PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

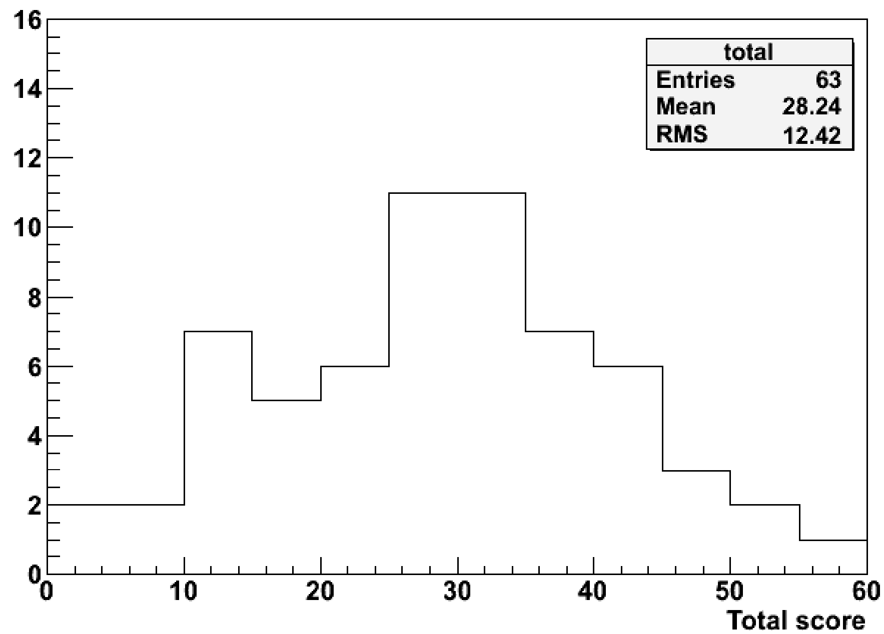
Waves & Oscillations

Lecture 26 – Propagation of Light

Spring 2013 Semester

Matthew Jones

Midterm Exam



Almost all grades have been uploaded to

http://chip.physics.purdue.edu/public/422/spring2013/

These grades have not been adjusted

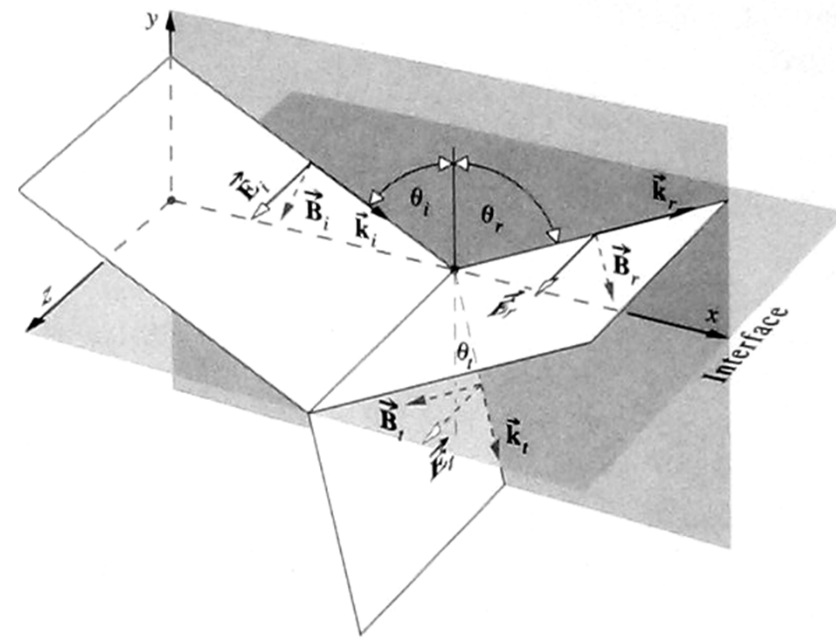
Exam questions and solutions are available on the Physics 42200 web page.

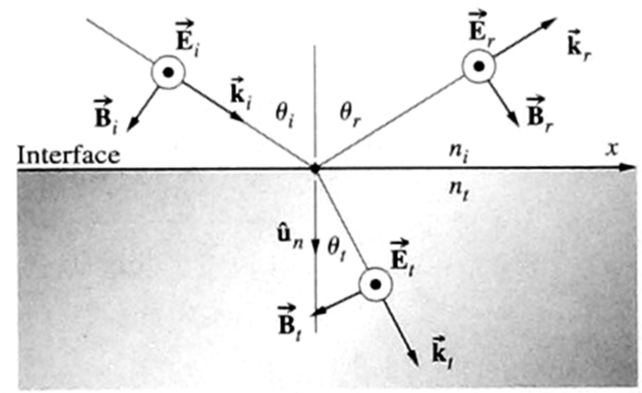
Outline for the rest of the course

- Polarization
- Geometric Optics
- Interference
- Diffraction
- Review

Polarization by Partial Reflection

- Continuity conditions for Maxwell's Equations at the boundary between two materials
- Different conditions for the components of $\vec{E}$ or $\vec{H}$ parallel or perpendicular to the surface.





Polarization by Partial Reflection

- Continuity of electric and magnetic fields were different depending on their orientation:
  - Perpendicular to surface

$$\varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$$
$$\mu_1 H_{1\perp} = \mu_2 H_{2\perp}$$

  - Parallel to surface

$$E_{1\parallel} = E_{2\parallel}$$
$$H_{1\parallel} = H_{2\parallel}$$

$\vec{\boldsymbol{E}}$ perpendicular to $\hat{\boldsymbol{n}}$

$$\frac{-E_i \cos\theta_i + E_r \cos\theta_i}{Z_1} = \frac{-E_t \cos\theta_t}{Z_2}$$

$$\frac{E_i \cos\theta_t + E_r \cos\theta_t}{Z_2} = \frac{E_t \cos\theta_t}{Z_2}$$

- Solve for E_r/E_i:

$$\boldsymbol{\frac{E_r}{E_t} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2\, cos\, \theta_i + Z_1\, cos\, \theta_t}}$$

- Solve for E_t/E_i:

$$\boldsymbol{\frac{E_t}{E_t} = \frac{2Z_2\, cos\, \theta_i}{Z_2\, cos\, \theta_i + Z_1\, cos\, \theta_t}}$$

$\overrightarrow{\boldsymbol{H}}$ perpendicular to $\widehat{\boldsymbol{n}}$

$$\frac{E_i \cos\theta_i - E_r \cos\theta_i}{Z_2} = \frac{E_t \cos\theta_t}{Z_2}$$

$$\frac{E_i \cos\theta_t + E_r \cos\theta_t}{Z_1} = \frac{E_t \cos\theta_t}{Z_2}$$

- Solve for E_r/E_i:

$$\boldsymbol{\frac{E_r}{E_t} = \frac{Z_1 \cos\theta_i - Z_2 \cos\theta_t}{Z_1\, cos\,\theta_i + Z_2\, cos\,\theta_t}}$$

- Solve for E_t/E_i:

$$\boldsymbol{\frac{E_t}{E_t} = \frac{2Z_1\, cos\,\theta_i}{Z_1\, cos\,\theta_i + Z_2\, cos\,\theta_t}}$$

Fresnel's Equations

- In most dielectric media, $\mu_1 = \mu_2$ and therefore

$$\frac{Z_1}{Z_2} = \sqrt{\frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1} = \frac{\sin\theta_i}{\sin\theta_t}$$

- After some trigonometry…

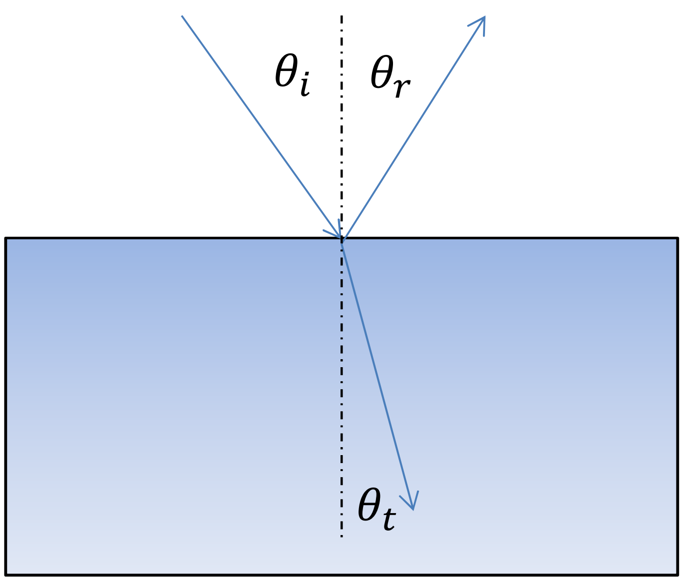
$$\left(\frac{E_r}{E_i}\right)_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad \left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\left(\frac{E_t}{E_i}\right)_{\perp} = \frac{(n_1/n_2)\sin(2\theta_i)}{\sin(\theta_i+\theta_t)} \qquad \left(\frac{E_t}{E_i}\right)_{\parallel} = \frac{2\cos(\theta_i)\sin(\theta_t)}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

For $\vec{E}$ perpendicular and parallel to plane of incidence.

Application of Fresnel's Equations

- Unpolarized light in air ($n = 1$) is incident on a surface with index of refraction $n' = 1.5$ at an angle $\theta_i = 30°$
- What are the magnitudes of the electric field components of the reflected light?



First calculate the angles of reflection and refraction…

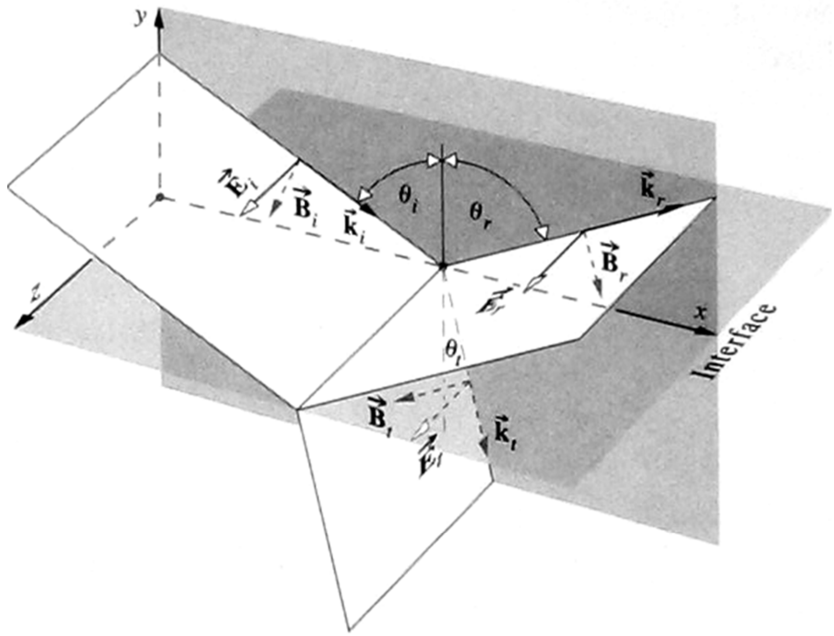
$$\boldsymbol{\theta_r = \theta_i = 30°}$$

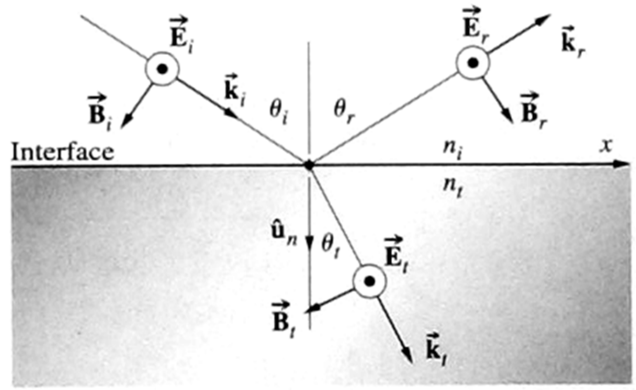
$$\sin\theta_i = n' \sin\theta_t$$

$$\sin\theta_t = \frac{\sin 30°}{1.5} = 0.333$$

$$\boldsymbol{\theta_t = 19.5°}$$

Application of Fresnel's Equations





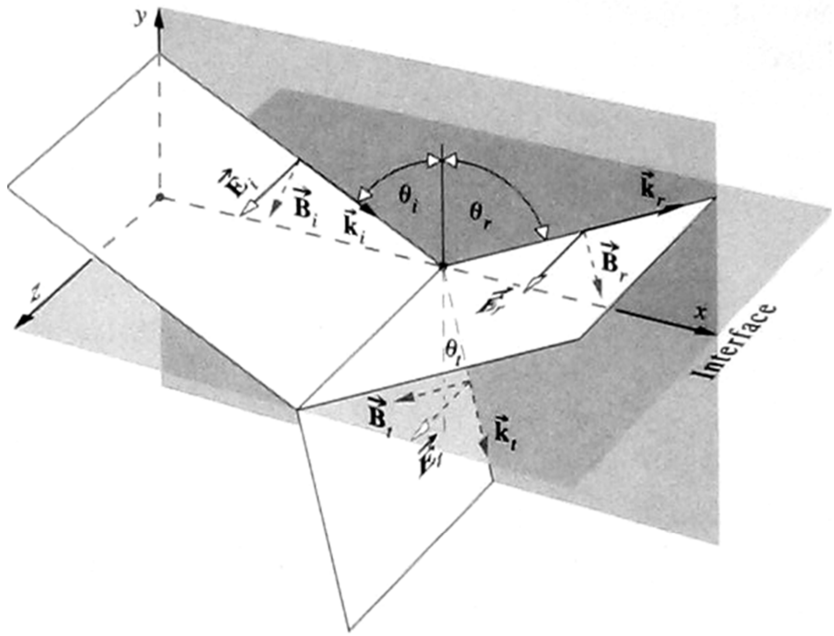
- Component of $\vec{E}$ parallel to the surface is perpendicular to the plane of incidence

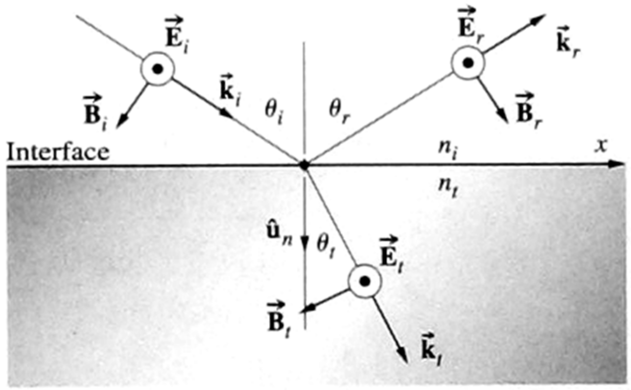
$$\left(\frac{E_r}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$= -\frac{\sin(30° - 19.5°)}{\sin(30° + 19.5°)}$$

$$= -0.240$$

Application of Fresnel's Equations





- Component of $\vec{E}$ perpendicular to the surface is parallel to the plane of incidence

$$\left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$= \frac{\tan(30° - 19.5°)}{\tan(30° + 19.5°)}$$

$$= 0.158$$

- Reflected light is preferentially polarized parallel to the surface.

Reflected Components

- Since $\theta_t < \theta_i$ the component perpendicular to the plane of incidence is always negative:

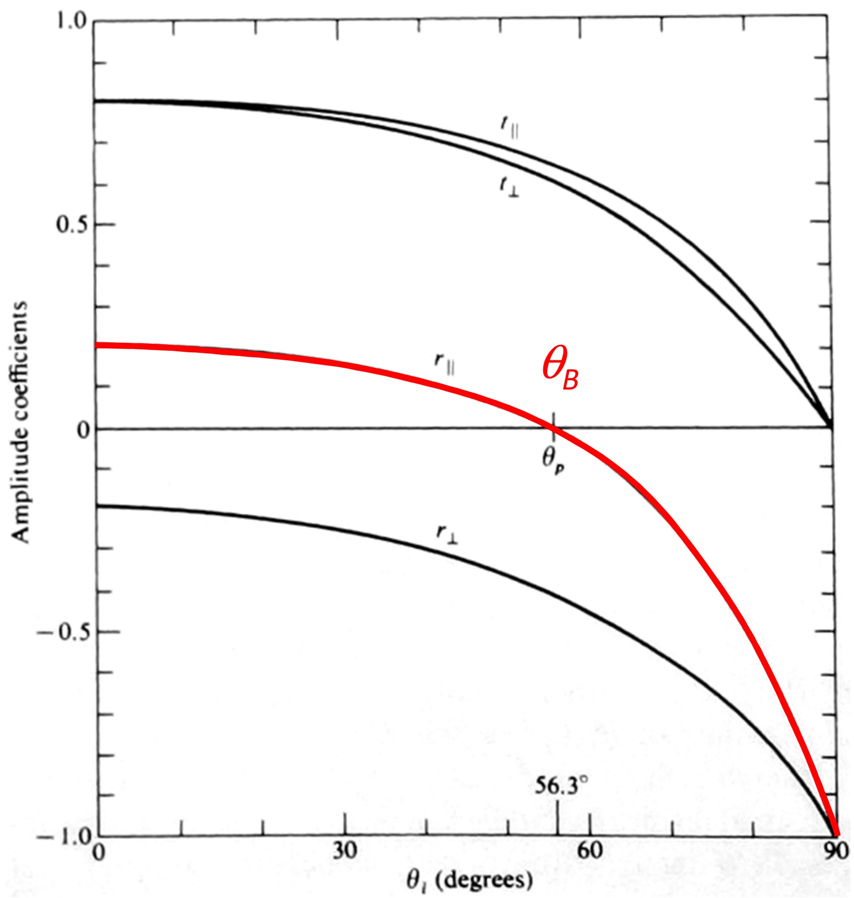
$$\left(\frac{E_r}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

- The component parallel to the plane of incidence could be positive or negative:

$$\left(\frac{E_r}{E_i}\right)_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

- What happens when $\theta_i + \theta_t = 90°$?
  - Can this happen? Sure… check when $\theta_i \to 90°$.

Brewster's Angle



$\tan\theta \to \infty$ as $\theta \to 90°$ while $\tan(\theta_i - \theta_t)$ remains finite.

Therefore,

$$\left(\frac{E_r}{E_i}\right)_{\|} \to 0$$

Brewster's angle, θ_B is the angle of incidence for which $r_{\|} \to 0$.

Brewster's Angle

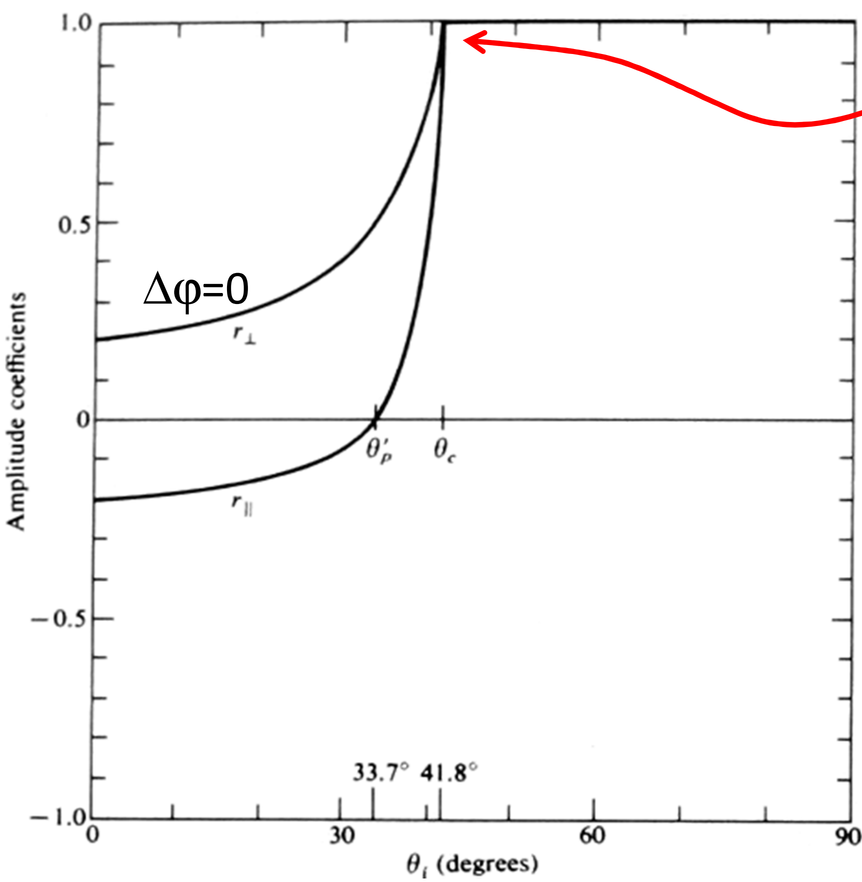
- Brewster's angle can be calculated from the relation $\theta_i + \theta_t = 90°$
- We can always calculate θ_t using Snell's law
- Good assignment question:

$$\textit{Show that } \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

- Light reflected from a surface at an angle θ_B will be linearly polarized parallel to the surface (perpendicular to the plane of incidence)

Total Internal Reflection

- Consider the other case when $n_i > n_t$, for example, glass to air:



At some incidence angle (critical angle θ_c) everything is reflected (and nothing transmitted).

It can be shown that for any angle larger than θ_c no waves are transmitted into media: **total internal reflection**.

No phase shift upon reflection.

Reflected Intensity

- Remember that the *intensity* (*irradiance*) is related to the energy carried by light:

$$I = \epsilon v \langle E^2 \rangle_T$$

(averaged over some time $T \gg 1/f$)

- Reflectance is defined as

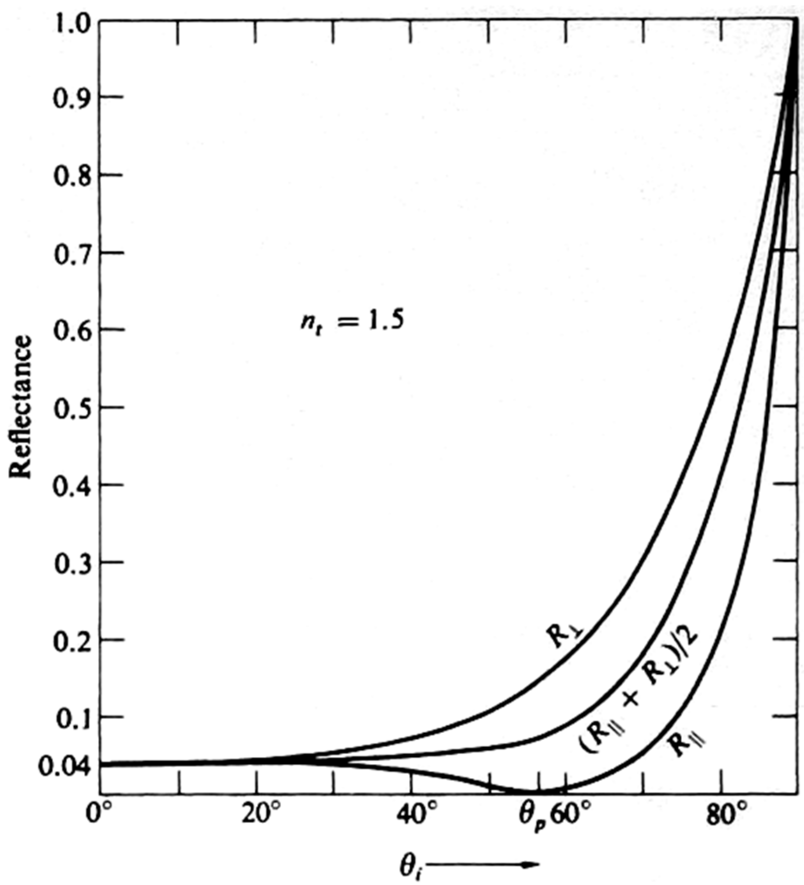
$$R_\perp = (r_\perp)^2 = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)}$$

$$R_\parallel = (r_\parallel)^2 = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}$$

- Unpolarized reflectance:

$$R = \frac{1}{2}(R_\perp + R_\parallel)$$

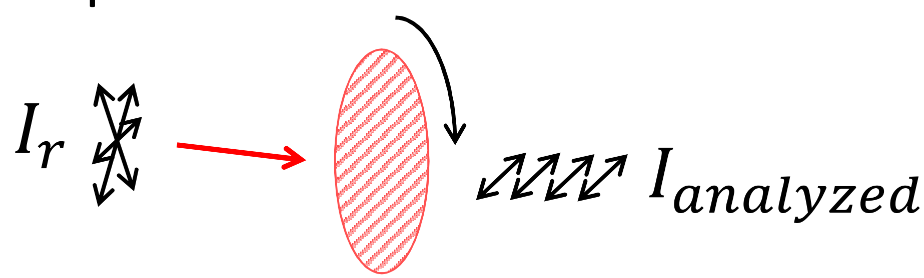
Reflected Intensity



- How polarized is the reflected light?
- Degree of polarization:

$$V = I_p / I_{total}$$

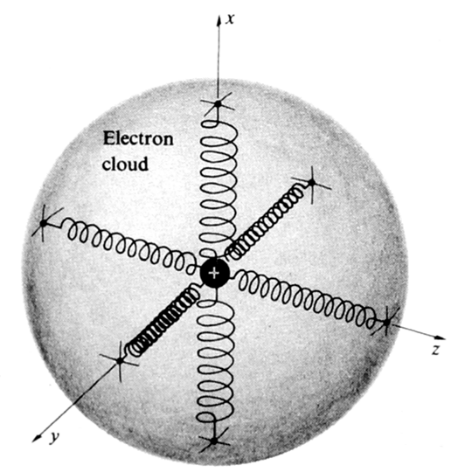
- Measured using an analyzing polarizer

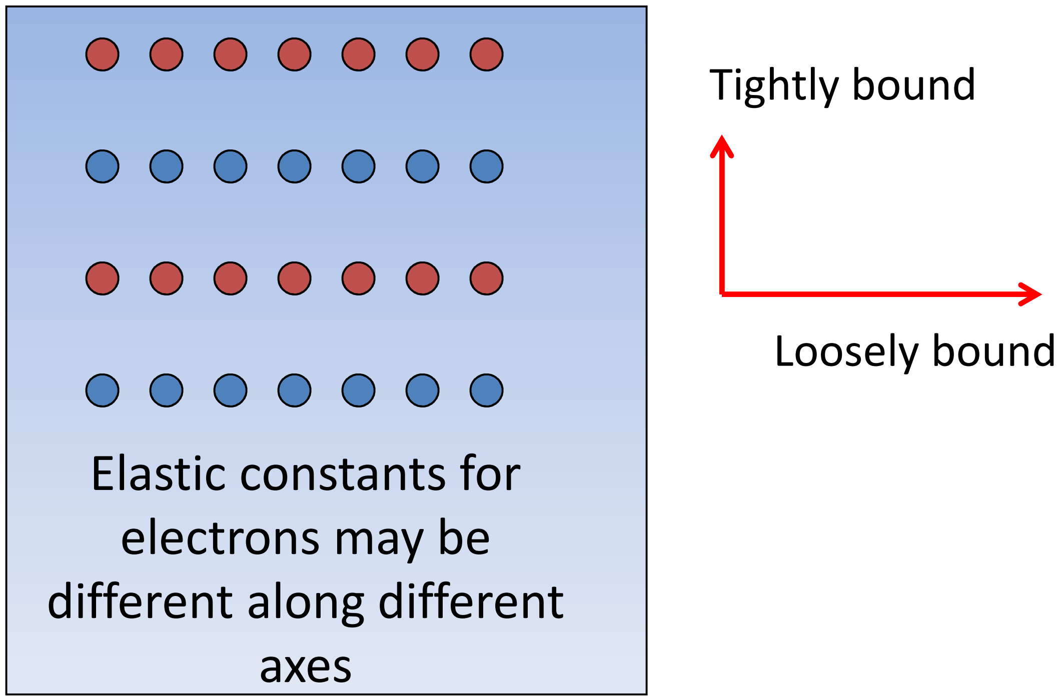


$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Birefringence

- In a crystal with a regular, repeating atomic lattice, electrons can be more tightly bound along certain axes:





Birefringence

- Electric fields oriented parallel or perpendicular to the planes in the crystal lattice interact with electrons differently.
- Depend on the atomic crystal structure
- Some crystals absorb light polarized along one axis:

Dichroic crystal: absorbs light polarized along one axis

- Other crystals transmit light, but the index of refraction depends on its polarization

Optic axis: the direction of linear polarization that differs from the other two axes

  – Assuming only one axis is special, the other two are the same.

Calcite ($CaCO_3$)

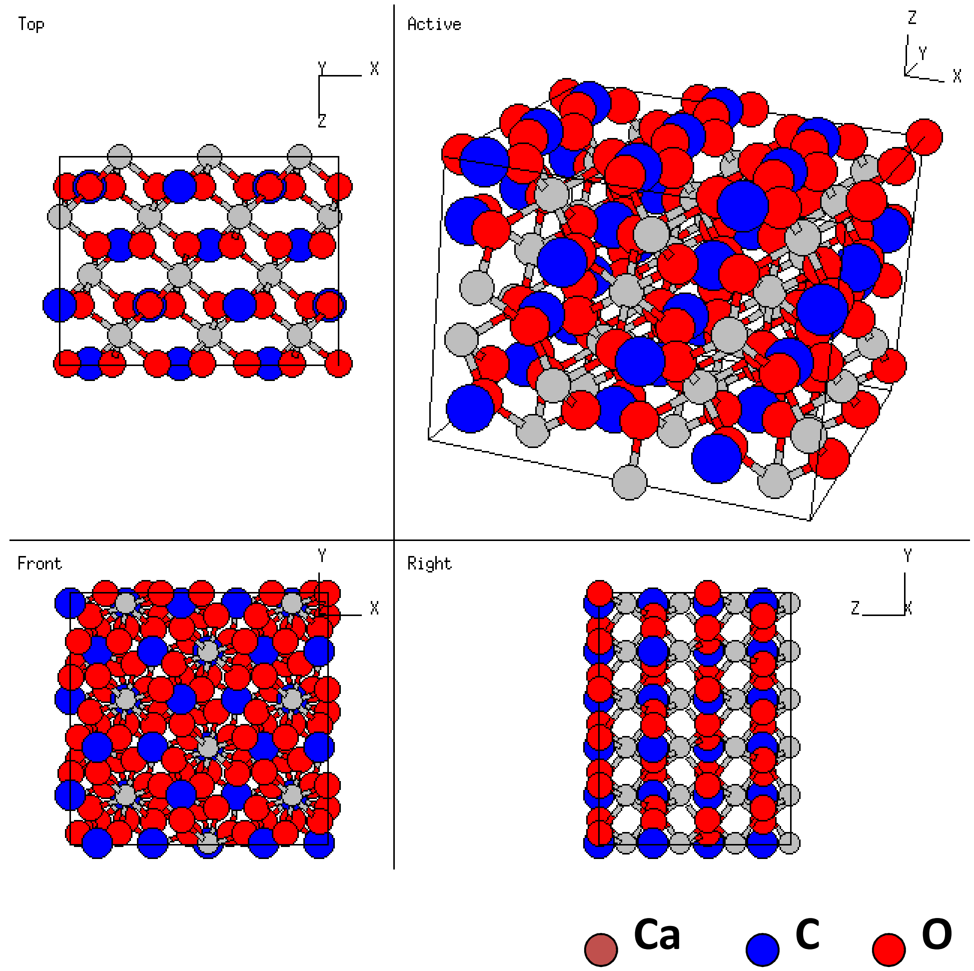
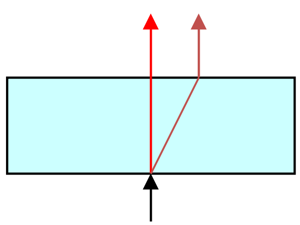


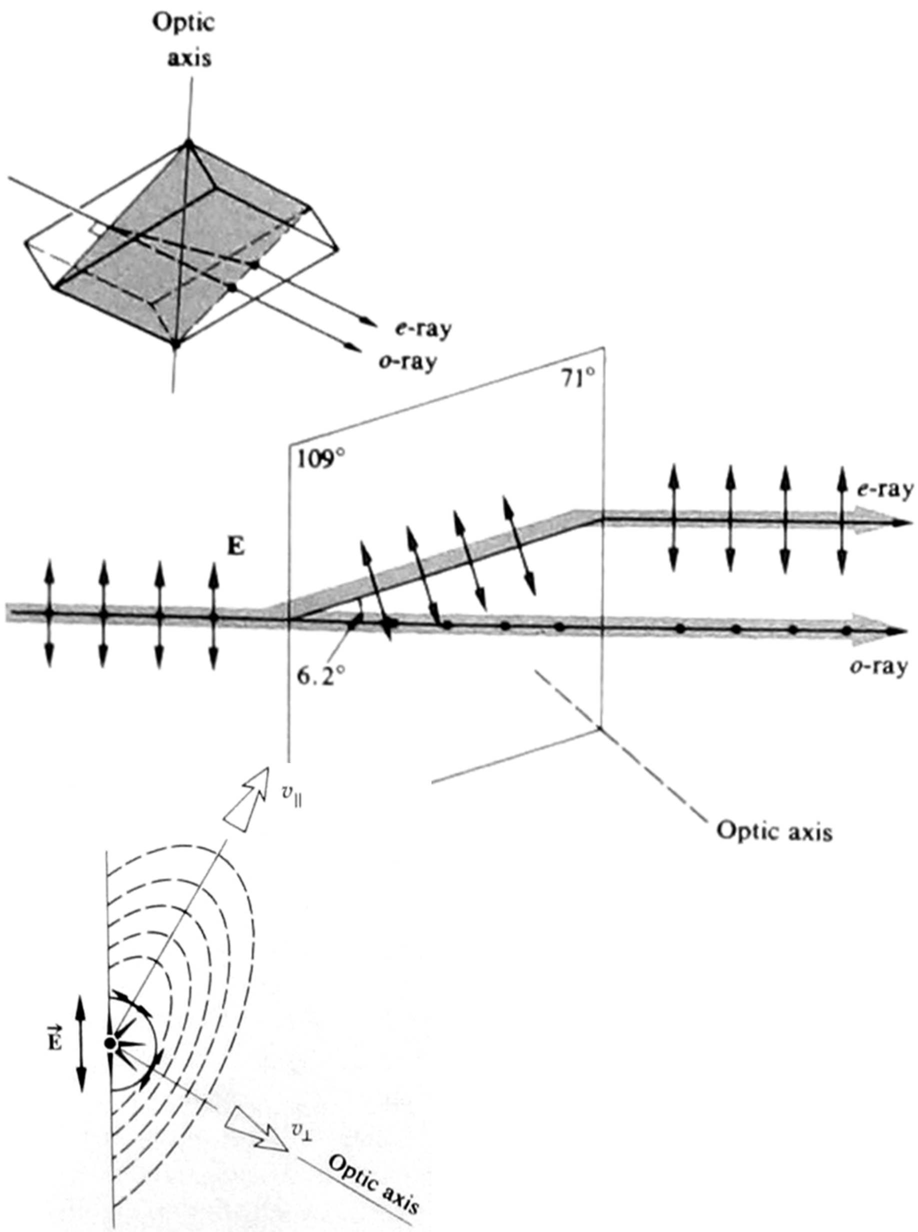
Image doubles:

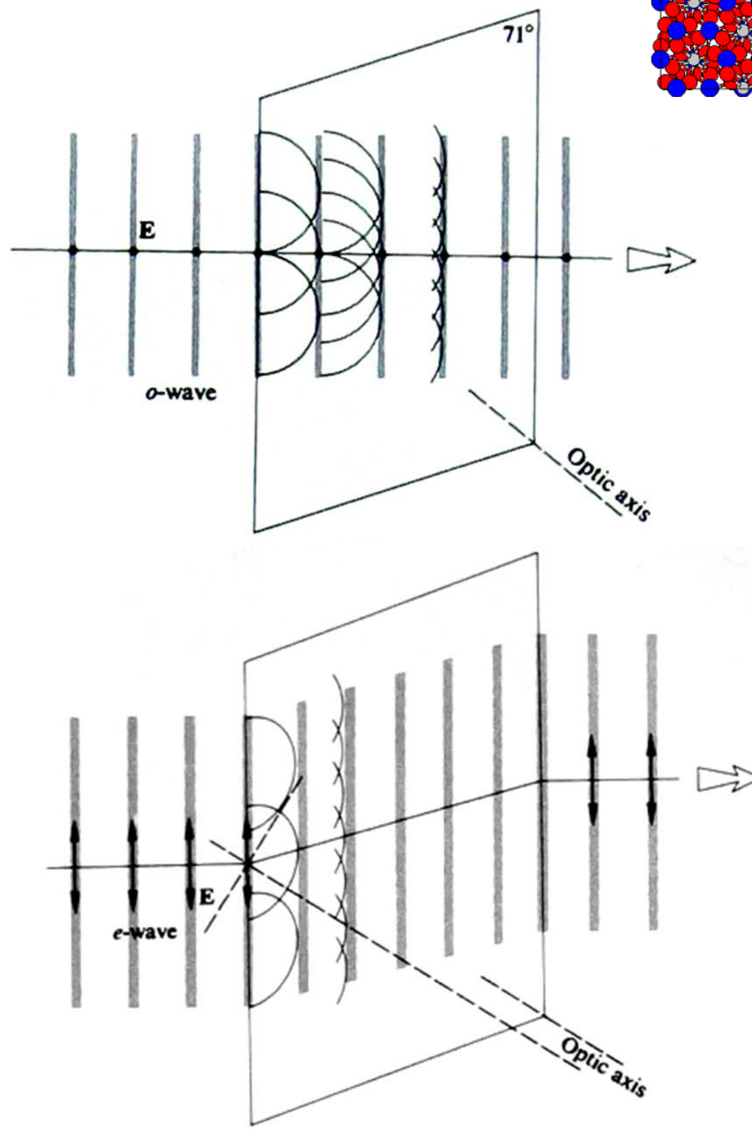


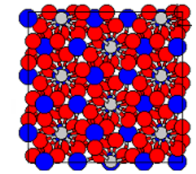
Ordinary rays (o-rays) - unbent

Extraordinary rays (e-rays) - bend

Birefringence and Huygens' Principle







Useful Applications

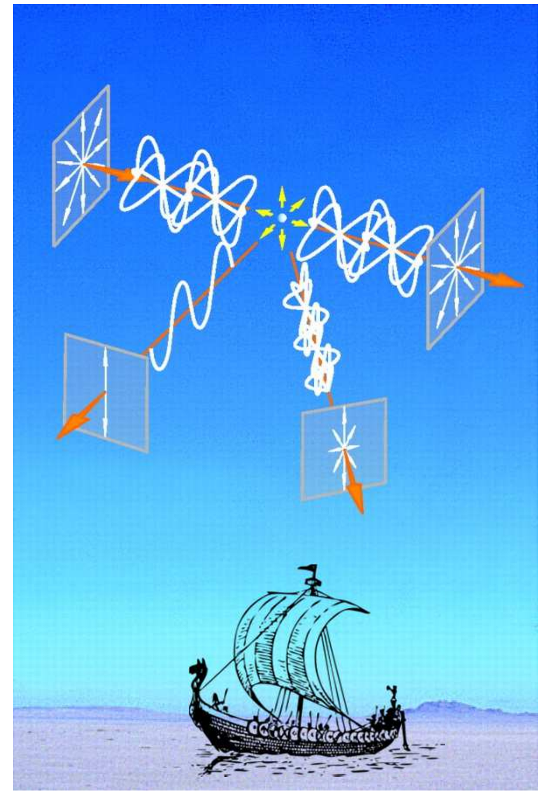
Categorized | Art, Iceland, Science

Stone found in English Channel could be Icelandic 'sunstone'

Posted on 14 March 2013. Tags: English Channel, Sun stone, sunstone, Vikings

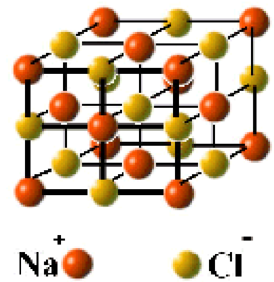
A crystal recovered from a shipwreck in the English Channel may turn out to be the fabled 'sunstone' said to be used by the Vikings. According to ancient folklore, the Vikings used a crystal called sunstone to help navigate during cloudy weather. However, it has long been debated on weather or not the substance actually exists.

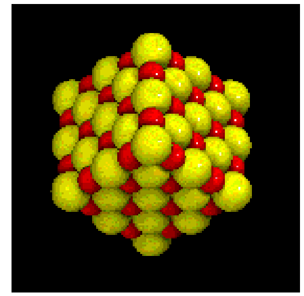
But researchers from the British-French group the Alderney Maritime Trust said in a new paper published the Proceedings of the Royal Society A journal that a large piece of Icelandic calcite recovered from the 16th century shipwreck might in fact be the fabled stone.



Crystal Structure and Birefringence

NaCl - cubic crystal

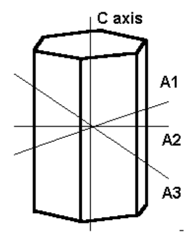


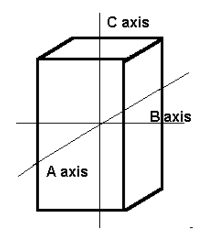


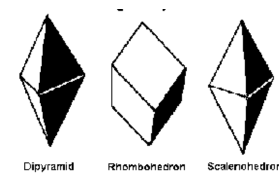
Four 3-fold symmetry axes - optically isotropic

Anisotropic, uniaxial birefringent crystals: hexagonal, tetragonal, trigonal

Uniaxial crystal: atoms are arranged symmetrically around optic axis







O-wave - $\vec{E}$ everywhere is perpendicular to the optic axis, $n_o \equiv c/v_\perp$

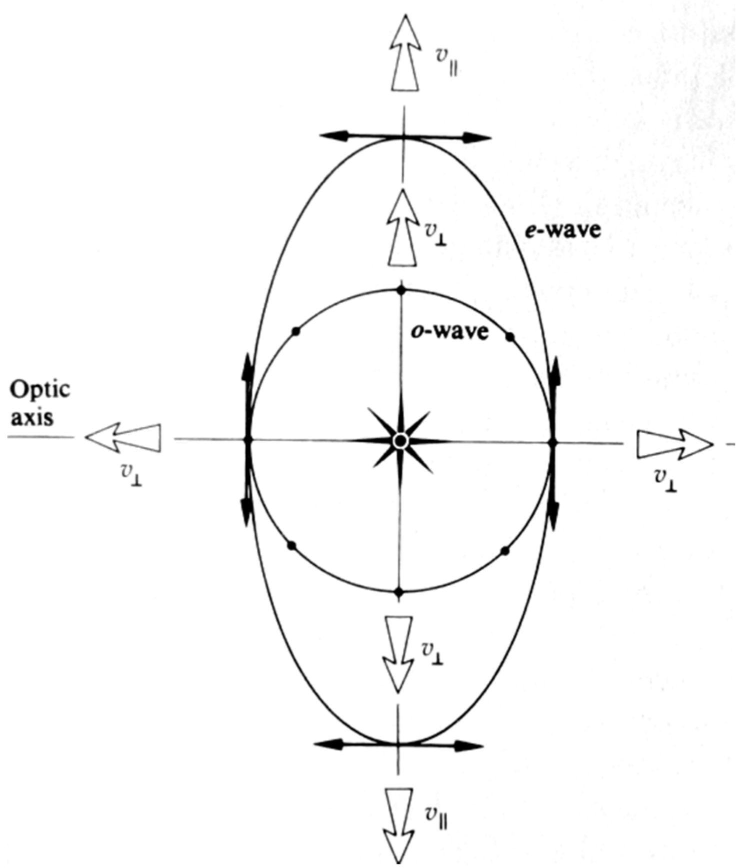
When $\vec{E}$ is parallel to optic axis: $n_e \equiv c/v_{||}$

Birefringence $\equiv n_e - n_o$

principal indices of refraction

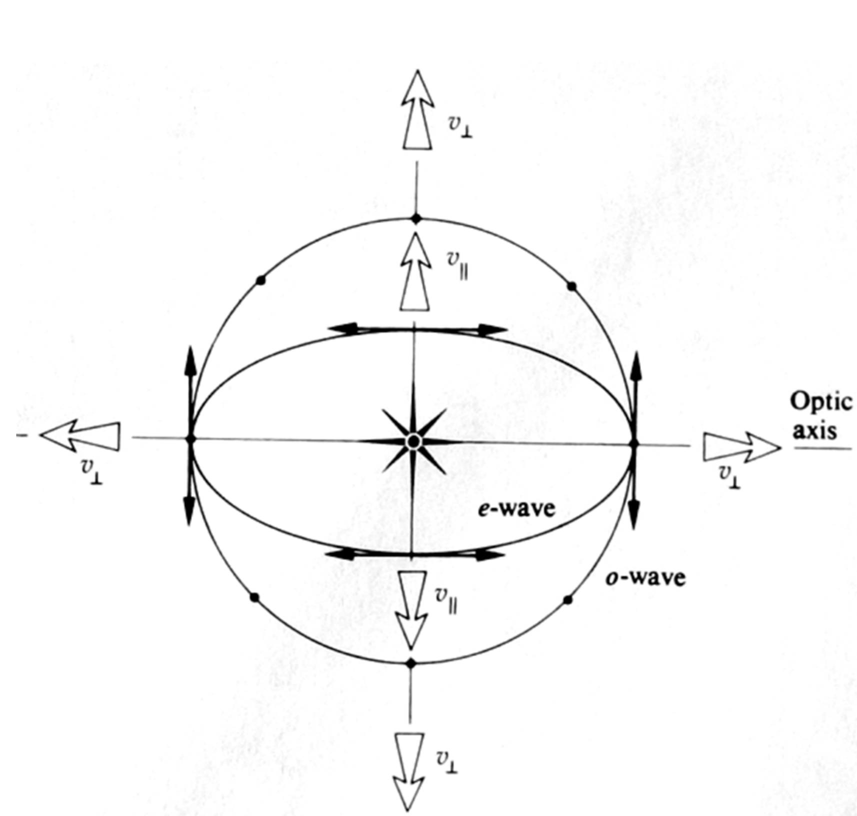
Calcite: $(n_e - n_o) = -0.172$ (negative uniaxial crystal)

Crystal Structure and Birefringence



Negative uniaxial crystal

$$n_e < n_o$$
$$\frac{c}{n_e} > \frac{c}{n_o}$$

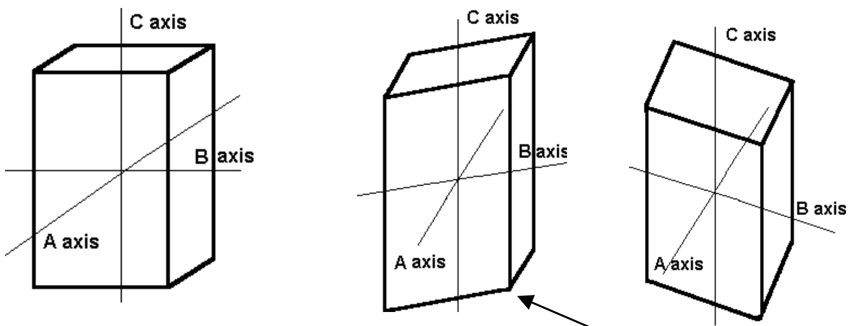


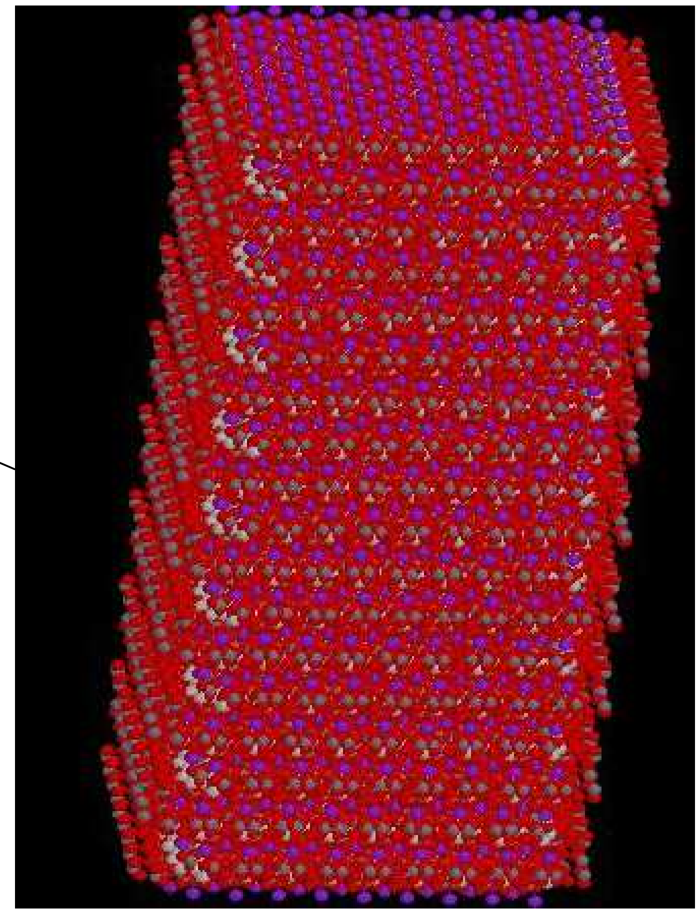
Positive uniaxial crystal

$$n_e > n_o$$
$$\frac{c}{n_e} > \frac{c}{n_o}$$

Biaxial Crystals

Two optic axes and three principal indices of refraction
Orthorhombic, monoclinic, triclinic





Example: mica, $KH_2Al_3(SiO_4)_3$