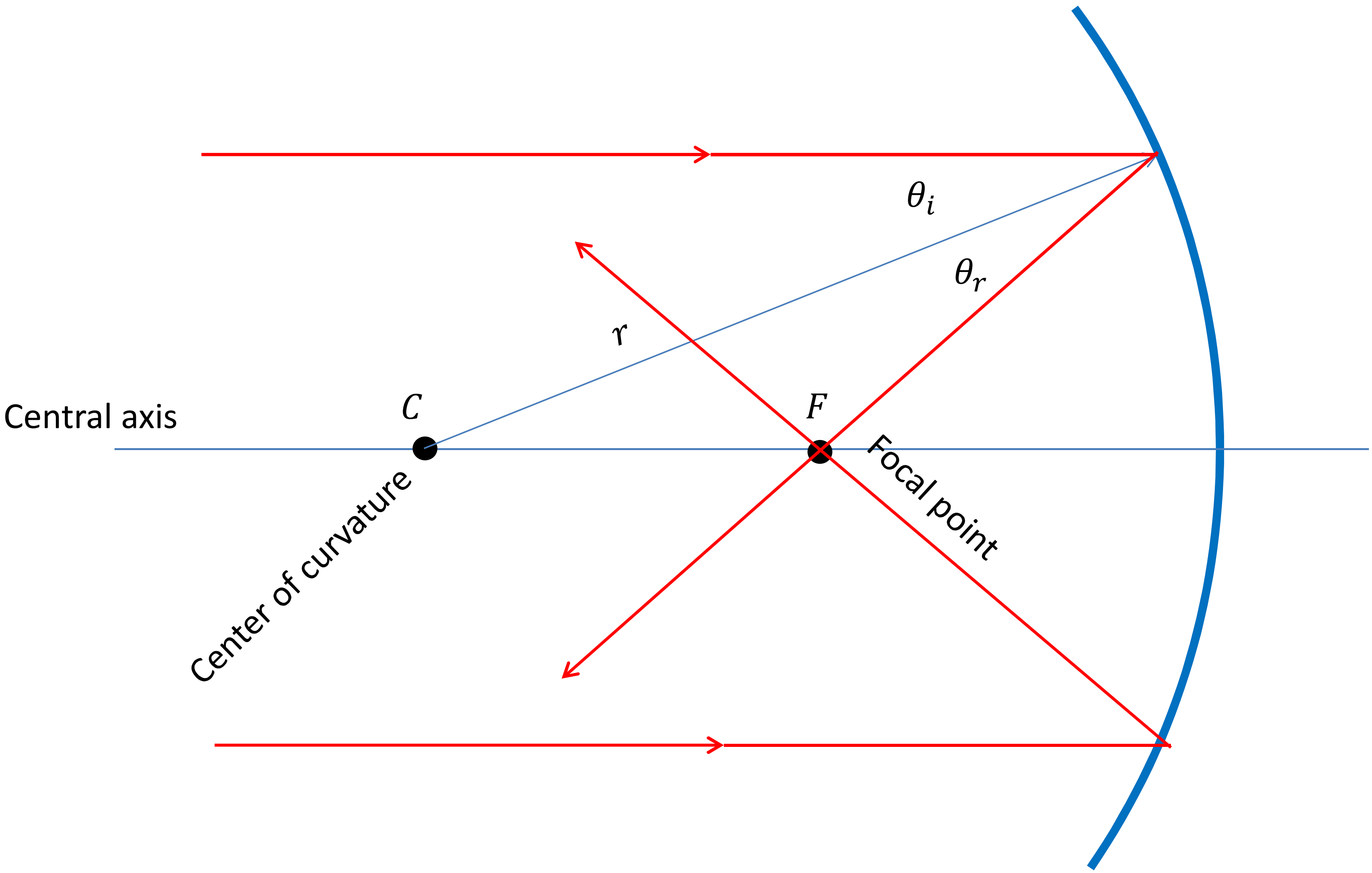
- Typical problems in geometric optics:
  - Given an optical system, what are the properties of the image that is formed (if any)?
  - What configuration of optical elements (if any) will produce an image with certain desired characteristics?
- No new physical principles: the laws of reflection and refraction are all we will use
- We need a method for analyzing these problems in a systematic an organized way

Types of Images

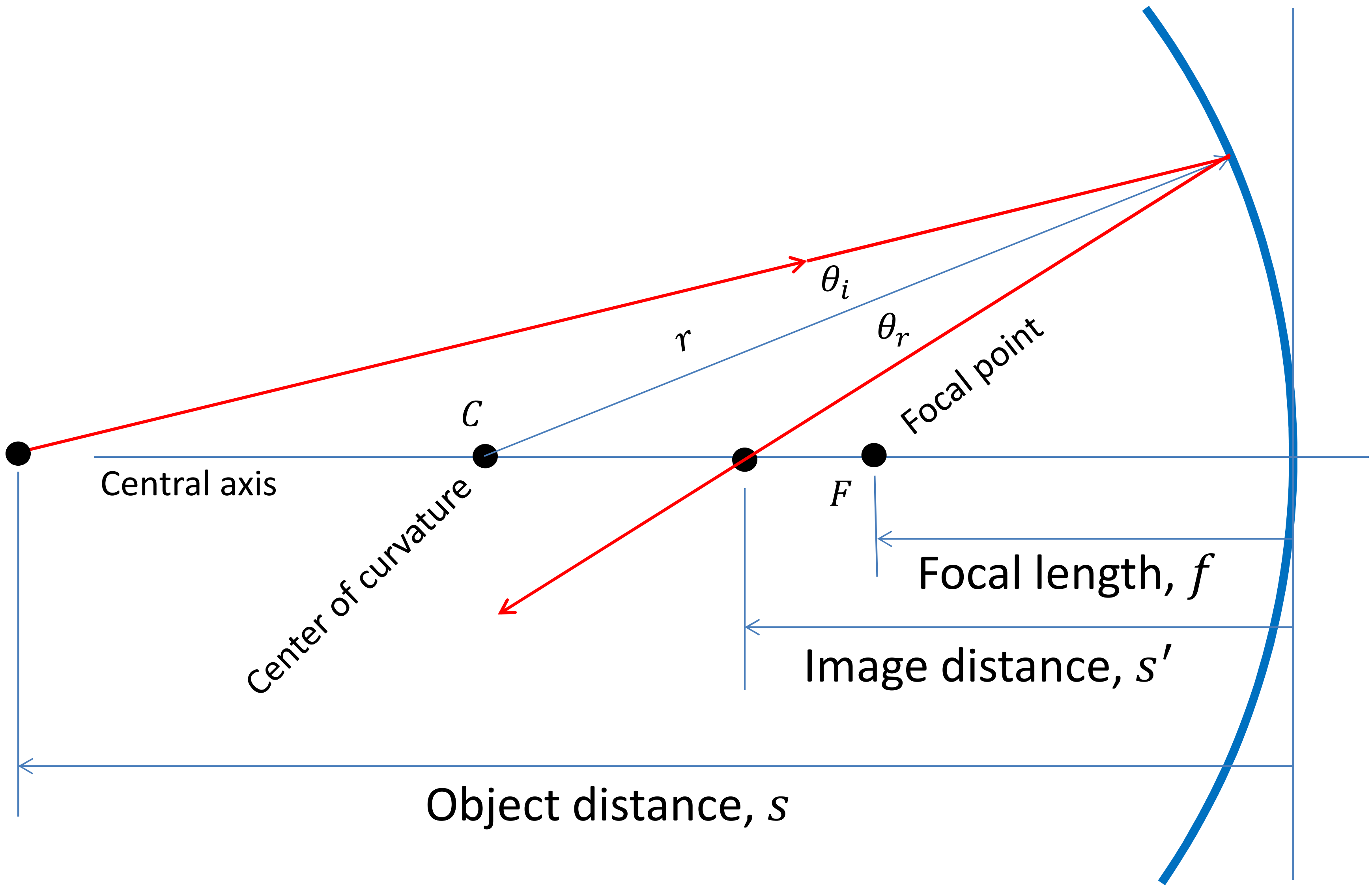
- **Real Image**: light emanates from points on the image
- **Virtual Image**: light *appears* to emanate from the image



Spherical Mirrors

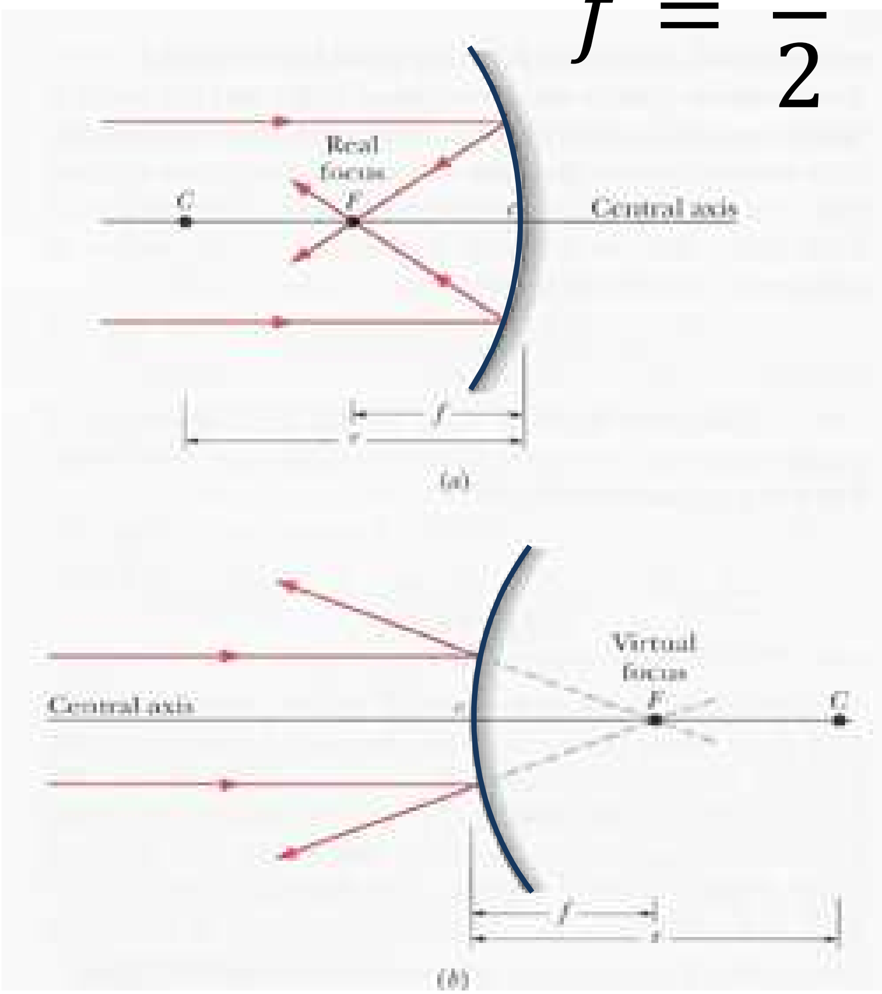


Spherical Mirrors



Focal Points of Spherical Mirrors

$$f = \frac{r}{2}$$



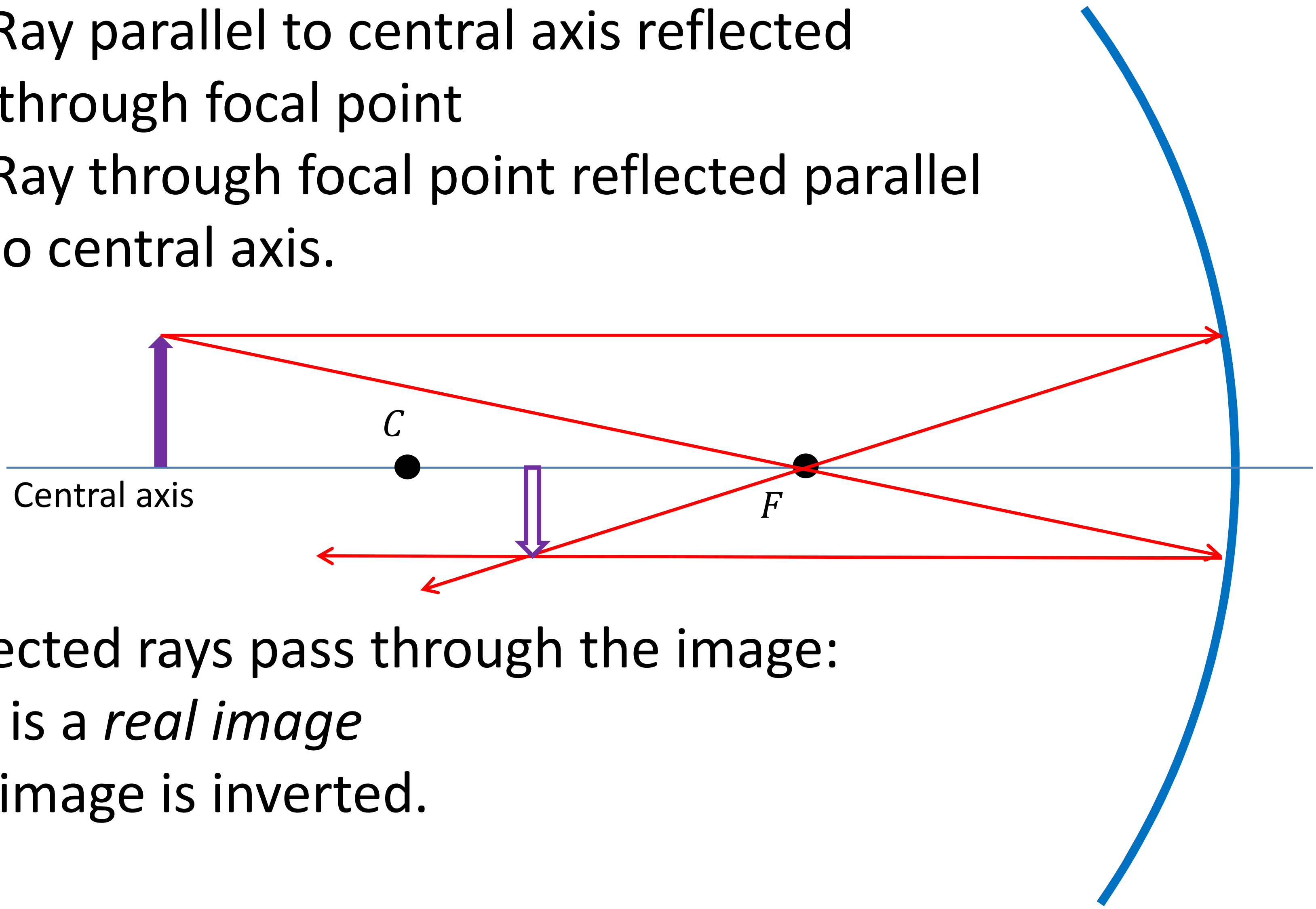
Sign convention:

- Concave:
  - Radius of curvature, $r > 0$
  - Focal length, $f > 0$
- Convex:
  - Radius of curvature, $r < 0$
  - Focal length, $f < 0$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Properties of Images

1. Ray parallel to central axis reflected through focal point
2. Ray through focal point reflected parallel to central axis.



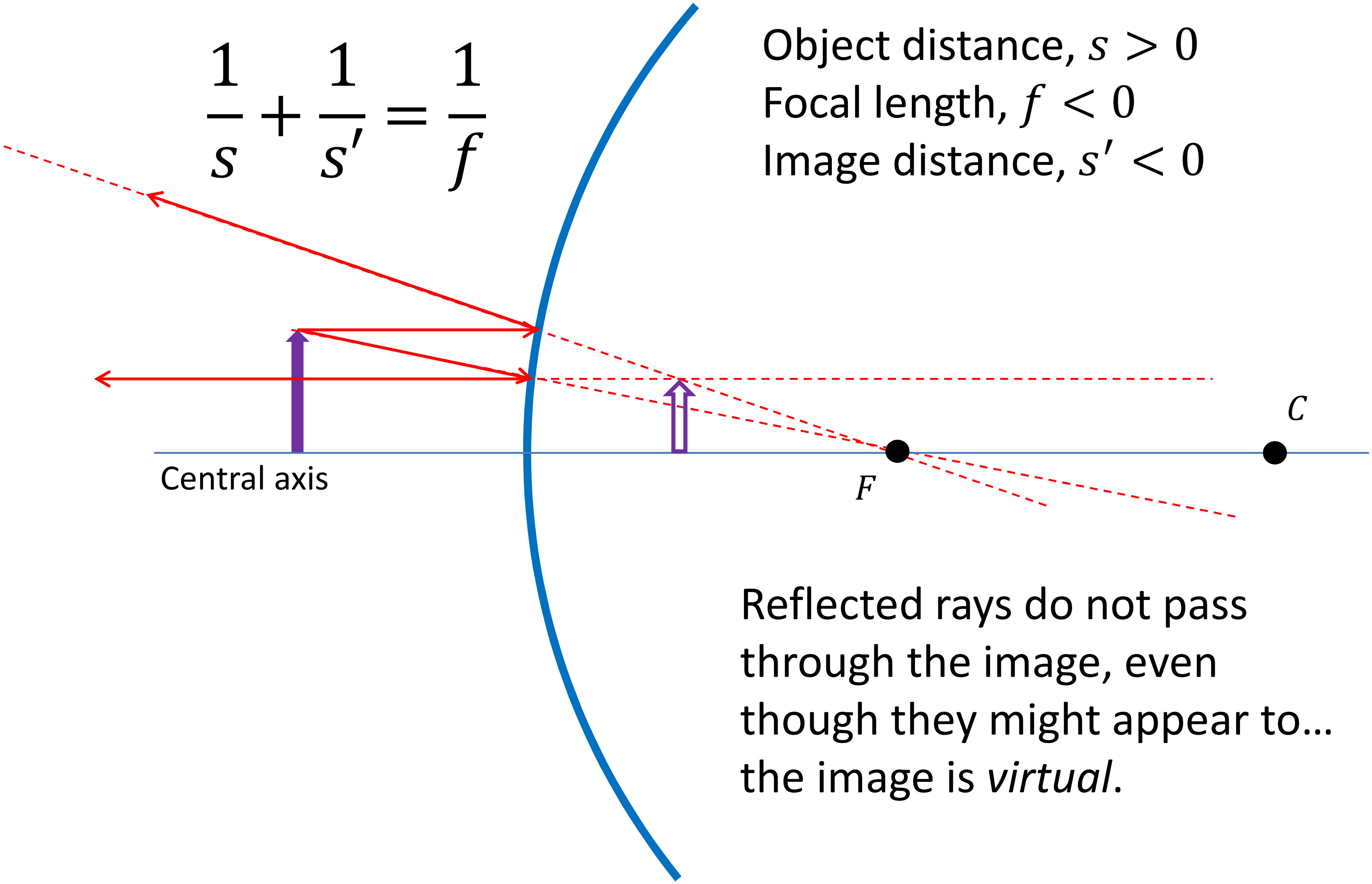
Reflected rays pass through the image:
it is a *real image*

The image is inverted.

Properties of Images

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

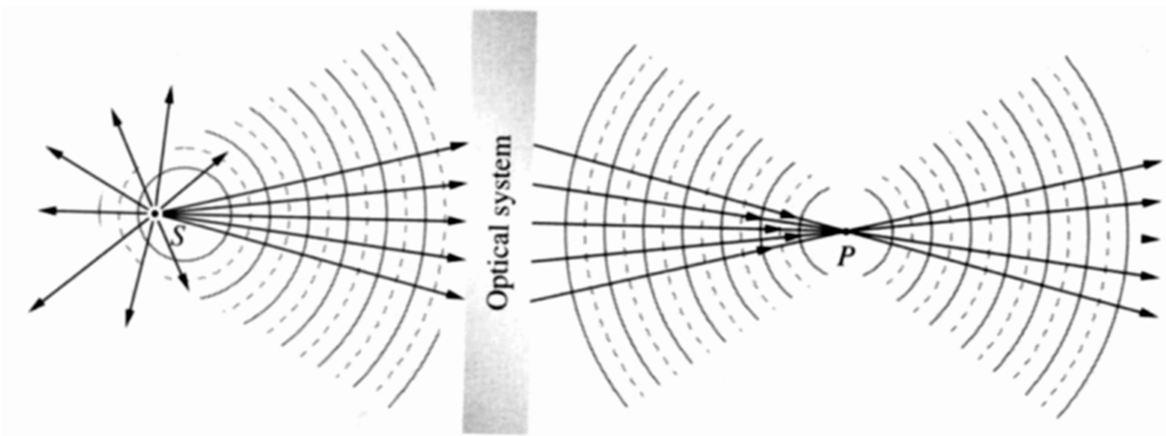
Object distance, $s > 0$
Focal length, $f < 0$
Image distance, $s' < 0$

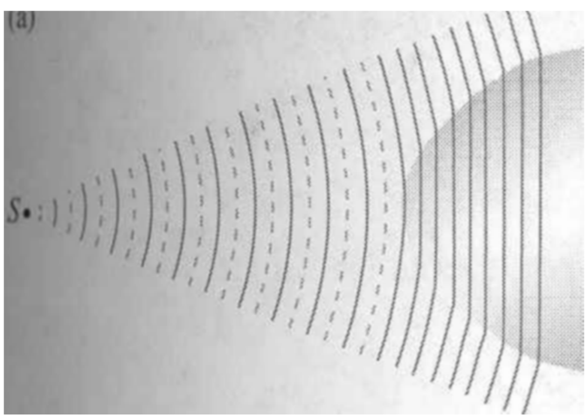


Reflected rays do not pass through the image, even though they might appear to… the image is *virtual*.

Lenses

- Insert a transparent object with $n > 1$ that is thicker in the middle and thinner at the edges

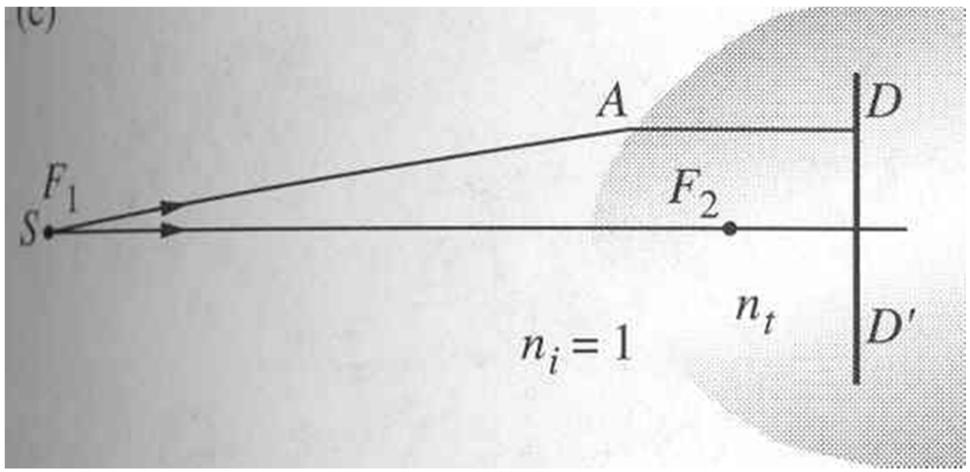




Spherical waves can be turned into plane waves.

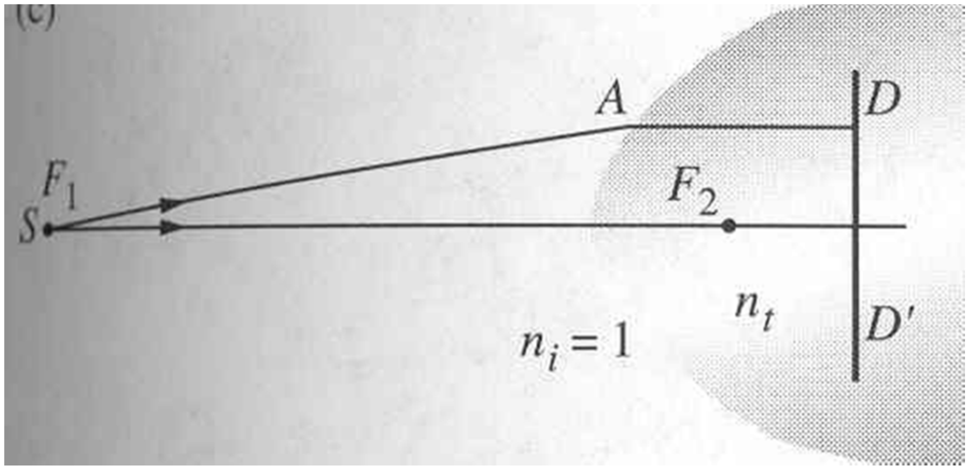
Aspherical Surfaces

- What shape of surface will change spherical waves to plane waves?



- Time to travel from S to plane DD’ must be equal for all points A on the surface.

Aspherical Surfaces

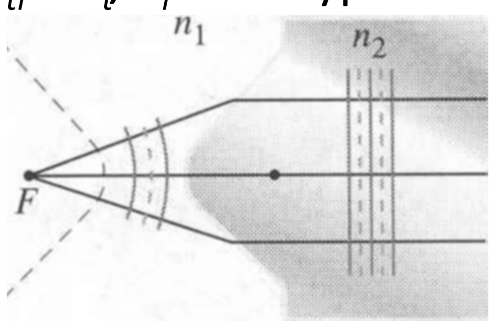


$$\frac{\overline{F_1A}}{v_i} + \frac{\overline{AD}}{v_t} = \frac{n_i(\overline{F_1A})}{c} + \frac{n_t(\overline{AD})}{c}$$

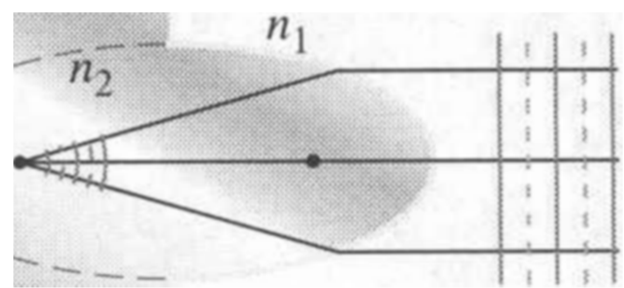
$$\overline{F_1A} + \frac{n_t}{n_i}\,\overline{AD} = \text{constant}$$

- This is the equation for a hyperbola if $n_t/n_i > 1$ and the equation for an ellipse if $n_t/n_i < 1$.

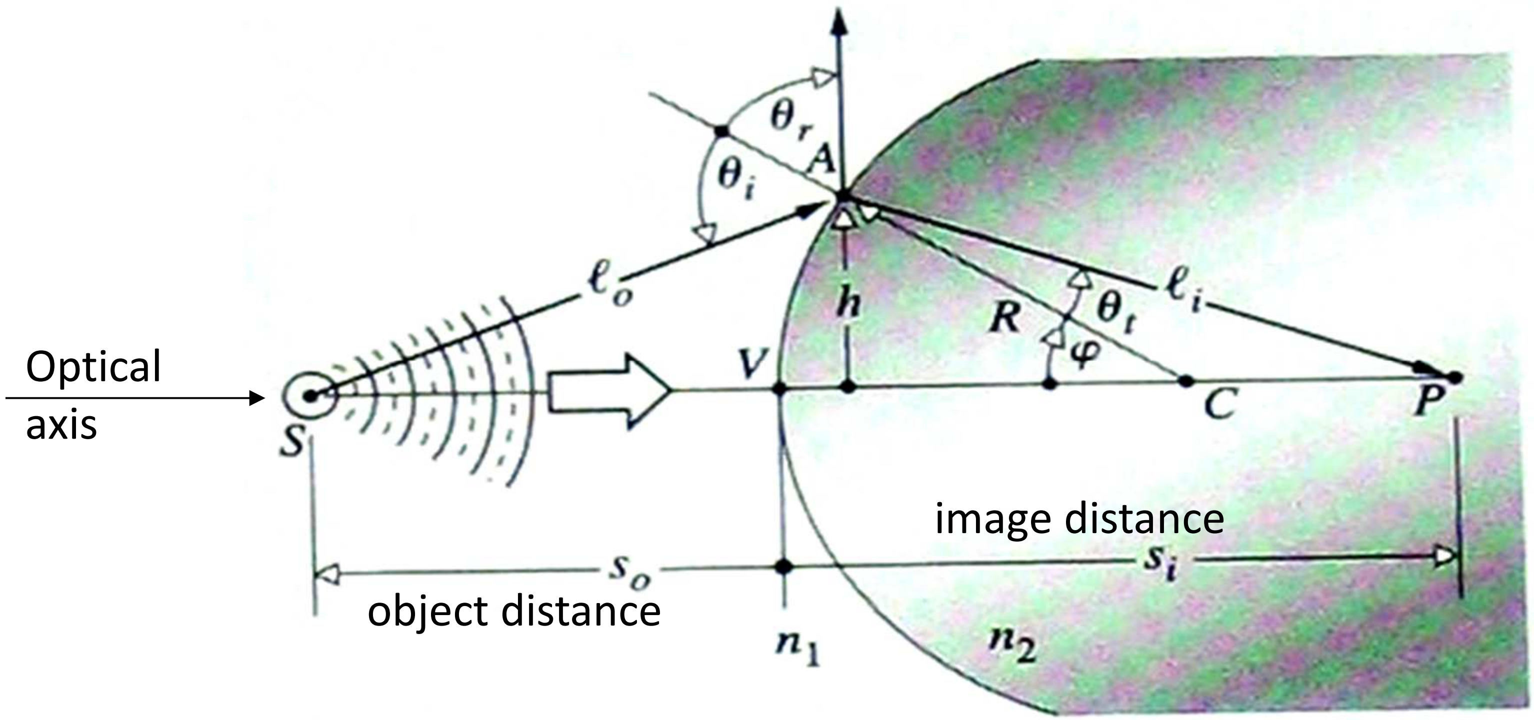
$n_{ti} \equiv n_t/n_i > 1$ - hyperbola



$n_{ti} \equiv n_t/n_i < 1$ - ellipsoid



Spherical Lens



- Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\ell_o = \sqrt{R^2 + (s_o + R)^2 - 2R(s_o + R)\cos\varphi}$$

$$\ell_i = \sqrt{R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\varphi}$$

Spherical Lens

Fermat's principle: *Light will travel on paths for which the optical path length is stationary* (ie, minimal, but possibly maximal)

$$\ell_o = \sqrt{R^2 + (s_o + R)^2 - 2R(s_o + R)\cos\varphi}$$

$$\ell_i = \sqrt{R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\varphi}$$

$$OPL = \frac{n_1 \ell_o}{c} + \frac{n_2 \ell_i}{c}$$

$$\frac{d(OPL)}{d\varphi} = \frac{n_1 R(s_o + R)\sin\varphi}{2\ell_o} - \frac{n_2 R(s_i - R)\sin\varphi}{2\ell_i} = 0$$

$$\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i} = \frac{1}{R}\left(\frac{n_2 s_i}{\ell_i} - \frac{n_1 s_o}{\ell_o}\right)$$

But P will be different for different values of φ…

Spherical Lens

$$\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i} = \frac{1}{R}\left(\frac{n_2 s_i}{\ell_i} - \frac{n_1 s_o}{\ell_o}\right)$$

- Approximations for small φ:

$$\cos\varphi = 1 \qquad \sin\varphi = \varphi$$

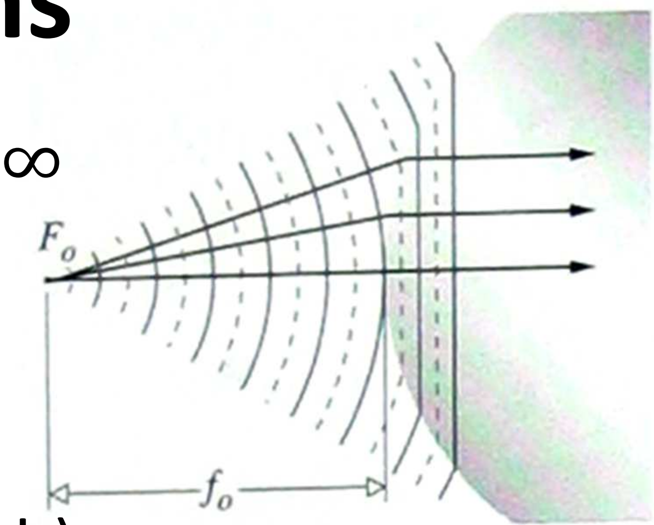
$$\ell_o = s_o \qquad \ell_i = s_i$$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

- The position of P is independent of the location of A over a small area close to the optical axis.
- **Paraxial rays**: rays that form small angles with respect to the optical axis.
- **Paraxial approximation**: consider paraxial rays only.

Spherical Lens

- For parallel transmitted rays, $s_i \to \infty$



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \to \frac{n_1}{f_o} = \frac{n_2 - n_1}{R}$$

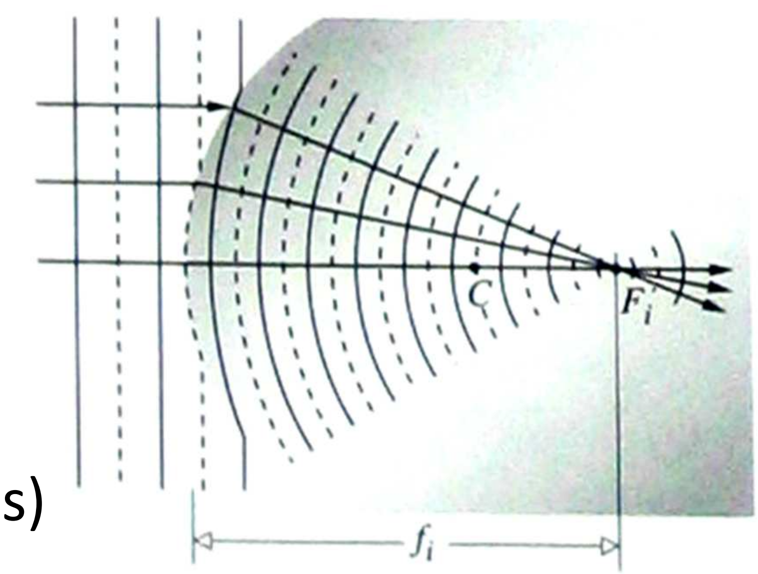
- First focal length (object focal length):

$$f_o = \frac{n_1}{n_2 - n_1} R$$

- Second focal length

(Image focal length)

$$f_i = \frac{n_2}{n_2 - n_1} R$$



$R > 0, n_2 > n_1 \to f > 0$ (converging lens)

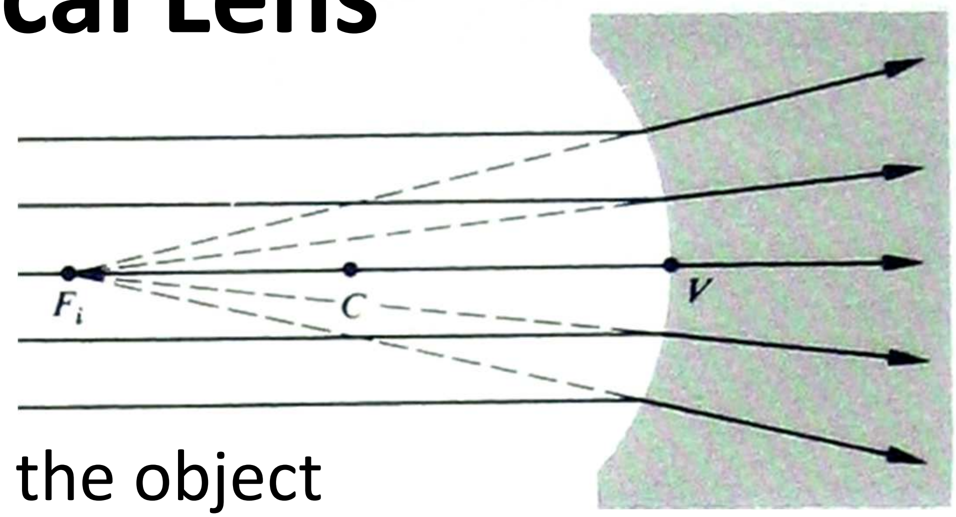
Spherical Lens

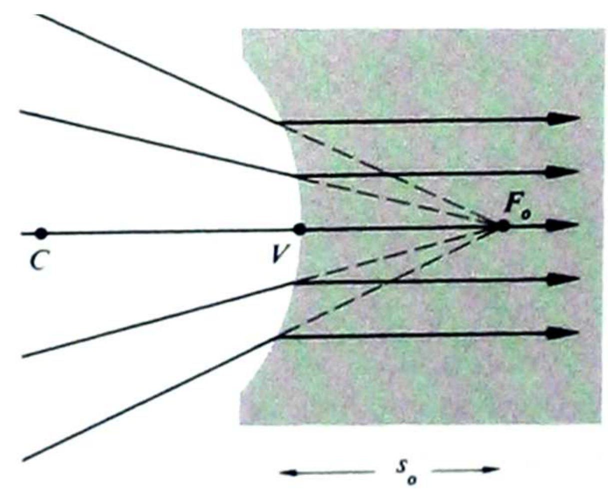
- When $R < 0$:

$$f_i = \frac{n_1}{n_2 - n_1} R$$

A virtual image appears on the object side.

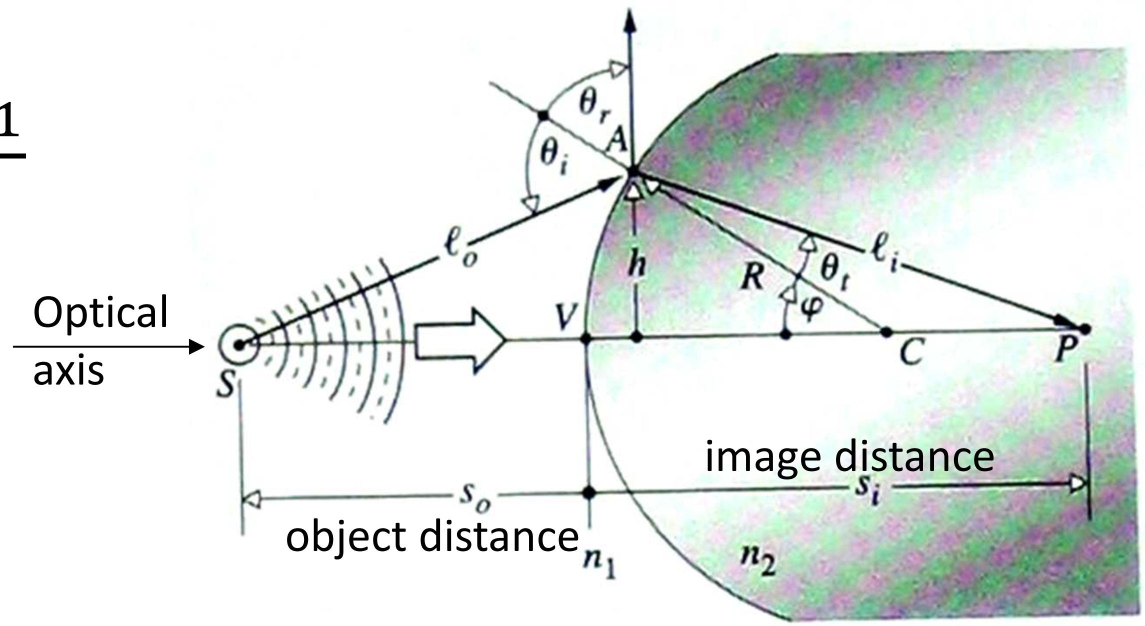
$$f_o = \frac{n_2}{n_2 - n_1} R$$





Sign Conventions

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$



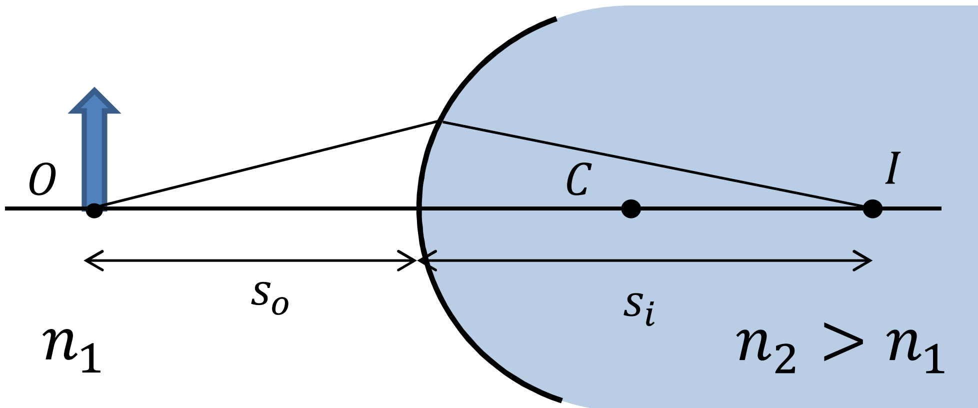
- Assuming light enters from the left:

$s_o, f_o > 0$ when left of vertex, V

$s_i, f_i > 0$ when right of vertex, V

$R > 0$ if C is on the right of vertex, V

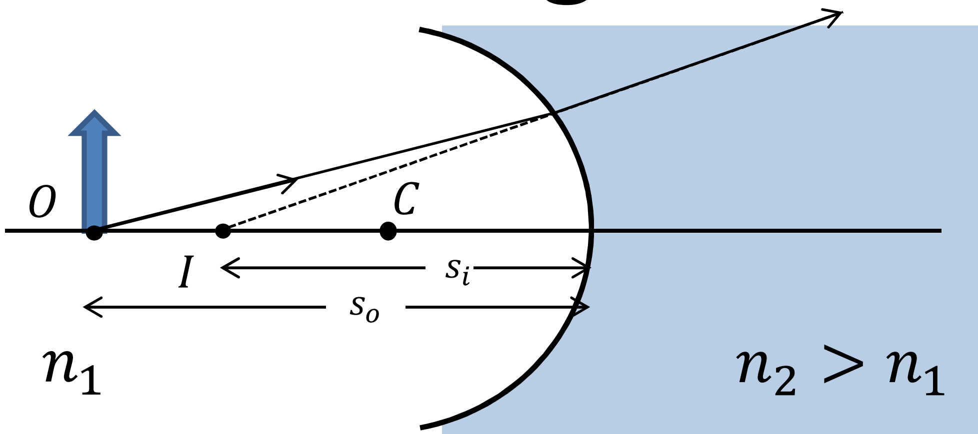
Sign Conventions



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

- Convex surface:
  - s_o is positive for objects on the incident-light side
  - s_i is positive for images on the refracted-light side
  - R is positive if C is on the refracted-light side

Sign Conventions



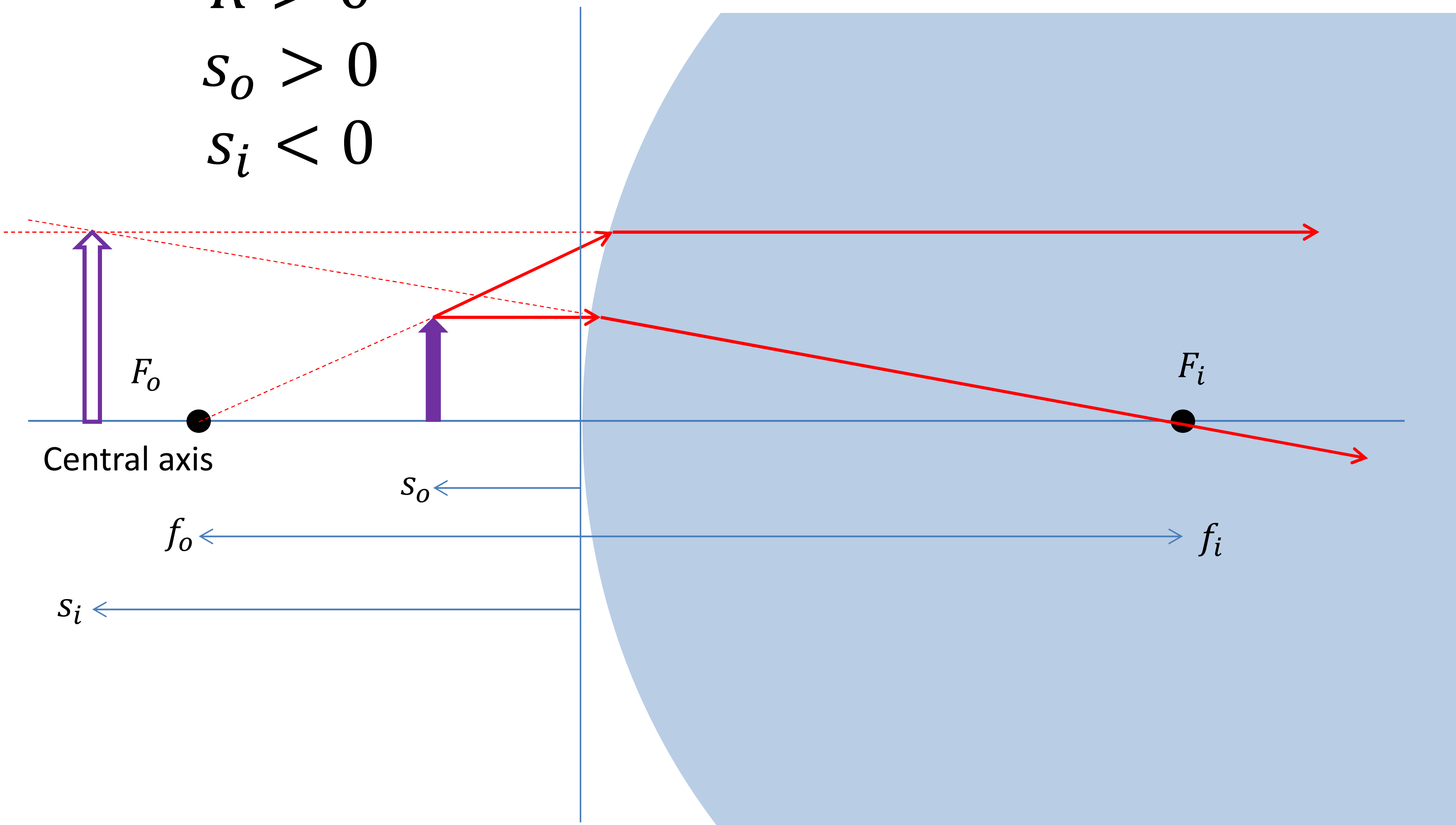
$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

(same formula)

- Concave surface:
  - s_o is positive for objects on the incident-light side
  - s_i is negative for images on the incident-light side
  - R is negative if C is on the incident-light side

Spherical Lens

$$R > 0$$
$$s_o > 0$$
$$s_i < 0$$



Magnification

- Using these sign conventions, the magnification is

$$m = -\frac{n_1 s_i}{n_2 s_o}$$

- Ratio of image height to object height
- Sign indicates whether the image is inverted