Physics 42200
Waves & Oscillations

Lecture 26 – Propagation of Light

Spring 2013 Semester
Almost all grades have been uploaded to http://chip.physics.purdue.edu/public/422/spring2013/

These grades have not been adjusted

Exam questions and solutions are available on the Physics 42200 web page.
Outline for the rest of the course

• Polarization
• Geometric Optics
• Interference
• Diffraction
• Review
Polarization by Partial Reflection

- Continuity conditions for Maxwell’s Equations at the boundary between two materials
- Different conditions for the components of $\vec{E}$ or $\vec{H}$ parallel or perpendicular to the surface.
Polarization by Partial Reflection

• Continuity of electric and magnetic fields were different depending on their orientation:
  – Perpendicular to surface
    \[ \varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp} \]
    \[ \mu_1 H_{1\perp} = \mu_2 H_{2\perp} \]
  – Parallel to surface
    \[ E_{1\parallel} = E_{2\parallel} \]
    \[ H_{1\parallel} = H_{2\parallel} \]
$\vec{E}$ perpendicular to $\hat{n}$

\[
\frac{-E_i \cos \theta_i + E_r \cos \theta_i}{Z_1} = \frac{-E_t \cos \theta_t}{Z_2} \\
\frac{E_i \cos \theta_t + E_r \cos \theta_t}{Z_2} = \frac{E_t \cos \theta_t}{Z_2}
\]

- Solve for $E_r/E_i$:
  \[
  \frac{E_r}{E_t} = \frac{Z_2 \cos \theta_i - Z_1 \cos \theta_t}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}
  \]

- Solve for $E_t/E_i$:
  \[
  \frac{E_t}{E_t} = \frac{2Z_2 \cos \theta_i}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}
  \]
\( \vec{H} \) perpendicular to \( \hat{n} \)

\[
\frac{E_i \cos \theta_i - E_r \cos \theta_i}{Z_2} = \frac{E_t \cos \theta_t}{Z_2}
\]

\[
\frac{E_i \cos \theta_t + E_r \cos \theta_t}{Z_1} = \frac{E_t \cos \theta_t}{Z_2}
\]

- Solve for \( E_r/E_i \):

\[
\frac{E_r}{E_t} = \frac{Z_1 \cos \theta_i - Z_2 \cos \theta_t}{Z_1 \cos \theta_i + Z_2 \cos \theta_t}
\]

- Solve for \( E_t/E_i \):

\[
\frac{E_t}{E_t} = \frac{2Z_1 \cos \theta_i}{Z_1 \cos \theta_i + Z_2 \cos \theta_t}
\]
Fresnel’s Equations

- In most dielectric media, $\mu_1 = \mu_2$ and therefore

$$\frac{Z_1}{Z_2} = \sqrt{\frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1} = \frac{\sin \theta_i}{\sin \theta_t}$$

- After some trigonometry...

$$\left(\frac{E_r}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \quad \left(\frac{E_r}{E_i}\right)_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\left(\frac{E_t}{E_i}\right)_\perp = \frac{(n_1/n_2)\sin(2\theta_i)}{\sin(\theta_i+\theta_t)} \quad \left(\frac{E_t}{E_i}\right)_\parallel = \frac{2 \cos(\theta_i) \sin(\theta_t)}{\sin(\theta_i+\theta_t) \cos(\theta_i-\theta_t)}$$

For $\vec{E}$ perpendicular and parallel to plane of incidence.
Application of Fresnel’s Equations

• Unpolarized light in air \((n = 1)\) is incident on a surface with index of refraction \(n' = 1.5\) at an angle \(\theta_i = 30^\circ\)

• What are the magnitudes of the electric field components of the reflected light?

First calculate the angles of reflection and refraction...

\[
\theta_r = \theta_i = 30^\circ \\
\sin \theta_i = n' \sin \theta_t \\
\sin \theta_t = \frac{\sin 30^\circ}{1.5} = 0.333 \\
\theta_t = 19.5^\circ
\]
Application of Fresnel’s Equations

- Component of $\vec{E}$ parallel to the surface is perpendicular to the plane of incidence

$$\left( \frac{E_r}{E_i} \right)_\perp = - \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$= - \frac{\sin(30° - 19.5°)}{\sin(30° + 19.5°)}$$

$$= -0.240$$
Application of Fresnel’s Equations

- Component of $\vec{E}$ perpendicular to the surface is parallel to the plane of incidence

\[
\left( \frac{E_r}{E_i} \right)_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}
\]

\[
= \frac{\tan(30^\circ - 19.5^\circ)}{\tan(30^\circ + 19.5^\circ)} = 0.158
\]

- Reflected light is preferentially polarized parallel to the surface.
Reflected Components

• Since $\theta_t < \theta_i$ the component perpendicular to the plane of incidence is always negative:

$$\left(\frac{E_r}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

• The component parallel to the plane of incidence could be positive or negative:

$$\left(\frac{E_r}{E_i}\right)_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

• What happens when $\theta_i + \theta_t = 90^\circ$?
  – Can this happen? Sure... check when $\theta_i \rightarrow 90^\circ$. 
Brewster’s Angle

\[ \tan \theta \to \infty \text{ as } \theta \to 90^\circ \]

while \[ \tan(\theta_i - \theta_t) \]
remains finite.

Therefore,

\[ \left( \frac{E_r}{E_i} \right)_\parallel \to 0 \]

Brewster’s angle, \( \theta_B \) is the angle of incidence for which \( r_\parallel \to 0 \).
Brewster’s Angle

• Brewster’s angle can be calculated from the relation $\theta_i + \theta_t = 90^\circ$

• We can always calculate $\theta_t$ using Snell’s law

• Good assignment question:

  $\textit{Show that } \theta_B = \tan^{-1} \left( \frac{n_2}{n_1} \right)$

• Light reflected from a surface at an angle $\theta_B$ will be linearly polarized parallel to the surface (perpendicular to the plane of incidence)
Total Internal Reflection

• Consider the other case when $n_i > n_t$, for example, glass to air:

At some incidence angle (critical angle $\theta_c$) everything is reflected (and nothing transmitted).

It can be shown that for any angle larger than $\theta_c$ no waves are transmitted into media: total internal reflection.

No phase shift upon reflection.
Reflected Intensity

- Remember that the *intensity* (*irradiance*) is related to the energy carried by light:
  \[ I = \epsilon \nu \langle E^2 \rangle_T \]
  (averaged over some time \( T \gg 1/f \))

- Reflectance is defined as
  \[ R_\perp = (r_\perp)^2 = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} \]
  \[ R_\parallel = (r_\parallel)^2 = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \]

- Unpolarized reflectance:
  \[ R = \frac{1}{2} (R_\perp + R_\parallel) \]
Reflected Intensity

- How polarized is the reflected light?
- Degree of polarization:
  \[ V = \frac{I_p}{I_{total}} \]
- Measured using an analyzing polarizer

\[ V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \]
Birefringence

• In a crystal with a regular, repeating atomic lattice, electrons can be more tightly bound along certain axes:

Elastic constants for electrons may be different along different axes.
Birefringence

• Electric fields oriented parallel or perpendicular to the planes in the crystal lattice interact with electrons differently.
• Depend on the atomic crystal structure
• Some crystals absorb light polarized along one axis: 
  **Dichroic crystal:** absorbs light polarized along one axis
• Other crystals transmit light, but the index of refraction depends on its polarization

**Optic axis:** the direction of linear polarization that differs from the other two axes
  – Assuming only one axis is special, the other two are the same.
Calcite ($\text{CaCO}_3$)

Image doubles:

**Ordinary rays** (o-rays) - unbent
**Extraordinary rays** (e-rays) - bend
Birefringence and Huygens’ Principle
Useful Applications

Stone found in English Channel could be Icelandic ‘sunstone’

Posted on 14 March 2013. Tags: English Channel, Sun stone, sunstone, Vikings

A crystal recovered from a shipwreck in the English Channel may turn out to be the fabled ‘sunstone’ said to be used by the Vikings. According to ancient folklore, the Vikings used a crystal called sunstone to help navigate during cloudy weather. However, it has long been debated on weather or not the substance actually exists.

But researchers from the British-French group the Alderney Maritime Trust said in a new paper published the Proceedings of the Royal Society A journal that a large piece of Icelandic calcite recovered from the 16th century shipwreck might in fact be the fabled stone.
**Crystal Structure and Birefringence**

NaCl - cubic crystal

Four 3-fold symmetry axes - optically isotropic

Anisotropic, uniaxial birefringent crystals: hexagonal, tetragonal, trigonal

Uniaxial crystal: atoms are arranged symmetrically around optic axis

O-wave - $\vec{E}$ everywhere is perpendicular to the optic axis, $n_o \equiv c/v_\perp$

When $\vec{E}$ is parallel to optic axis: $n_e \equiv c/v_\parallel$

**Birefringence** $\equiv n_e - n_o$

Calcite: $(n_e - n_o) = -0.172$ (negative uniaxial crystal)
Crystal Structure and Birefringence

Negative uniaxial crystal

\[ \frac{n_e}{c} < \frac{n_o}{c} \]

Positive uniaxial crystal

\[ \frac{n_e}{c} > \frac{n_o}{c} \]
Biaxial Crystals

Two optic axes and three principal indices of refraction
Orthorhombic, monoclinic, triclinic

Example: mica, $\text{KH}_2\text{Al}_3(\text{SiO}_4)_3$