1. Consider Young's double slit experiment without a glass sheet in front of one slit:

The number of wavelengths in \( r_1 \) is \( r_1 / \lambda \) and the phase advance is \( kr_1 \). Likewise, the phase advance of the second path is \( kr_2 \) so the phase difference is \( \delta = k(r_2 - r_1) \). The difference in path length is \( \Delta y = \frac{kd\delta}{s} \) so the phase difference is \( \delta \approx \frac{kdy}{s} \). The \( m^{th} \) maximum occurs at \( kdy = 2\pi m \).

So \( y = \frac{2\pi ms}{kd} = \frac{2\lambda ms}{d} \).

Now, if a glass sheet is placed in front of one of the slits, then the phase advance will be \( k't + k(r - t) \) where \( k' = nk \) is the wavenumber in the glass sheet of thickness \( t \) with index of refraction \( n \). In this case,

\[
\delta' = k't + k(r - t) - kr,
\]

\[
= kt(n-1) + \delta = \delta + kt(n-1) = \frac{kdy'}{s} + kt(n-1) = 2\pi m
\]

Thus, \( y' = \frac{\lambda ms}{d} - \frac{st(n-1)}{d} \).

Here, we assumed that \( y/s \ll 1 \).
2. Consider the change in angle as a ray passes through a prism with angle $\alpha$.

At the first surface, the transmitted angle is given by Snell's law:

$$n' \theta = n \theta_t \implies \theta_t = \frac{n'}{n} \theta$$

The ray then impinges on the second surface with an angle of incidence $\theta'_i = \theta_t - \alpha$ and is refracted to an angle

$$\theta'_t = \frac{n'}{n} \theta'_i = \frac{n'}{n} (\theta_t - \alpha) = \theta - \frac{\alpha n}{n'}$$

with respect to the second surface. This angle is then

$$\theta' = \theta'_t + \alpha = \theta - \frac{\alpha n}{n'} + \alpha = \theta - \alpha \left( \frac{n - n'}{n'} \right)$$

When passing through the bottom part of the biprism, the angle of the refracted beam is

$$\theta' = \theta + \alpha \left( \frac{n - n'}{n'} \right)$$

The system acts like a double-slit experiment. The separation between the vertical sources can be calculated using $\theta' = 0$.

\[
\alpha \quad \theta' \quad \rightarrow \quad \theta \quad d 
\]
Thus, \( \frac{1}{2} a = d\Theta = d\alpha \left( \frac{n - n'}{n'} \right) \)

So \( \alpha = 2d\alpha \left( \frac{n - n'}{n'} \right) \).

Once we treat the problem as a double-slit experiment we can calculate the path length difference at a point \( y \) on a screen a distance \( s \) from the source.

\[
S = \frac{kay}{s} = \frac{2\pi n'\alpha y}{\lambda_0 s} = 2\pi M
\]

the fringe separation is then

\[
\frac{n'\alpha}{\lambda_0 s} \Delta y = 1 \quad \text{or} \quad \Delta y = \frac{\lambda_0 s}{n'\alpha} = \frac{\lambda_0 s}{2d\alpha (n - n')}
\]

where \( \lambda_0 \) is the wavelength in free space.
Separation between the fringes is a which corresponds to one wavelength difference in the optical path lengths.

The geometric path length difference is

\[ \Delta d = 2a \alpha \]

So \[ 2a \alpha = \frac{\lambda_0}{\pi} \]

and \[ \alpha = \frac{\lambda_0}{2a \pi} = \frac{(500 \text{ nm})}{(2) (\frac{1}{3} \text{ cm}) (1.5)} \]

\[ = \frac{500 \times 10^{-7} \text{ cm}}{2 \times \frac{1}{3} \times \frac{3}{2} \text{ cm}} = 500 \times 10^{-7} \]

\[ = 5 \times 10^{-5} \text{ radians}. \]
4. The optical path length of the chamber is 2nd and the number of fringes that shift as the optical path length is reduced to 2d will be

\[ m = \frac{2(n - 1)d}{\lambda_0} = \frac{2(1.00029 - 1)(10 \text{ cm})}{600 \times 10^{-7} \text{ cm}} = 96.67 \approx 97. \]

5. Since \( n_1 > n \) and \( n_2 > n_1 \), there is a 180° phase shift at each interface. Thus, the phase difference is just due to the optical path length in the film: \( 2n_1t \).

If the reflected light is to be 1/2 a wavelength out of phase, then

\[ 2n_1t = \frac{\lambda_0}{2} \quad \Rightarrow \quad t = \frac{\lambda_0}{4n_1} \approx \frac{500 \text{ nm}}{4(1.30)} = 96.2 \text{ nm}. \]

The cryolite film should be 96.2 nm thick.
\[
\begin{align*}
\alpha & \quad \frac{R+l}{R} \quad a \\
(R+l)^2 &= R^2 + a^2 \\
R^2 + 2RL + l^2 &= R^2 + a^2
\end{align*}
\]

Fraunhofer diffraction occurs when

\[
l^2 + 2RL - a^2 = 0
\]

\[
l = -R + \sqrt{R^2 + a^2}
\]

\[
= -R + R \sqrt{1 + \frac{a^2}{R^2}}
\]

\[
= \frac{a^2}{2R} \ll \lambda
\]

So \( \frac{a^2}{2} \ll 2R \)

Smallest \( R \) when \( a = 1 \text{ mm} \), \( \lambda = 500 \text{ nm} \)

\[
\frac{\lambda}{10} = \frac{a^2}{2R}
\]

\[
R = 10a^2 = 5 \left( \frac{10^{-3}}{500 \times 10^{-3}} \right)^2 = \frac{10^{-6}}{10^{-7}} = 10 \text{ m}
\]