Physics 310 - Assignment #6 - Due Thursday, December 9^{th}

1. (Fowles and Cassiday, 10.3)

Find the acceleration of a solid uniform sphere that rolls without slipping down an incline plane that is tilted at an angle θ with respect to the horizontal plane.

2. (Fowles and Cassiday, example 10.5.4 and problem 10.17) Use the method of Lagrange multipliers to find the equations of motion for the three masses and the tensions in the strings of the "Double Atwood Machine":



in which the strings have length l_1 and l_2 and the pulleys are assumed to be massless.

3. A bead of mass m, which is acted on by gravity, slides on a thin, circular hoop of radius a that spins about the z axis with constant angular velocity ω as shown:



Find the equation of motion of the bead expressed in terms of the angle θ . What is the period of small oscillations about its equilibrium position?

4. The awesome suspension system of the 1999 Toyota Corolla can be approximate by two springs with equal spring constants, k, attached to a body of mass M and moment of inertia, I, about the center of mass as shown:



The generalized coordinates x and θ represent the height of the center of mass and the angle that the body makes with respect to the horizontal plane.

(a) Express the kinetic energy of the system in terms of x and θ , taking into account the vertical translation and the rotation about the center of mass.

(b) Express the potential energy of the system in terms of x and θ , taking into account the gravitational potential energy of the body and the compression of the springs.

(c) Write Lagrange's equations for the system.

(d) Calculate the frequencies of the normal modes of vibration.