PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 25 – Propagation of Light

Spring 2013 Semester

Matthew Jones

Midterm Exam:

Date: Wednesday, March 6^{th}

Time: 8:00 – 10:00 pm

Room: PHYS 203

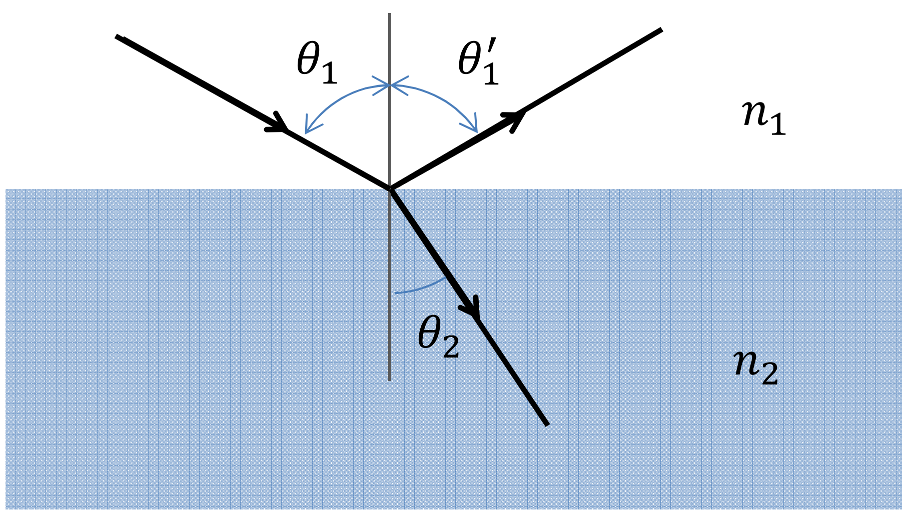
Material: French, chapters 1-8

You can bring one page of notes/formulas.

No lecture on Friday.

Geometric Optics

- When the wavelength of light is much shorter than the sizes of objects it interacts with, we can ignore the wave-like nature and treat it as rays that propagate in straight lines.



Reflection:

$$\theta_1' = \theta_1$$

Refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Polarization

- Light is an oscillating electromagnetic field
- The electric field has a direction

$$\vec{E}(x,t) = \vec{E}_0 \cos\left(\vec{k} \cdot \vec{r} - \omega t\right)$$

- No need to specify the magnetic field direction:

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{H} = (\hat{k} \times \vec{E})/Z \qquad \text{where } Z = \sqrt{\mu/\varepsilon}$$

- $\vec{H}$ refers to the magnetic field due to the light, not including any induced magnetic fields in the presence of matter.
- *Coherent* light has the same phase over macroscopic distances and time
- *Polarized* light has the electric field aligned over macroscopic distances and time

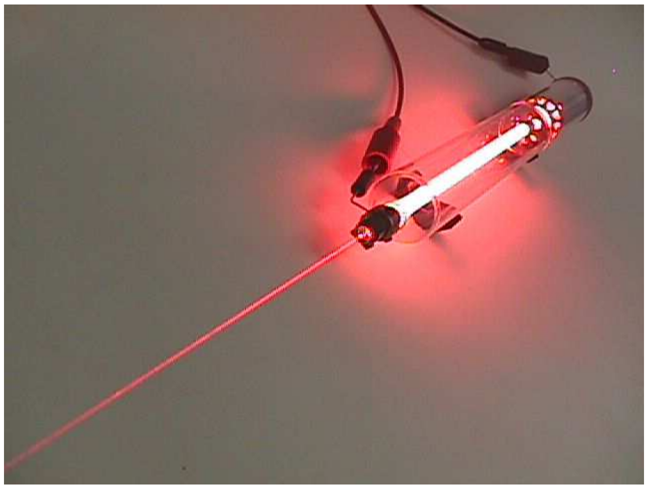
Polarization

Sources of un-polarized light

- Hot atoms transfer kinetic energy to electrons randomly
- Electrons randomly de-excite, emitting incoherent light – uncorrelated in phase and polarization

Polarization

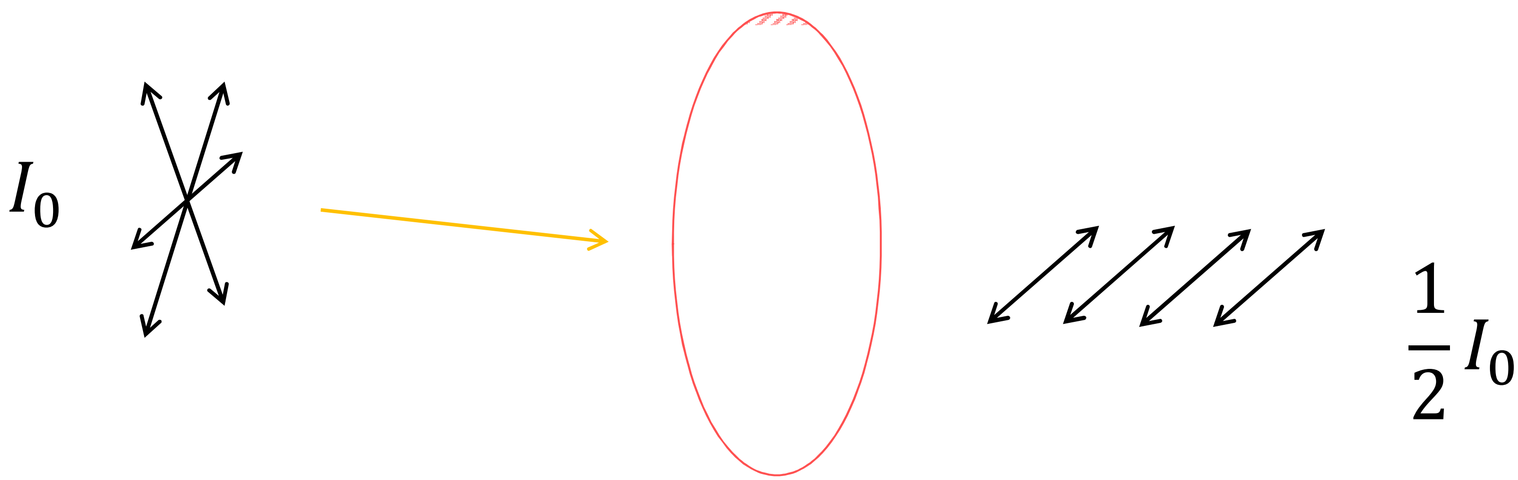
Sources of polarized light



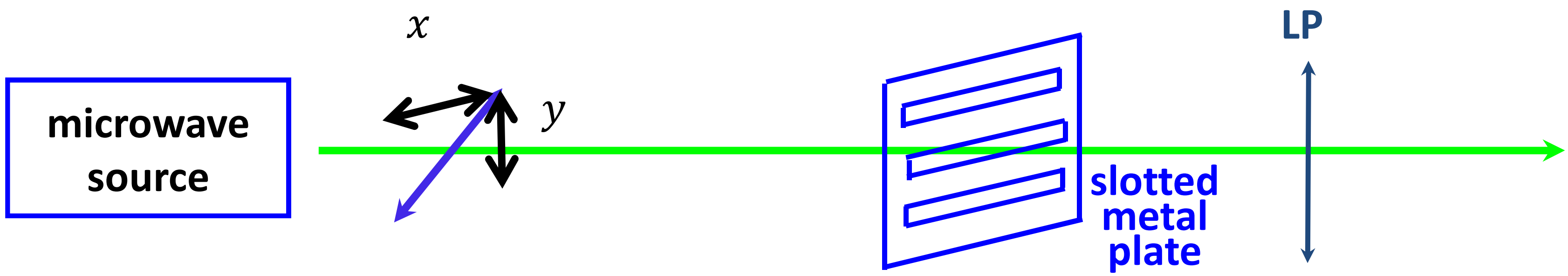
- Lasers produce light by stimulated emission
  - A photon causes an excited atom to emit another photon
  - The photon is emitted in phase and with the same polarization
- The resulting beam is highly coherent and polarized

Polarization by Absorption

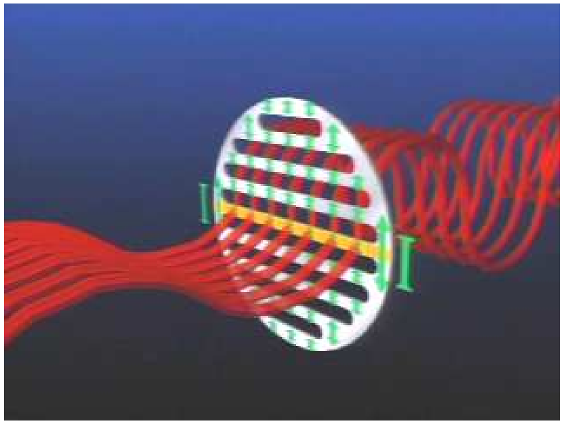
- A polarizer absorbs the component with $\vec{E}$ oriented along a particular axis.
- The light that emerges is linearly polarized along the perpendicular axis.
- If the light is initially un-polarized, half the light is absorbed.



Example with Microwaves



- The electric field in the x-direction induces currents in the metal plate and loses energy:
  - Horizontally polarized microwaves are absorbed
- No current can flow in the y-direction because of the slots
  - Vertically polarized microwaves are transmitted

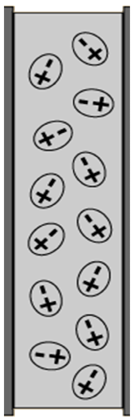


Polarization by Reflection

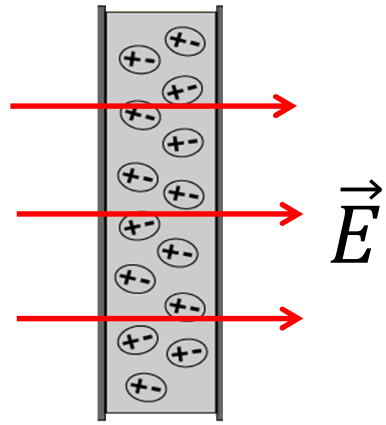
- Reflected light is preferentially polarized
- The other component must be transmitted
- Transmission and reflection coefficients must depend on the polarization

Continuity Conditions

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{E}}\, d\boldsymbol{A} = \frac{\boldsymbol{Q_{total}}}{\boldsymbol{\varepsilon_0}}$$



$$\vec{D} = \varepsilon \vec{E}$$

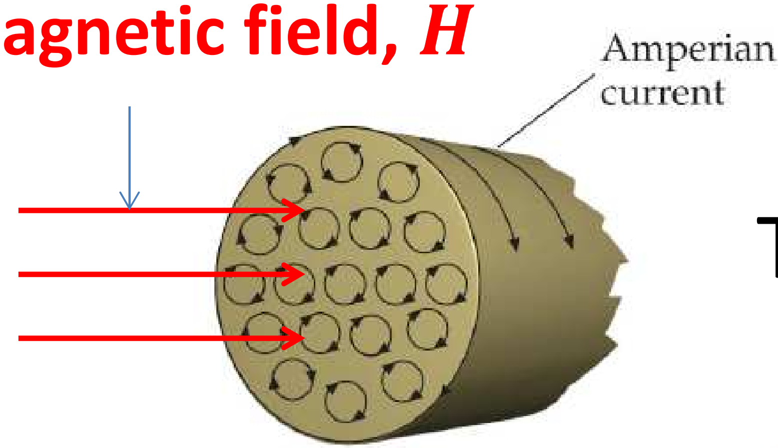


$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{D}}\, d\boldsymbol{A} = \boldsymbol{Q_{free}}$$

Continuity Conditions

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{B}}\, \boldsymbol{dA} = \boldsymbol{0}$$

Externally applied magnetic field, $\vec{H}$



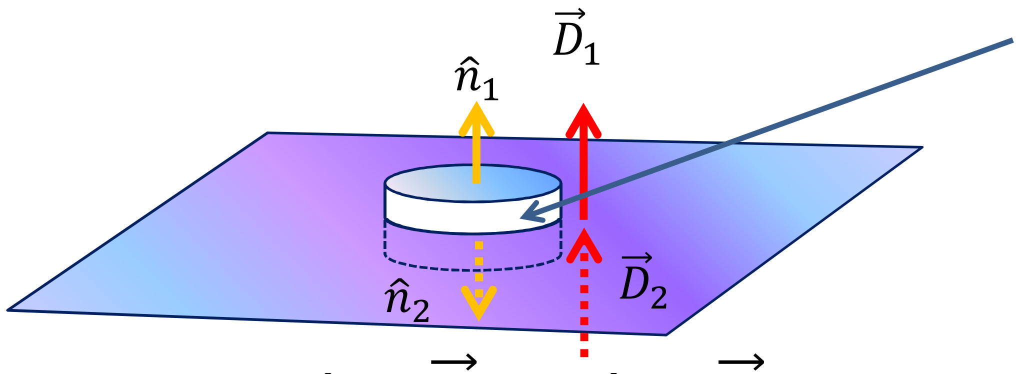
Total magnetic field,

$$\vec{B} = \mu \vec{H}$$

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{H}}\, \boldsymbol{dA} = \boldsymbol{0}$$

Continuity Conditions

- If there are no free charges at the boundary then $\vec{D}$ must be the same on both sides:



$$Q_{free} = 0$$

$$\vec{D} = \varepsilon \vec{E}$$

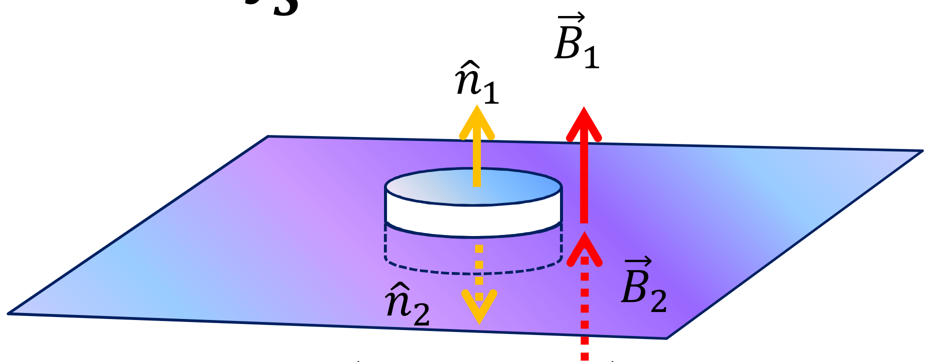
$$\hat{n}_1 \cdot \vec{D}_1 - \hat{n}_2 \cdot \vec{D}_2 = 0$$

$$\varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$$

- The components of $\vec{D}$ perpendicular to the surface are the same on each side.
- The component of $\vec{E}$ perpendicular to the surface is different on each side of the interface.

Continuity Conditions

$$\oint_{S} \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{B}}\, d\boldsymbol{A} = \boldsymbol{0}$$



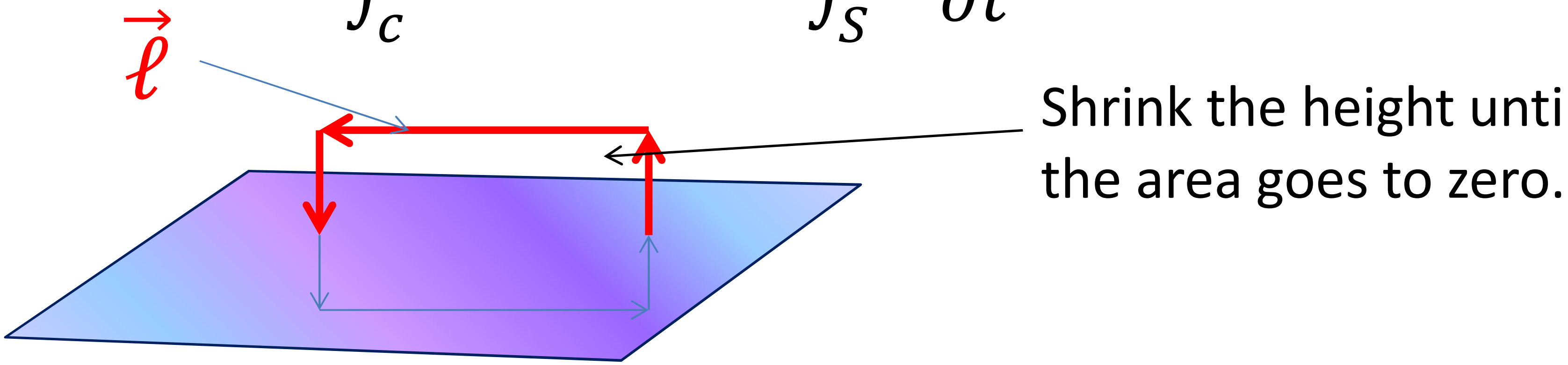
$$\hat{n}_1 \cdot \vec{B}_1 - \hat{n}_2 \cdot \vec{B}_2 = 0$$

$$\mu_1 H_{1\perp} = \mu_2 H_{2\perp}$$

- The component of $\vec{B}$ perpendicular to the surface is the same on both sides.
- The component of $\vec{H}$ perpendicular to the surface is different on each side of the interface.

Continuity Conditions

$$\oint_c \vec{E} \cdot d\vec{\ell} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$



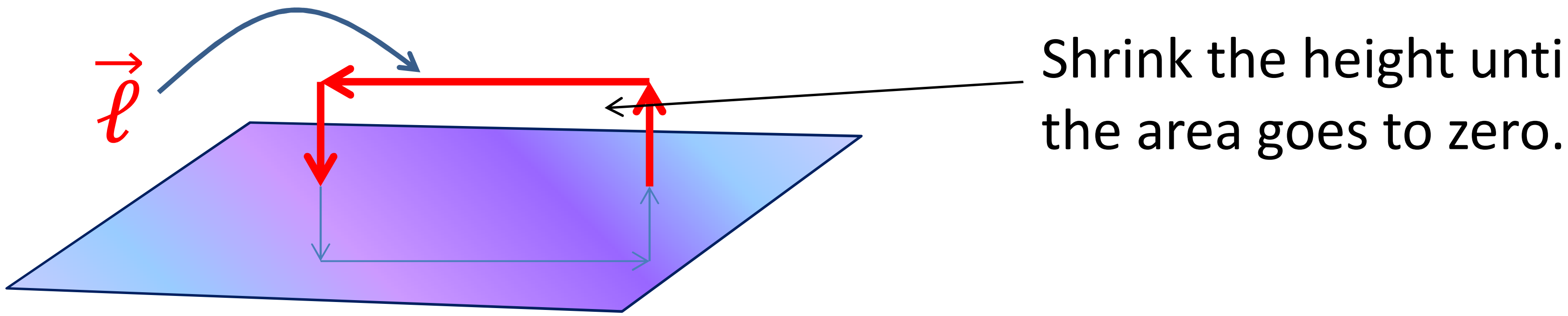
$$\vec{E}_1 \cdot \vec{\ell} - \vec{E}_2 \cdot \vec{\ell} = 0$$

$$E_{1\parallel} = E_{2\parallel}$$

- The component of $\vec{E}$ parallel to the surface is the same on both sides.

Continuity Conditions

$$\oint_c \vec{H} \cdot d\vec{\ell} = \int_S \vec{J}_{free} \cdot d\vec{a} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{a}$$



$$\vec{H}_1 \cdot \vec{\ell} - \vec{H}_2 \cdot \vec{\ell} = 0$$

$$H_{1\parallel} = H_{2\parallel}$$

- The component of $\vec{H}$ parallel to the surface is the same on both sides.

Boundary Conditions

- Summary:
  - Perpendicular to surface

$$\varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$$
$$\mu_1 H_{1\perp} = \mu_2 H_{2\perp}$$

  - Parallel to surface

$$E_{1\parallel} = E_{2\parallel}$$
$$H_{1\parallel} = H_{2\parallel}$$

Reflection From a Surface

First case:

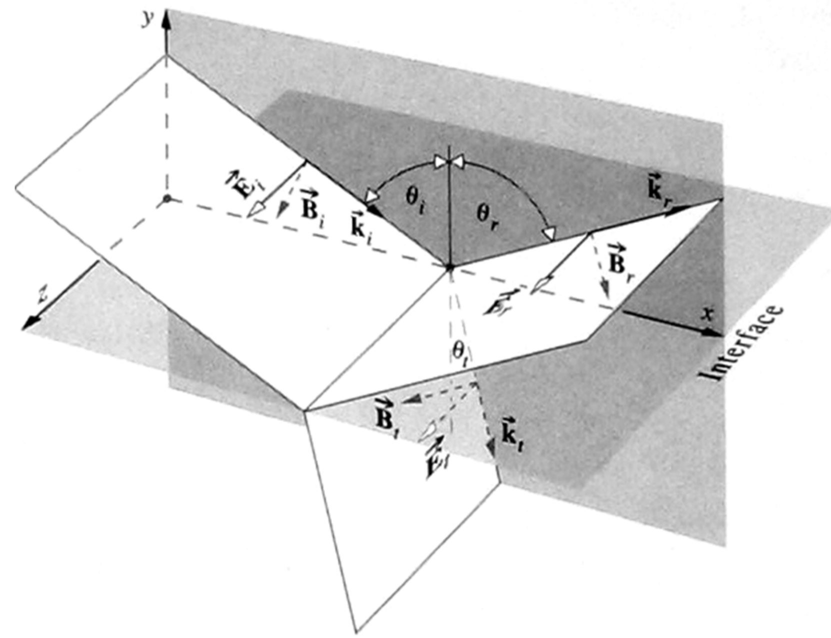
$\vec{E}$ is parallel to the surface…

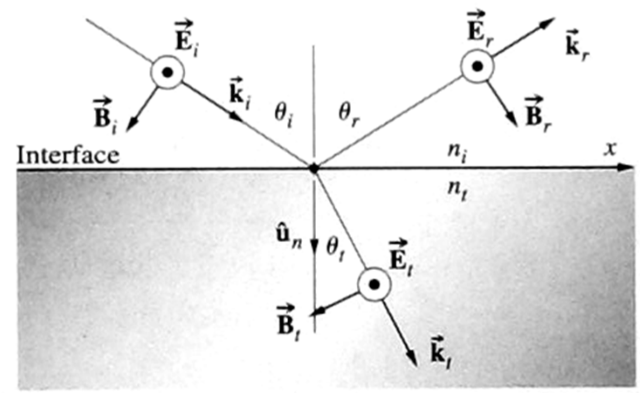
$$E_i + E_r = E_t$$

$\vec{H}$ has components parallel and perpendicular to the surface

$$H_{\parallel i} + H_{\parallel r} = H_{\parallel t}$$

But $H_{\parallel} = \vec{H} \cdot \hat{\tau}$…





Reflection From a Surface

$\vec{E}$ is perpendicular to $\hat{n}$

$$\hat{\tau} = \frac{\vec{E} \times \hat{n}}{|\vec{E}|}$$

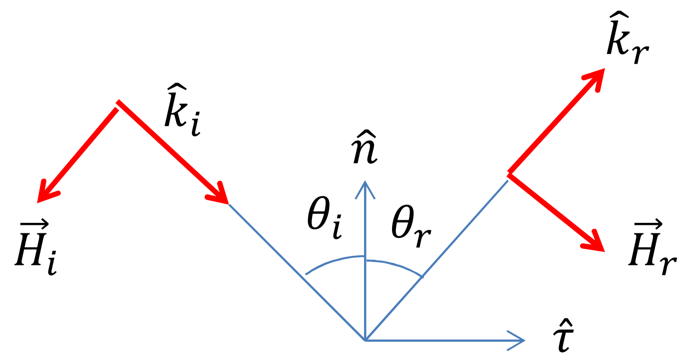
$\vec{H}$ can be written

$$\vec{H} = \frac{\hat{k} \times \vec{E}}{Z}$$

So we can write

$$\vec{H}_i \cdot \hat{\tau} = \frac{1}{ZE_i}(\hat{k} \times \vec{E}_i) \cdot (\vec{E}_i \times \hat{n})$$

$$= -\frac{E_i}{Z}(\hat{k} \cdot \hat{n}) = -\frac{E_i}{Z}\cos\theta_i$$



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Likewise,

$$\vec{H}_r \cdot \hat{\tau} = \frac{E_r}{Z}\cos\theta_r$$

$\vec{E}$ perpendicular to $\hat{n}$

- Boundary condition for $\vec{H}$:

$$H_{\parallel i} + H_{\parallel r} = H_{\parallel t}$$

- Previous results:

$$H_{i\parallel} = -\frac{E_i}{Z_1}\cos\theta_i \qquad H_{r\parallel} = \frac{E_r}{Z_1}\cos\theta_r = \frac{E_r}{Z_1}\cos\theta_i$$

$$\frac{-E_i\cos\theta_i + E_r\cos\theta_i}{Z_1} = \frac{-E_t\cos\theta_t}{Z_2}$$

$$E_i + E_r = E_t$$

- Two equations in two unknowns…

$\vec{E}$ perpendicular to $\hat{n}$

$$\frac{-E_i \cos\theta_i + E_r \cos\theta_i}{Z_1} = \frac{-E_t \cos\theta_t}{Z_2}$$

$$\frac{E_i \cos\theta_t + E_r \cos\theta_t}{Z_2} = \frac{E_t \cos\theta_t}{Z_2}$$

- Solve for E_r/E_i:

$$\boldsymbol{\frac{E_r}{E_t} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2\, cos\, \theta_i + Z_1\, cos\, \theta_t}}$$

- Solve for E_t/E_i:

$$\boldsymbol{\frac{E_t}{E_t} = \frac{2Z_2\, cos\, \theta_i}{Z_2\, cos\, \theta_i + Z_1\, cos\, \theta_t}}$$

Reflection From A Surface

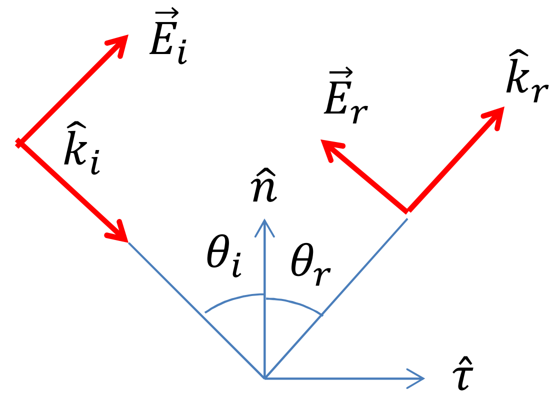
$\vec{H}$ parallel to surface

$$H_i + H_r = H_t$$

$$E_{\parallel i} + E_{\parallel r} = E_{\parallel t}$$

$$E_i \cos\theta_i - E_r \cos\theta_r = E_t \cos\theta_t$$

$$\frac{E_i}{Z_1} + \frac{E_r}{Z_1} = \frac{E_t}{Z_2}$$



$\vec{H}$ is out of the page

- Two equations in two unknowns…

$\vec{H}$ perpendicular to $\hat{n}$

$$\frac{E_i \cos\theta_i - E_r \cos\theta_i}{Z_2} = \frac{E_t \cos\theta_t}{Z_2}$$

$$\frac{E_i \cos\theta_t + E_r \cos\theta_t}{Z_1} = \frac{E_t \cos\theta_t}{Z_2}$$

- Solve for E_r/E_i:

$$\frac{\boldsymbol{E_r}}{\boldsymbol{E_t}} = \frac{\boldsymbol{Z_1 \cos\theta_i - Z_2 \cos\theta_t}}{\boldsymbol{Z_1\, cos\,\theta_i + Z_2\, cos\,\theta_t}}$$

- Solve for E_t/E_i:

$$\frac{\boldsymbol{E_t}}{\boldsymbol{E_t}} = \frac{\boldsymbol{2Z_1\, cos\,\theta_i}}{\boldsymbol{Z_1\, cos\,\theta_i + Z_2\, cos\,\theta_t}}$$

Fresnel's Equations

- In most dielectric media, $\mu_1 = \mu_2$ and therefore

$$\frac{Z_1}{Z_2} = \sqrt{\frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1} = \frac{\sin\theta_i}{\sin\theta_t}$$

- After some trigonometry…

$$\left(\frac{E_r}{E_i}\right)_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad \left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\left(\frac{E_t}{E_i}\right)_{\perp} = \frac{(n_1/n_2)\sin(2\theta_i)}{\sin(\theta_i+\theta_t)} \quad \left(\frac{E_t}{E_i}\right)_{\parallel} = \frac{2\cos(\theta_i)\sin(\theta_t)}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

For $\vec{E}$ perpendicular and parallel to plane of incidence.

Normal Incidence

- At normal incidence, $\cos\theta = 1$…

$$\left(\frac{E_r}{E_i}\right)_\perp = \frac{n_1 - n_2}{n_1 + n_2} \qquad \left(\frac{E_r}{E_i}\right)_\parallel = \frac{n_2 - n_1}{n_1 + n_2}$$

$$\left(\frac{E_t}{E_i}\right)_\perp = \frac{2n_1}{n_1 + n_2} \qquad \left(\frac{E_t}{E_i}\right)_\parallel = \frac{2n_2}{n_1 + n_2}$$

- See you at 8:00 pm this evening…