Assignment #8

1. When $\theta_i = \theta_B$ (Brewster's angle) then
   $\theta_i + \theta_e = \frac{\pi}{2}$.

   But $n_1 \sin \theta_i = n_2 \sin \theta_e$ (Snell's law)
   and $\theta_e = \frac{\pi}{2} - \theta_i$.

   So when $\theta_i = \theta_B$,
   $n_1 \sin \theta_B = n_2 \sin \left( \frac{\pi}{2} - \theta_B \right)$
   
   But $\sin \left( \frac{\pi}{2} - \theta_B \right) = \cos \theta_B$

   so $n_1 \sin \theta_B = n_2 \cos \theta_B$

   $\frac{\sin \theta_B}{\cos \theta_B} = \frac{n_2}{n_1}$

   or $\tan \theta_B = \frac{n_2}{n_1}$

2. We expect that the specific rotation will be a linear function of the D-glucose fraction $f_D$. Thus, we can write
   $\alpha = a + bf_D$.

   When $f_D = 1$ (all D-glucose), $\alpha = +52^\circ$

   When $f_D = 0$ (all L-glucose), $\alpha = -52^\circ$.

   Thus, $a = -52^\circ$ and $b = 2 \times 52^\circ = 104^\circ$.

   So $\alpha = (-52^\circ) + (104^\circ)f_D$.
3. First, we need to calculate the angle \( \Theta_t \) using Snell's law:

\[
\eta_1 \sin \Theta_i = \eta_2 \sin \Theta_t
\]

where \( \eta_1 = 1 \), \( \eta_2 = 1.5 \) and \( \Theta_i = 30^\circ \).

This gives

\[
\sin \Theta_t = \frac{\sin 30^\circ}{1.5} = 0.333
\]

so

\[
\Theta_t = 19.47^\circ
\]

(a) Using

\[
R_1 = \frac{\sin^2(\Theta_i - \Theta_t)}{\sin^2(\Theta_i + \Theta_t)}
\]

and

\[
R_\eta = \frac{\tan^2(\Theta_i - \Theta_t)}{\tan^2(\Theta_i + \Theta_t)}
\]

we have

\[
R_1 = \left(\frac{\sin(30^\circ - 19.47^\circ)}{\sin(30^\circ + 19.47^\circ)}\right)^2
\]

\[
= \left(\frac{\sin 10.53^\circ}{\sin 99.47^\circ}\right)^2
\]

\[
= 0.0578
\]

\[
R_\eta = \left(\frac{\tan 10.53^\circ}{\tan 49.47^\circ}\right)^2
\]

\[
= 0.0253
\]
(b) To calculate the effective reflection coefficient when the light also reflects off the back surface, we will first need to calculate the intensity of light in each path shown below:

\[ I_2 = I_1 T_{12} \]
\[ I_3 = I_2 R_{21} = I_1 T_{12} R_{21} \]
\[ I_4 = I_3 T_{21} = I_1 T_{12} R_{21} T_{21} \]

So we need to calculate \( T_{12} \), \( T_{21} \) and \( R_{21} \) for each polarization state.

\[ T_{12} = 1 - R_{12} \]
\[ T_{\perp,12} = 1 - 0.0578 = 0.9422 \]
\[ T_{\parallel,12} = 1 - 0.0253 = 0.9747 \]
\[ R_{1,21} = \left( \frac{\sin(19.47^\circ - 30^\circ)}{\sin(19.47^\circ + 30^\circ)} \right)^2 = 0.0578 \]
\[ R_{\parallel,21} = \left( \frac{\tan(19.47^\circ - 30^\circ)}{\tan(19.47^\circ + 30^\circ)} \right)^2 = 0.0253 \]
\[ T_{\perp,21} = 1 - R_{\perp,21} = 0.9422 \]
\[ T_{\parallel,21} = 1 - R_{\parallel,21} = 0.9747 \]
So \[ R'_\perp = 0.0578 + (0.9422)(0.0578)(0.9422) \]
\[ = 0.0578 + 0.0513 \]
\[ = 0.109 \]

\[ R'_\parallel = 0.0253 + (0.9747)(0.0253)(0.9747) \]
\[ = 0.0253 + 0.0240 \]
\[ = 0.0493 \]

In principle, light could be reflected multiple times inside the slice of material so the effective reflection coefficients would be even larger than these.