Purdue University Department of Physics

Physics 42200

Waves & Oscillations

Lecture 23 – Electromagnetism

Spring 2013 Semester

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Midterm Exam:

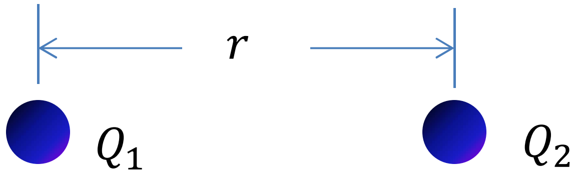
Date:	Wednesday, March 6th
Time:	8:00 – 10:00 pm
Room:	PHYS 203
Material:	French, chapters 1-8

Electromagnetism

- Previous examples of wave propagation:
  - String with tension T and mass per unit length μ
  - Elastic rod with Young's modulus Y and density ρ
  - Sound propagating in air…
  - Waves in water…
  - Etc…
- The rest of the course is devoted to the study of one very important example of wave phenomena: **LIGHT**

Forces on Charges

- Coulomb's law of electrostatic force:



- The magnitude of the attractive/repulsive force is

$$\vec{F} = k\,\frac{|Q_1||Q_2|}{r^2}\,\hat{r}$$

where

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ N \cdot m^2 \cdot C^{-2}$$

and therefore

$$\epsilon_0 = 8.85 \times 10^{-12}\ C^2 \cdot N^{-1} \cdot m^{-2}$$

(This constant is called the "permittivity of free space")

Electric Field

- An electric charge changes the properties of the space around it.
  - It is the source of an "electric field".
  - It could be defined as the "force per unit charge":

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad \Rightarrow \quad \vec{F} = q\vec{E}$$

  - Quantum field theory provides a deeper description…
- Gauss's Law:

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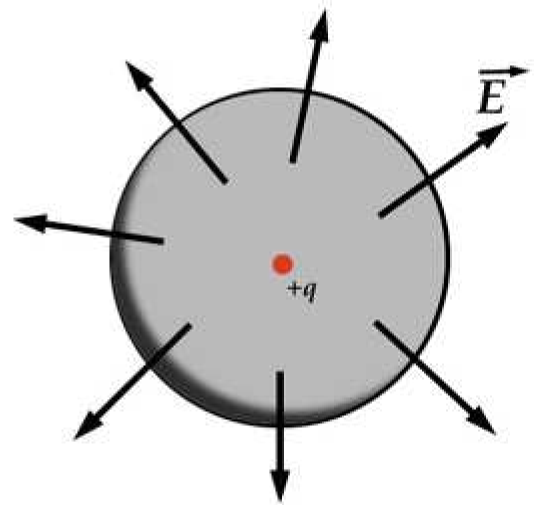
$$\underbrace{\int_S \hat{n}\cdot\vec{E}\,dA}_{\text{Electric "flux" through surface S}} = \frac{Q_{inside}}{\epsilon_0}$$

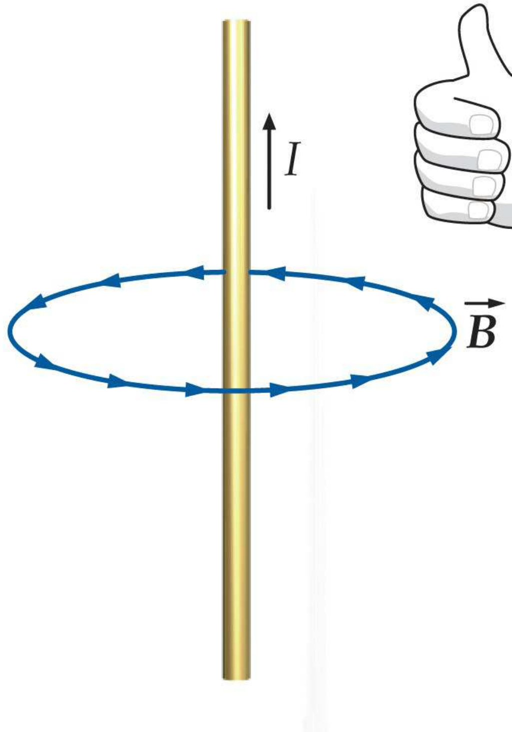
Electric "flux" through surface S



Magnetic Field

- Moving charges (ie, electric current) produce a magnetic field:

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$



A moving charge in a magnetic field experiences a force:

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Lorentz force law:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Gauss's Law for Magnetism

- Electric charges produce electric fields:

$$\int_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$$

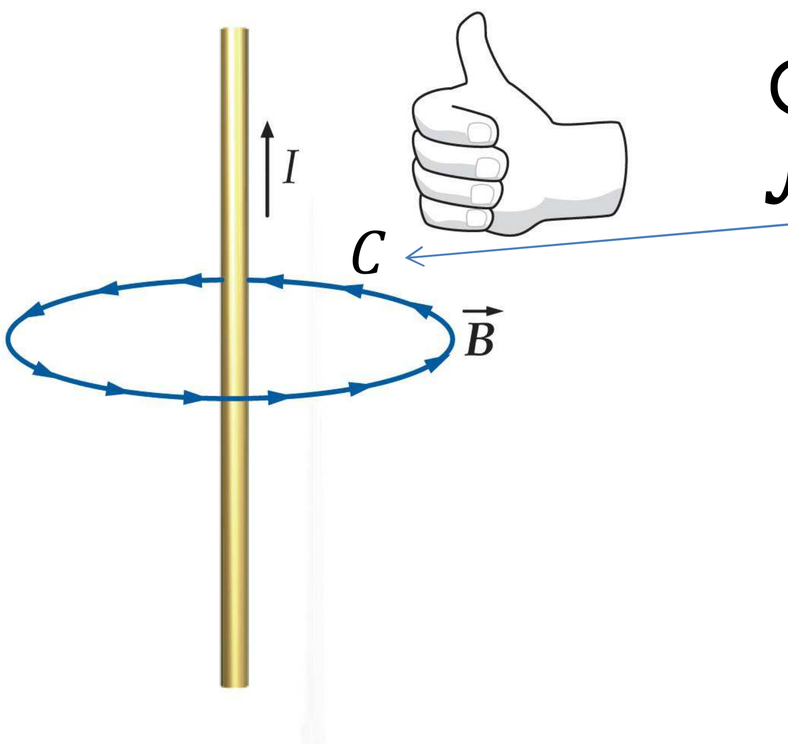
- But there are no "magnetic charges":

2

$$\underbrace{\int_S \hat{n} \cdot \vec{B}\, dA}_{\text{Magnetic "flux" through surface S}} = 0$$

Ampere's Law

- An electric current produces a magnetic field that curls around it:



$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_C$$

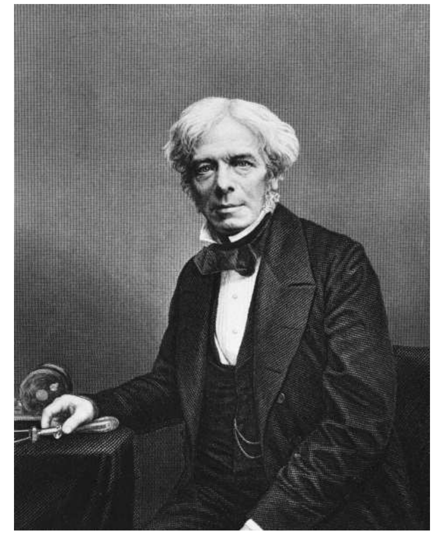
(Close, but not quite the whole story)

Faraday's Law of Magnetic Induction

- Magnetic flux: $\phi_m = \int_S \vec{B} \cdot \hat{n}\, dA$
- Faraday observed that a changing magnetic flux through a wire loop induced a current
  - It transferred energy to the charge carriers in the wire

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \hat{n} \cdot \vec{B}\, dA$$



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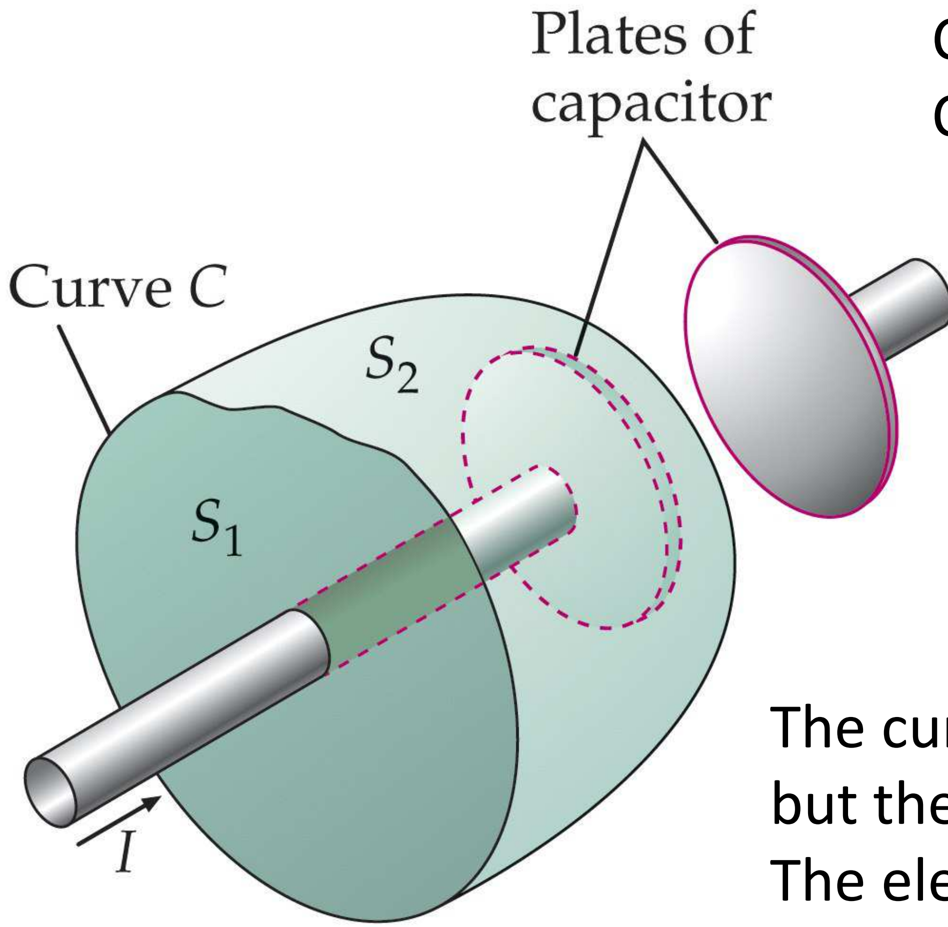
The Problem with Ampere’s Law



Current through S_1 is I
Current through S_2 is 0!

The current through S_2 is zero, but the electric flux is not zero. The electric flux changes as charge flows onto the capacitor.

Maxwell's Displacement Current

- We can think of the changing electric flux through S_2 as if it were a current:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \epsilon_0 \frac{d}{dt}\int_{S_2} \vec{E}\cdot\hat{n}\,dA$$

$$\oint_C \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} = \boldsymbol{\mu_0}(\boldsymbol{I}+\boldsymbol{I_d}) = \boldsymbol{\mu_0}\mathbf{I} + \boldsymbol{\mu_0\epsilon_0}\frac{\boldsymbol{d}}{\boldsymbol{dt}}\int_{S_2}\vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}\,\boldsymbol{dA}$$

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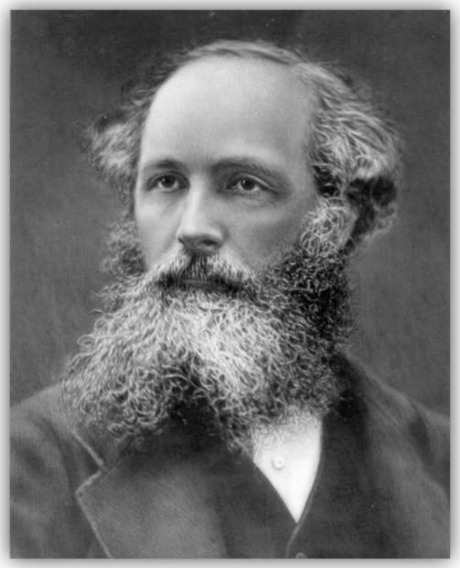
Maxwell's Equations (1864)



$$\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0} \quad (1)$$

$$\oint_S \hat{n} \cdot \vec{B}\, dA = 0 \quad (2)$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt} \quad (3)$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt} \quad (4)$$

Maxwell's Equations in Free Space

In "free space" where there are no electric charges or sources of current, Maxwell's equations are quite symmetric:

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{E}}\, d\boldsymbol{A} = \boldsymbol{0}$$

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{B}}\, d\boldsymbol{A} = \boldsymbol{0}$$

$$\oint_C \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\boldsymbol{\phi}_{\boldsymbol{m}}}{d\boldsymbol{t}}$$

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \boldsymbol{\mu}_0 \boldsymbol{\varepsilon}_0 \frac{d\boldsymbol{\phi}_{\boldsymbol{e}}}{d\boldsymbol{t}}$$

Maxwell's Equations in Free Space

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \varepsilon_0 \frac{d\phi_e}{dt}$$

A changing magnetic flux induces an electric field.

A changing electric flux induces a magnetic field.

Will this process continue indefinitely?

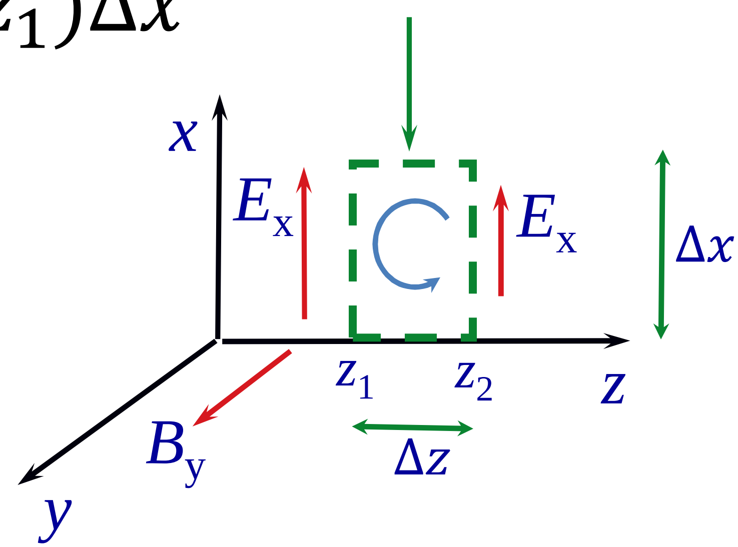
Light is an Electromagnetic Wave

- Faraday's Law:

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt} = -\frac{\partial B_y}{\partial t} \Delta x \Delta z$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = E_x(z_2)\Delta x - E_x(z_1)\Delta x$$

$$\approx \frac{\partial E_x}{\partial z} \Delta z \Delta x$$



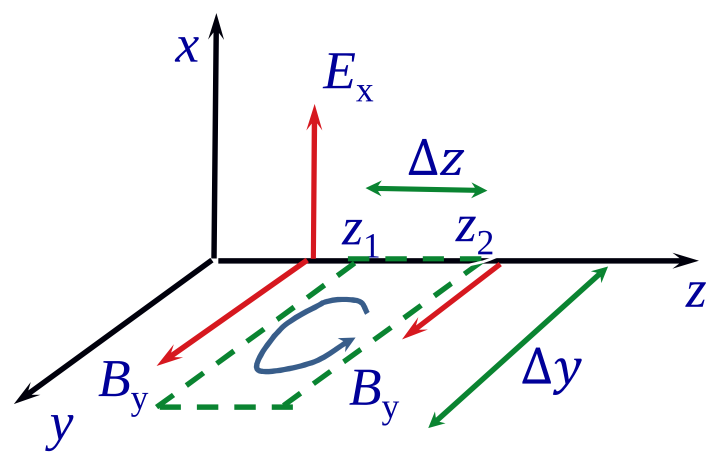
Light is an Electromagnetic Wave

- Ampere's law:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\phi_e}{dt} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} \Delta y \Delta z$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = B_y(z_1)\Delta y - B_y(z_2)\Delta y$$

$$\approx -\frac{\partial B_y}{\partial z} \Delta z \Delta y$$



Putting these together…

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$-\frac{\partial B_y}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$$

Differentiate the first with respect to z:

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial^2 B_y}{\partial z \partial t}$$

Differentiate the second with respect to t:

$$-\frac{\partial^2 B_y}{\partial z \partial t} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Velocity of Electromagnetic Waves

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Speed of wave propagation is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= \frac{1}{\sqrt{(4\pi \times 10^{-7} N/A^2)(8.854 \times 10^{-12}\ C^2/N \cdot m)}}$$

$$= \mathbf{2.998 \times 10^8\ m/s}$$

(speed of light)

Light is an Electromagnetic Wave

Speed of light was measured by Fizeau in 1849:

$$v = 315{,}300\ \mathrm{km/s}$$

Maxwell wrote:

This velocity is so nearly that of light, that it seems we have strong reason to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws.

Electromagnetic Waves

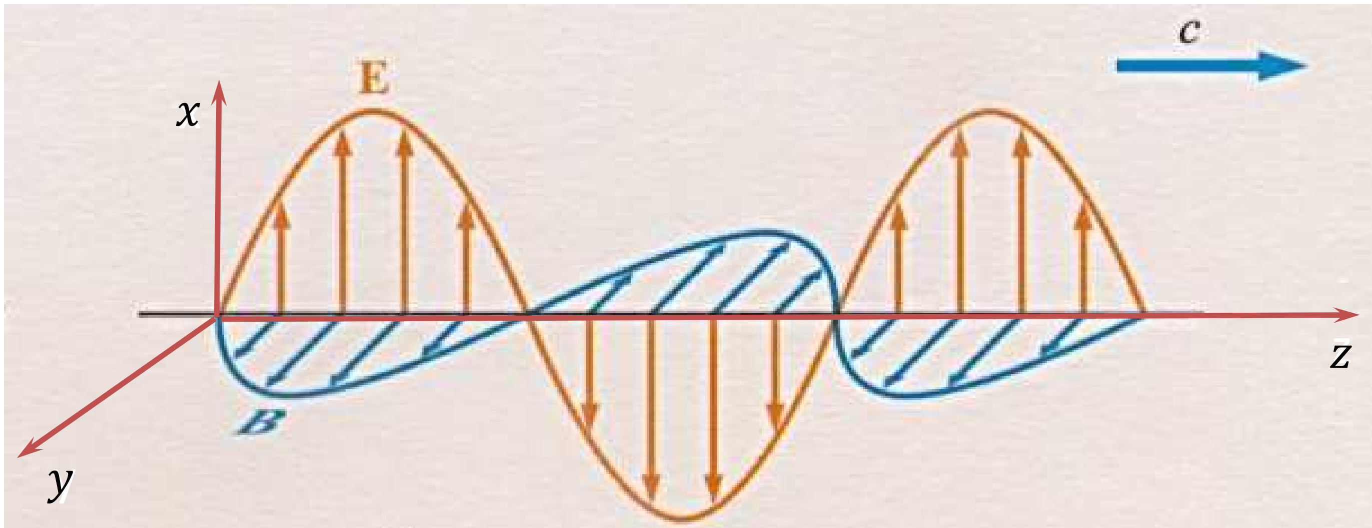
$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

- A solution is $E_x(z,t) = E_0 \sin(kz - \omega t)$ where $\omega = kc = 2\pi c/\lambda$
- What is the magnetic field?

$$\frac{\partial B_y}{\partial t} = -\frac{\partial E_x}{\partial z} = -kE_0 \cos(kz - \omega t)$$

$$B_y(x,t) = \frac{k}{\omega} E_0 \sin(kx - \omega t)$$

Electromagnetic Waves



- $\vec{E}, \vec{B}$ and $\vec{v}$ are mutually perpendicular.
- In general, the direction is

$$\hat{s} = \hat{E} \times \hat{B}$$

Energy in Electromagnetic Waves

Energy density of electric and magnetic fields:

$$u_e = \frac{1}{2}\epsilon_0 E^2 \qquad\qquad u_m = \frac{1}{2\mu_0}B^2$$

For an electromagnetic wave, $B = E/c = E\sqrt{\mu_0 \epsilon_0}$

$$u_m = \frac{1}{2\mu_0}B^2 = \frac{1}{2}\epsilon_0 E^2 = u_e$$

The total energy density is

$$u = u_m + u_e = \epsilon_0 E^2$$

Intensity of Electromagnetic Waves

- Intensity is defined as the average power transmitted per unit area.

Intensity = Energy density × wave velocity

$$I = \epsilon_0 c \langle E^2 \rangle = \frac{\langle E^2 \rangle}{\mu_0 c}$$

$$\mu_0 c = 377\ \Omega \equiv Z_0$$

(Impedance of free space)

Poynting Vector

- We can construct a vector from the intensity and the direction $\hat{s} = \hat{E} \times \hat{B}$:

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\langle \vec{S} \rangle = \frac{\langle E^2 \rangle}{Z_0} = I$$

- This represents the flow of power in the direction $\hat{s}$
- Average electric field: $E_{rms} = E_0/\sqrt{2}$

$$\langle \vec{S} \rangle = \frac{(E_0)^2}{2Z_0}$$

- Units: Watts/m^2

Optics

- If light is an electromagnetic wave, and we just finished reading that yellow book, why are there still 7 weeks left in the course?
- In practice, the boundary value problem is too complicated to solve exactly
- We develop approximation techniques instead
- These can be excellent approximations when used appropriately
- These techniques are so useful for many practical applications that they deserve specific attention