Purdue University Department of Physics

Physics 42200

# Waves & Oscillations

Lecture 24 – Propagation of Light

Spring 2013 Semester

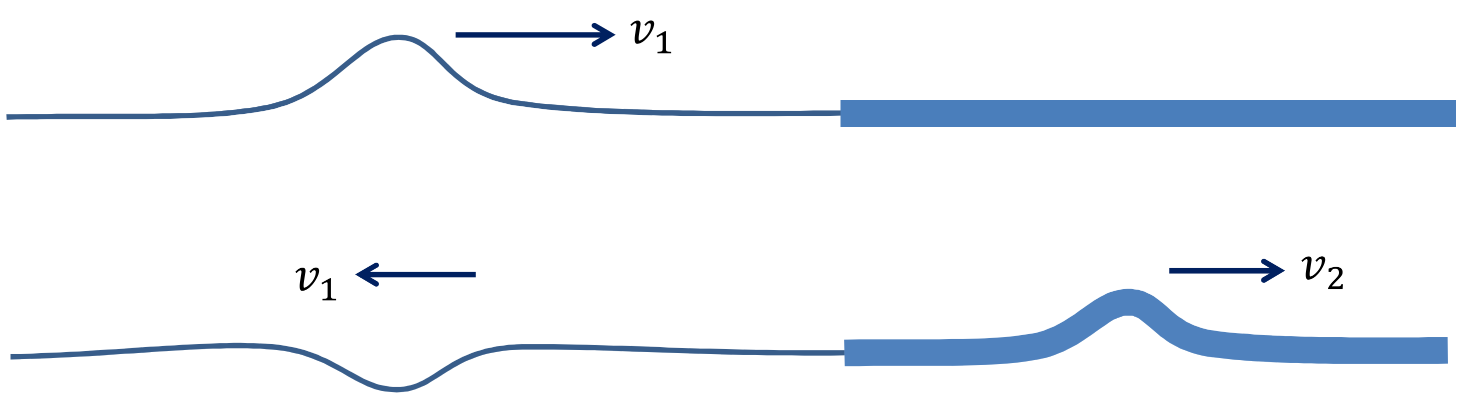
Matthew Jones

# Midterm Exam:

| | |
|---|---|
| Date: | Wednesday, March 6th |
| Time: | 8:00 – 10:00 pm |
| Room: | PHYS 203 |
| Material: | French, chapters 1-8 |

# Reflection and Transmission Coefficients

- In Lecture 19 we derived the relation between the amplitudes of reflected and transmitted waves:
  - Reflection coefficient: $\rho = (v_2 - v_1)/(v_2 + v_1)$
  - Transmission coefficient: $\tau = 2v_2/(v_2 + v_1)$
  - Example: $\mu_2 > \mu_1$ so $v_2 < v_1$



# Reflection and Transmission Coefficients

- The relation between the power and the amplitude of the wave was:

$$P = \frac{1}{2} Z \omega^2 A^2$$

- In the case of the string,

$$Z_1 = T/v_1 = \sqrt{T\mu_1} \qquad Z_2 = T/v_2 = \sqrt{T\mu_2}$$

$$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$\tau = \frac{2v_2}{v_2 + v_1} = \frac{2Z_1}{Z_1 + Z_2}$$

# Reflection and Transmission Coefficients

- Let's check whether this makes sense…
- Suppose the string were attached to an immovable object.
  - This can be approximated by letting $\mu \to \infty$
  - Impedance is $Z_2 = \sqrt{T\mu_2} \to \infty$
  - Reflection coefficient for the amplitude:

$$\rho = \frac{Z_1 - Z_2}{Z_1 + Z_2} \to -1$$

  - Reflected pulse is inverted, as expected.

# Reflection and Transmission Coefficients

- In transmission lines:

$$\frac{\partial V}{\partial x} = -I(x)X = -i\omega L' \, I(x)$$

- Reflected pulse: $V(x,t) = V(x+vt) = V(x)e^{i\omega t}$

$$\frac{\partial V}{\partial x} = \frac{1}{v}\frac{\partial V}{\partial t} = \frac{i\omega}{v}V(x) = -i\omega L' I(x)$$

- Relation between current and voltage of reflected pulse:

$$V(x) = -vL' I(x) = \frac{L'}{\sqrt{L'C'}} I(x) = \sqrt{\frac{L'}{C'}} I(x) = -Z \, I(x)$$

- Voltage reflection coefficient should be opposite the current reflection coefficient.

# Reflection and Transmission Coefficients

- Current reflection coefficient:

$$\rho_I = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

- Voltage reflection coefficient:

$$\rho_V = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

- Transmitted current:

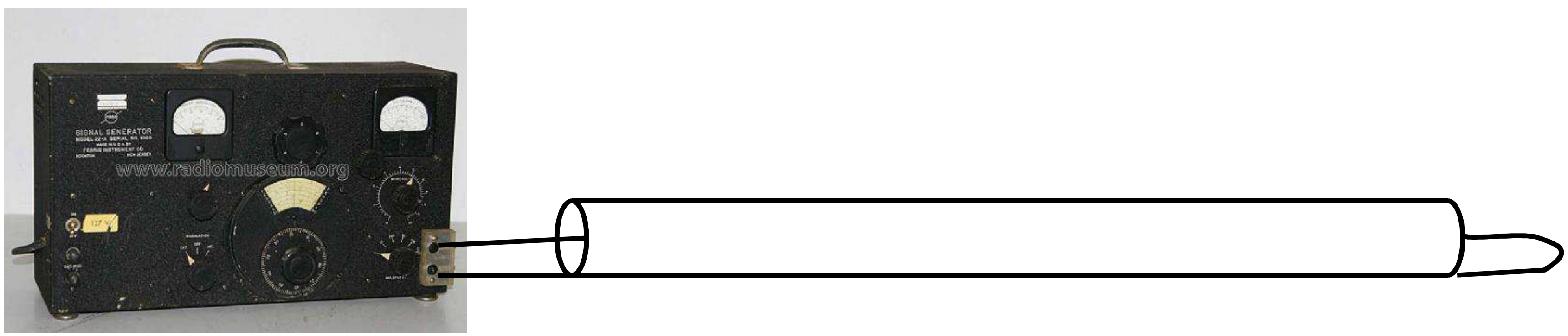
$$I_t = I_i \, \tau = I_i \, \frac{2Z_1}{Z_1 + Z_2}$$

- Transmitted voltage:

$$V_t = Z_2 I_t = \frac{Z_2}{Z_1} V_i \frac{2Z_1}{Z_1 + Z_2} = V_i \, \frac{2Z_2}{Z_1 + Z_2}$$

# Reflection and Transmission Coefficients

- Does this make sense?
- Reflection from a shorted transmission line: $Z_2 = 0$



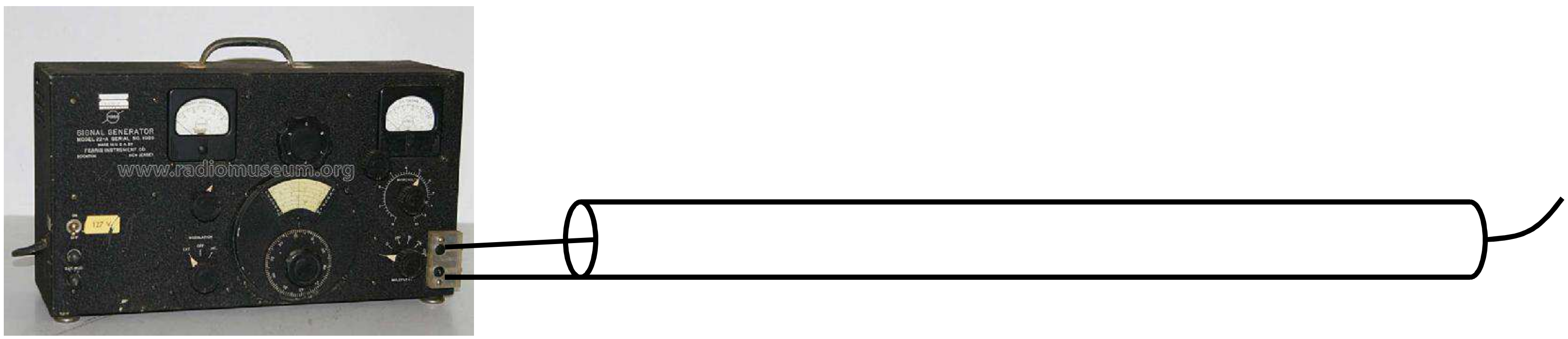

- Voltage across the wire at the end must be zero: incident and reflected voltages cancel

$$\rho_V = \frac{Z_2 - Z_1}{Z_2 + Z_1} = -1$$

Yes!

# Reflection and Transmission Coefficients

- Does this make sense?
- Reflection from an open transmission line: $Z_2 = \infty$



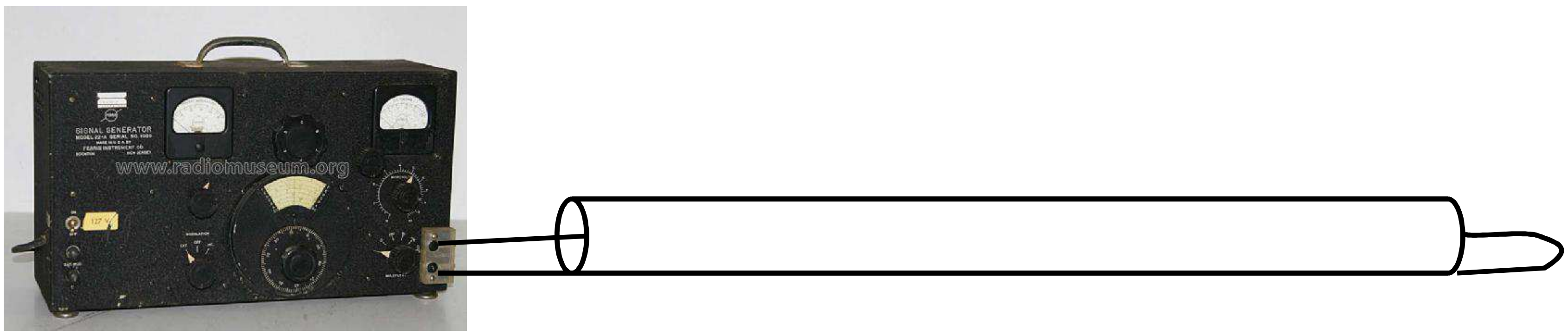

- Voltage across the wire at the end must be zero: incident and reflected voltages cancel

$$\rho_V = \frac{Z_2 - Z_1}{Z_2 + Z_1} = +1$$

Yes!

# Reflection and Transmission Coefficients

- Does this make sense?
- Reflection from a shorted transmission line: $Z_2 = 0$



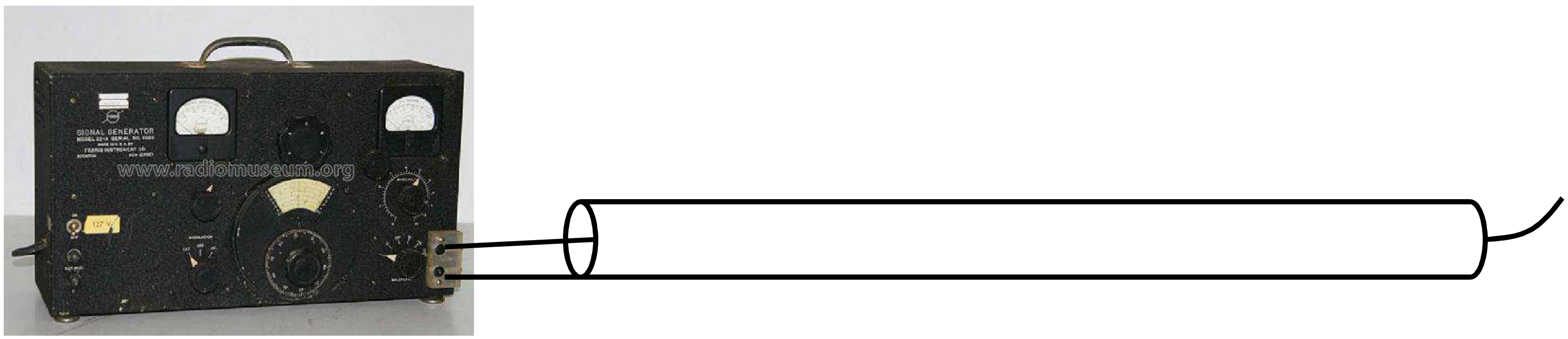

- Current across the wire at the end should be maximal: incident and reflected currents don't cancel

$$\rho_I = \frac{Z_1 - Z_2}{Z_1 + Z_1} = +1$$

Yes!

# Reflection and Transmission Coefficients

- Does this make sense?
- Reflection from an open transmission line: $Z_2 = \infty$




- Current across the wire at the end should be zero: incident and reflected currents must cancel

$$\rho_I = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -1$$

Yes!

# Geometric Optics

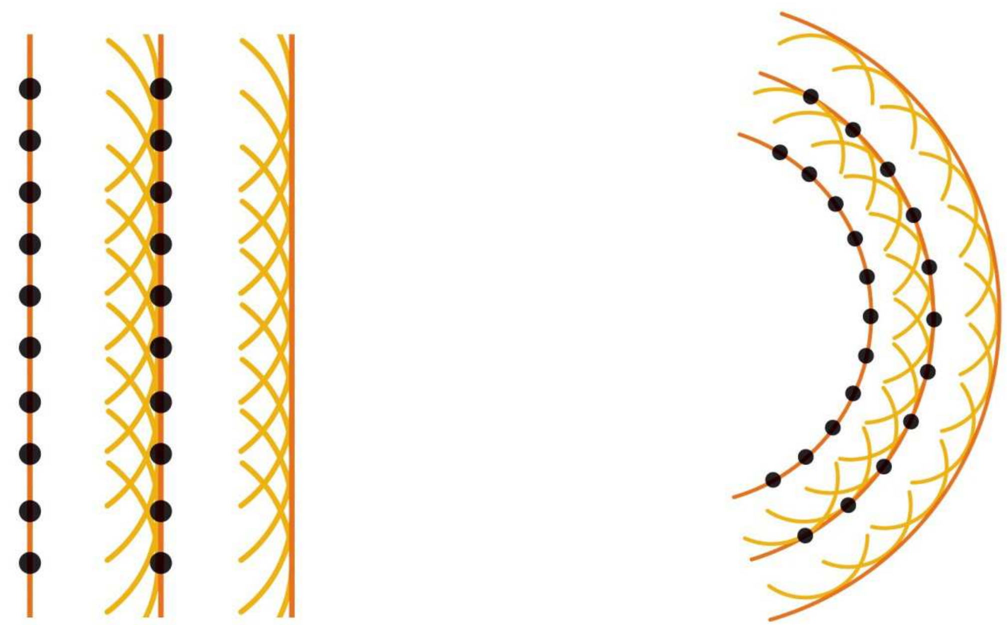
- Wave equation in free space:

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2}$$

- Impedance of free space: $Z_0 = \mu_0 c = 377\ \Omega$
- Free space is isotropic, so $\rho = 0$ and $\tau = 1$
- Waves propagate in the forward direction according to the wave equation.
- What about wave propagation in transparent media?

# Geometric Optics

- Huygens' principle: Light is continuously re-emitted in all directions from all points on a wave front
  - Further developed by Fresnel and Kirchhoff
  - Destructive interference except in the forward direction



# Forward Propagation

- Remember the forced harmonic oscillator problem:

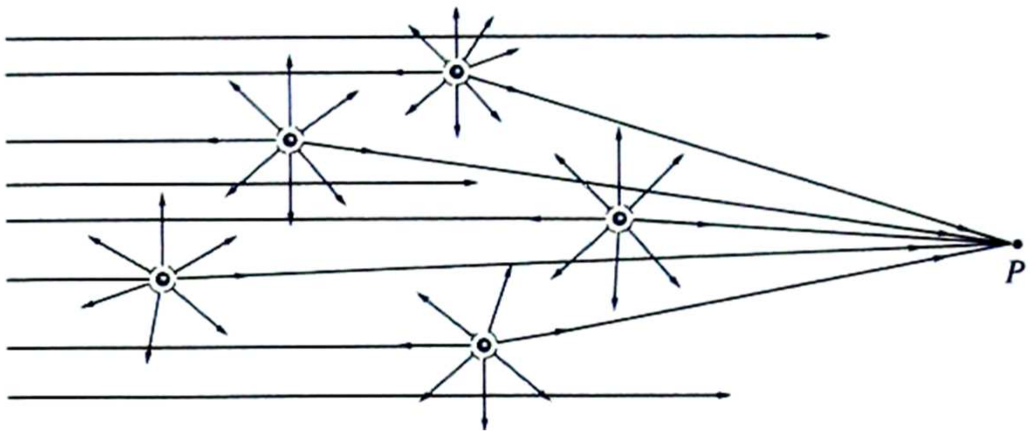
$$m\ddot{x} + b\dot{x} + kx = F_0 e^{i\omega t}$$

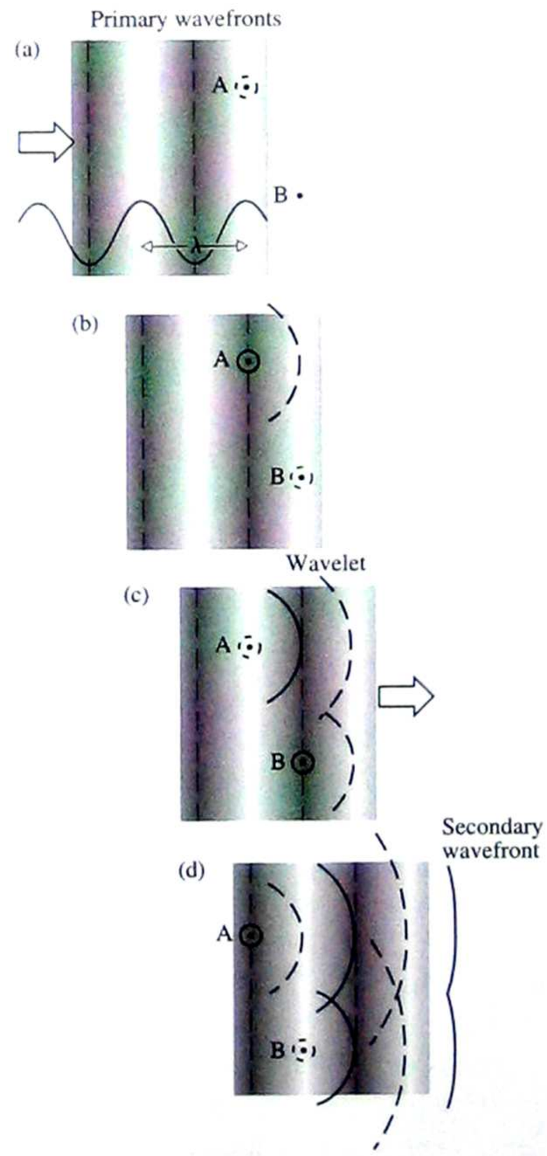
$$A = \frac{F_0/m}{\sqrt{\left((\omega_0)^2 - \omega^2\right)^2 + (\omega\gamma)^2}}$$

$$\delta = \tan^{-1}\left(\frac{\omega\gamma}{(\omega_0)^2 - \omega^2}\right)$$

- When the driving force is much greater than the resonant frequency, $\delta \rightarrow 180°$
- Electrons bound to molecules act as tiny oscillators that are driven by the electric field.

# Forward Propagation



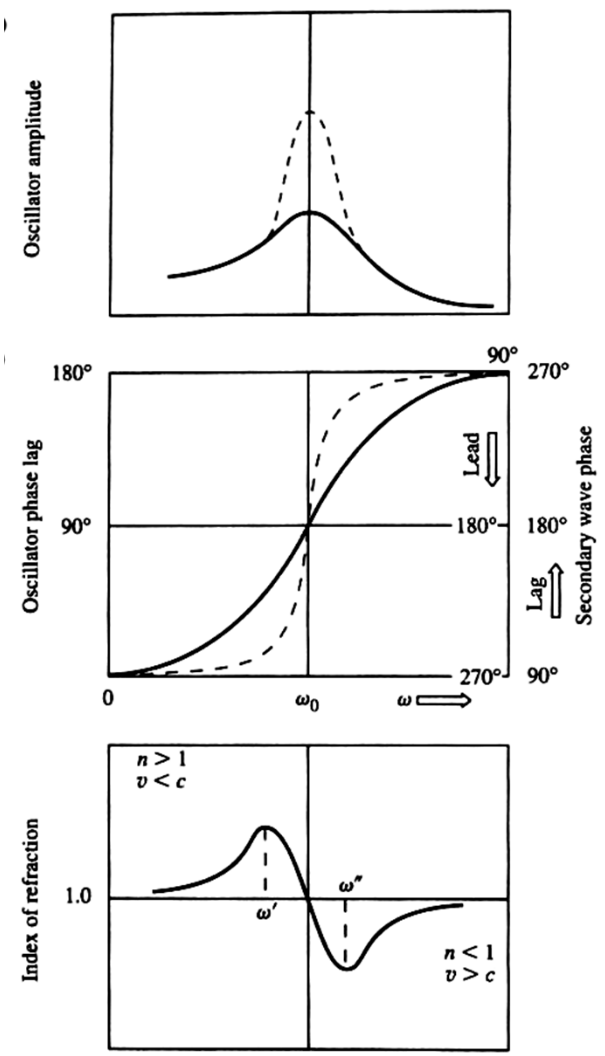




At point P the scattered waves are more or less in-phase:

- constructive interference of wavelets scattered in forward direction.
- Destructive interference of wavelets and incident light in the backward direction.

True for low and high density substance

# Forward Propagation




- At lower frequencies the phase shift is less than $180°$
- The phase lag results in the scattered waves having a shorter wavelength

$$\lambda' = \frac{v}{f} < \frac{c}{f}$$

- Index of refraction:

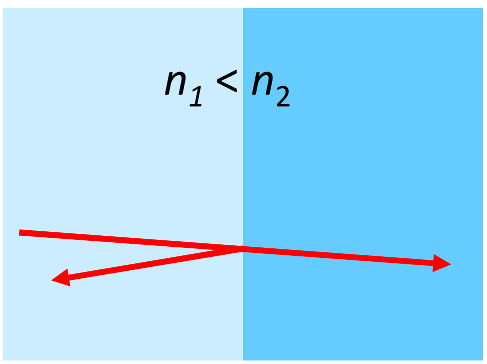
$$n = \frac{c}{v}$$

- Applies to the *phase* velocity

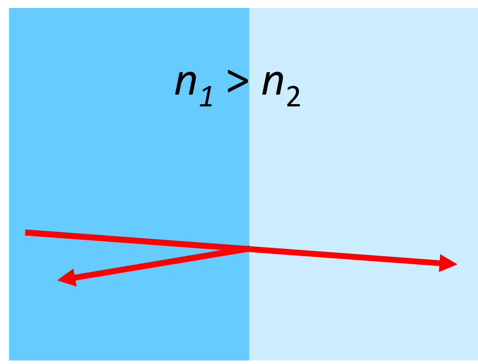
# Reflection

- Energy is reflected from an interface between materials with different indices of refraction
- Reflection coefficient:

$$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{1/n_2 - 1/n_1}{1/n_2 + 1/n_1} = \frac{n_1 - n_2}{n_1 + n_2}$$

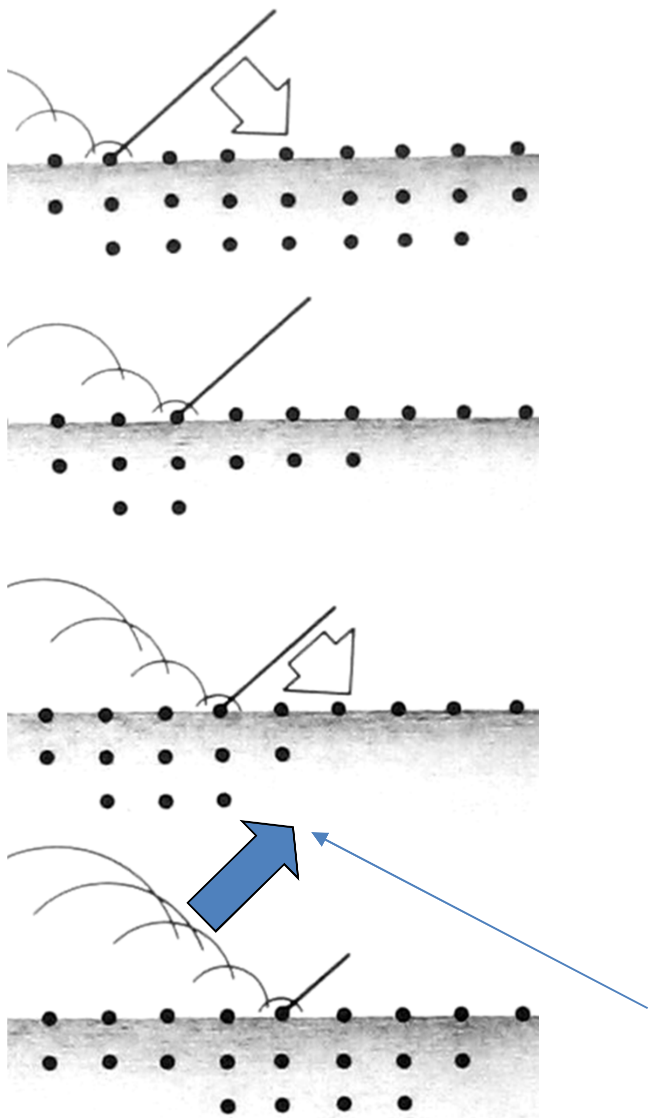



External reflection:
180° phase shift




Internal reflection:
No phase shift

# Reflection: Microscopic View



When an incident plane wave front strikes the surface at some angle it does not reach all the atoms along the surface simultaneously.

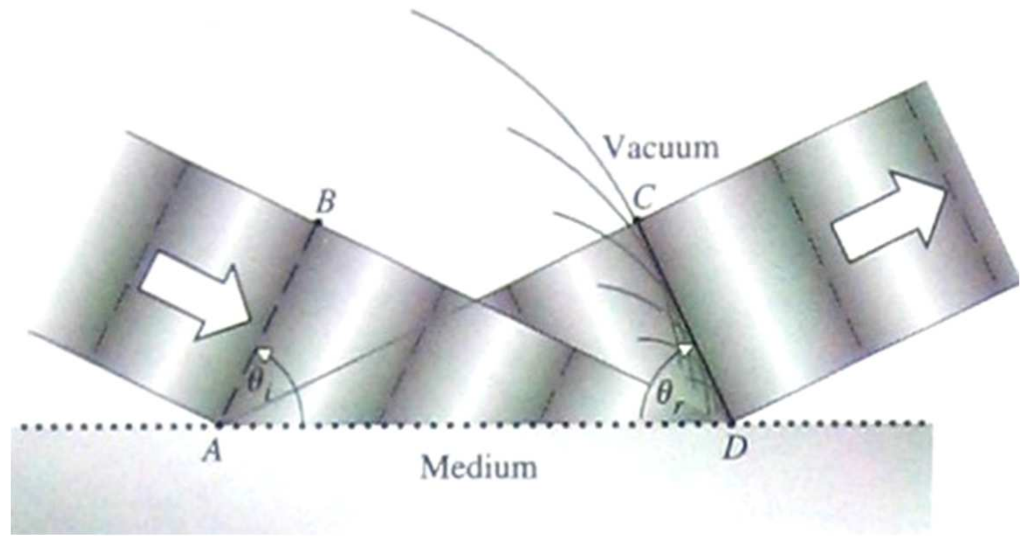
Each consequent atom will scatter at slightly different phase, while the spherical wave created by previous atom had a chance to move away some distance.

The resulting reflected wave front created as a superposition of all scattered wavelets will emerge also at an angle to the surface.

# Reflection: Constructive Interference

For constructive interference spherical waves created by the atoms on the surface must arrive in-phase.

Consider two atoms on the surface.




Wave function depends only on $\vec{k}\cdot\vec{r}-\omega t$: $E=f(\vec{k}\cdot\vec{r}-\omega t)$

Wave phase along the incident wavefront is the same.

Scattered wavefront: Points C and D must also have the same phase.

$$E_C = f(\xi + k\cdot\overline{AC} - \omega t)$$
$$E_D = f(k\cdot\overline{BD} + \xi - \omega t)$$

$\Rightarrow$ $\overline{AC}=\overline{BD}$

↑ phase shift due to scattering atom

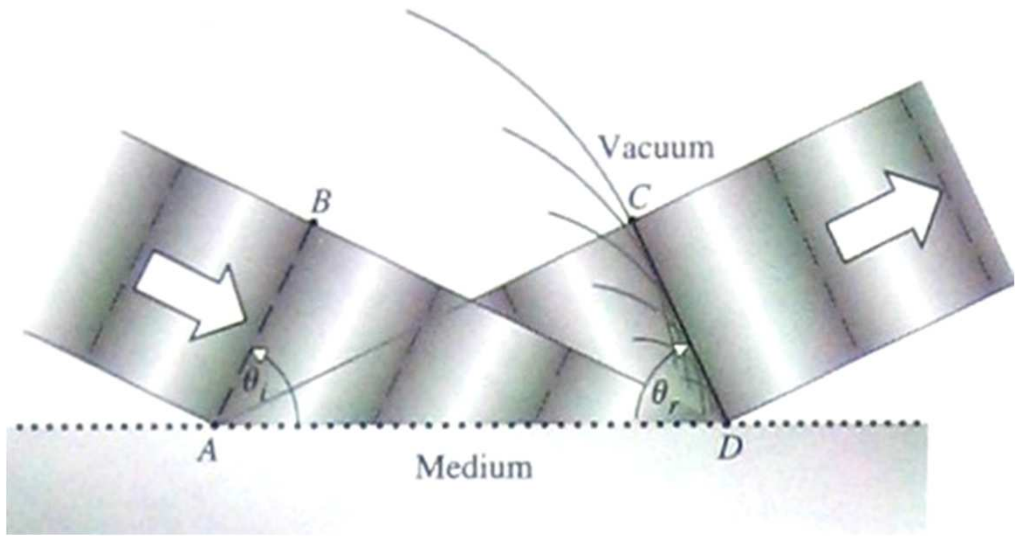
# The Angle of Reflection

Triangles ABD and ACD:

$$\overline{AC} = \overline{BD}$$

$$\overline{AD} = \overline{AD}$$

$$\angle B = \angle C = 90^{\circ}$$



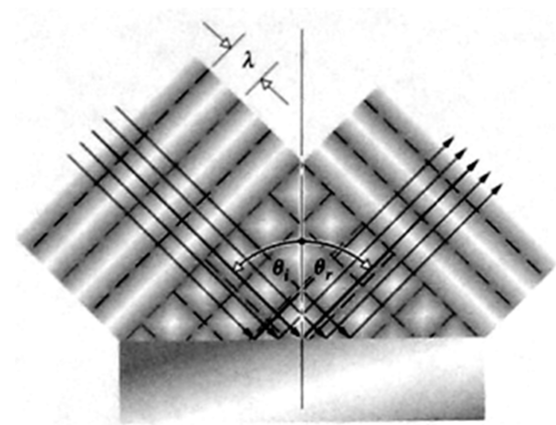

**The Law of Reflection (1st part)**:
The angle-of-incidence equals the angle-of-reflection:

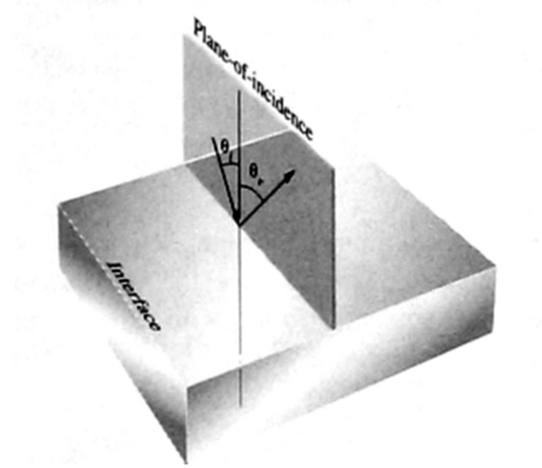
$$\theta_r = \theta_i$$

# Rays and the Law of Reflection

**A ray** is a line drawn in space along the direction of flow of the radiant energy. Rays are straight and they are perpendicular to the wavefront

Conventionally we talk about rays instead of wavefronts

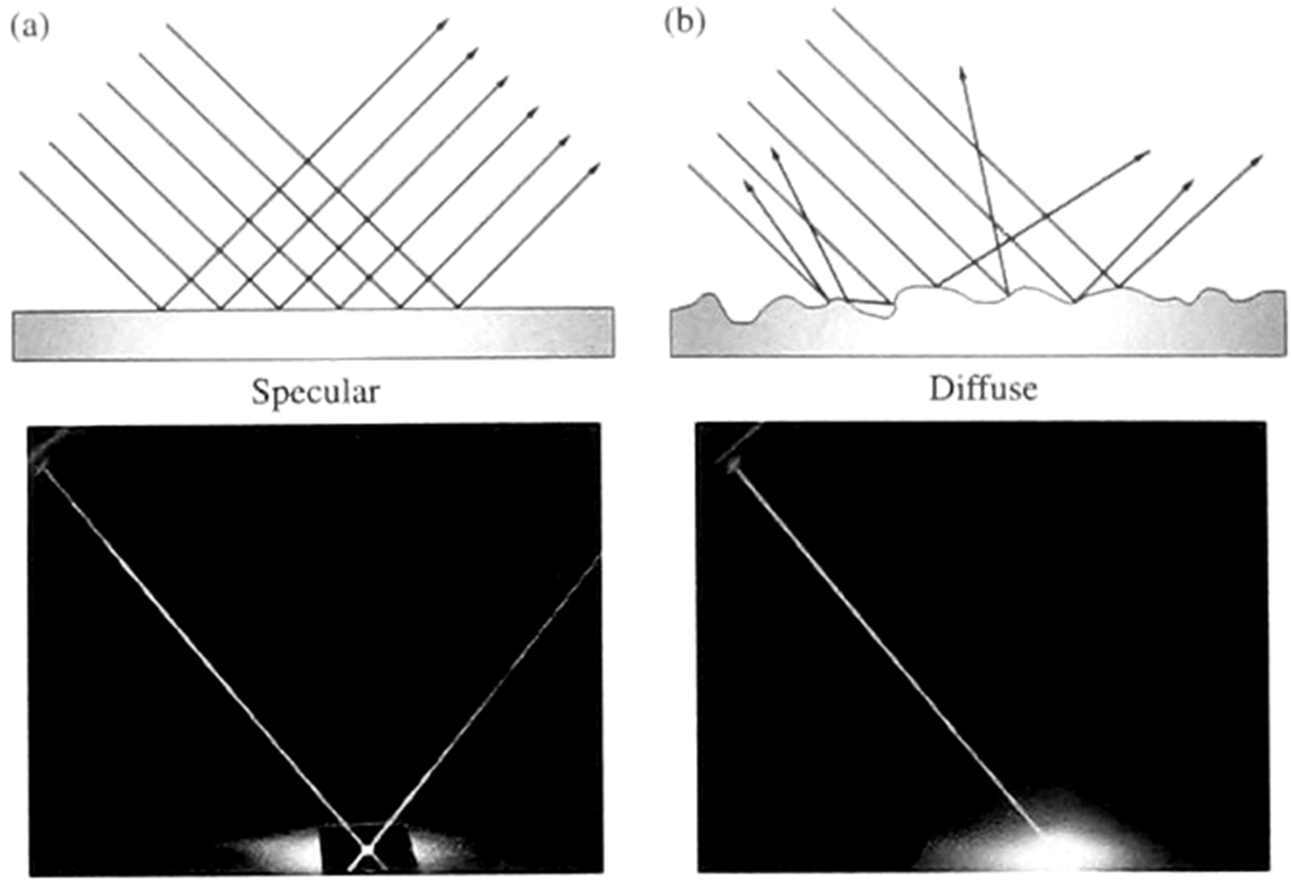



**The Law of Reflection**

1. The angle-of-incidence equals the angle-of-reflection $\theta_r = \theta_i$
2. The incident ray, the perpendicular to the surface and the reflected ray all lie in a plane (**plane-of-incidence**)

# Specular and Diffuse Reflection




Smooth surface:
specular reflection

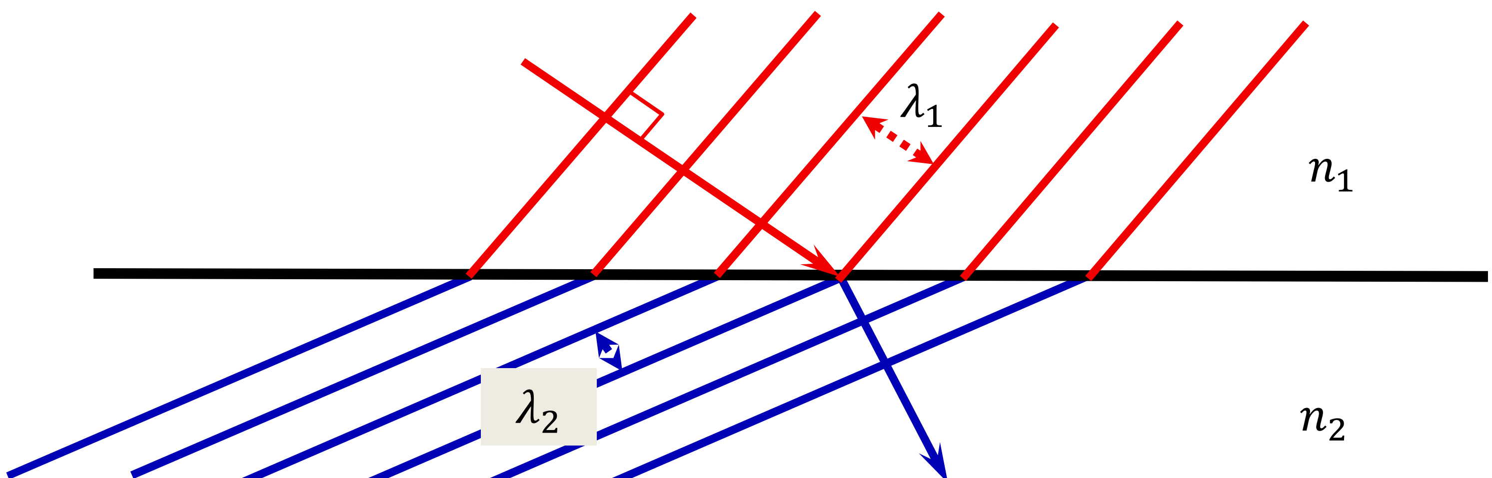
Rough surface:
diffuse reflection

# Refraction

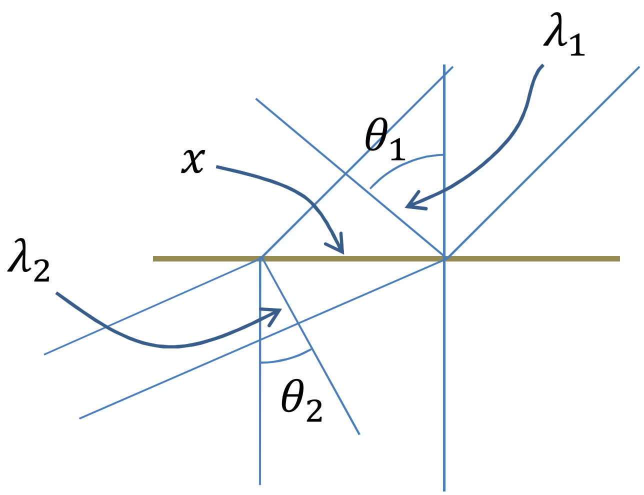
- The speed changes in different media, but the frequency remains the same
  - Wavelength is shorter in the "slow" medium:

$$\lambda = \frac{v}{f} = \frac{c}{nf}$$

- The phases must be constant everywhere on a wave front:



# Refraction

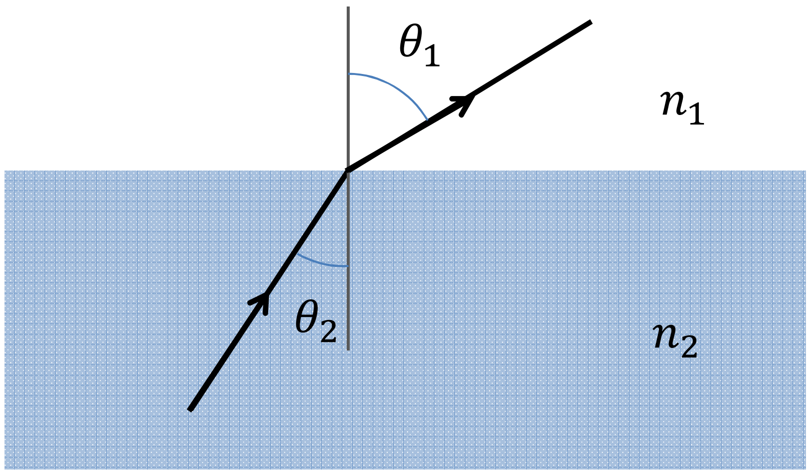


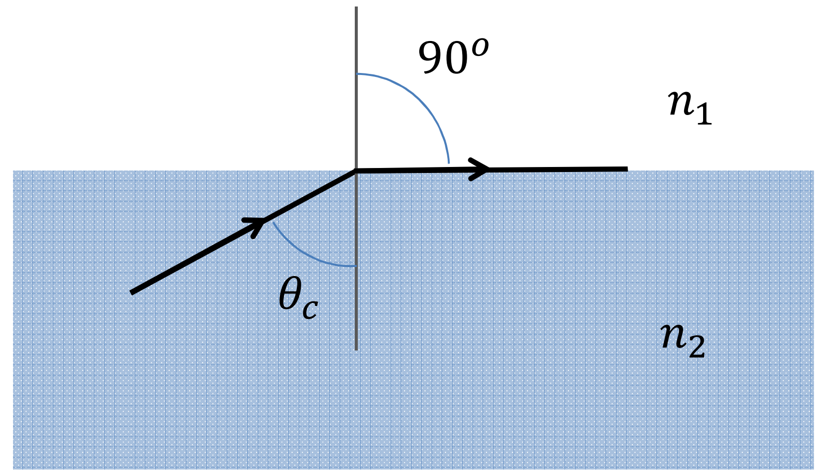

$$x\cos(90^o - \theta_1) = x\sin\theta_1 = \lambda_1$$
$$x\cos(90^o - \theta_2) = x\sin\theta_2 = \lambda_2$$
$$x = \frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$$
$$\lambda = \frac{c}{nf}$$
$$\frac{c}{f}\frac{1}{n_1\sin\theta_1} = \frac{c}{f}\frac{1}{n_2\sin\theta_2}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

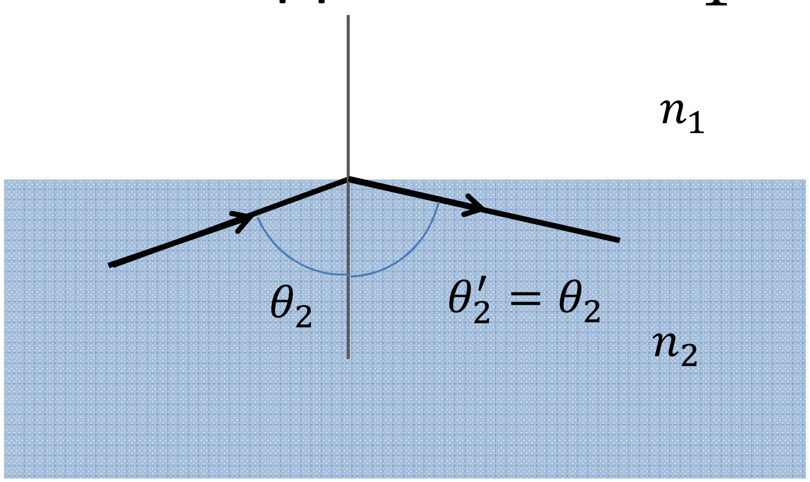
- When light enters a medium with a larger index of refraction, it bends towards the direction that is normal to the surface.

# Total Internal Reflection




- What happens when $\theta_1 \geq 90^o$?




Critical angle:

$$n_2 \sin\theta_c = n_1$$

$$\sin\theta_c = \frac{n_1}{n_2}$$