Physics 310 - Assignment #2 - Due September 28^{th}

1. My 1999 Corolla needs its wheels blanaced. The wheels have a diameter d, and the steering wheel vibrates with a maximum amplitude of A_{max} when I drive with a speed v.
   (a) If the amplitude is reduced to $A_{\text{max}}/\sqrt{2}$ when I drive at $v + \Delta v$, find an expression for the Q value of my car.
   (b) Calculate the numerical value of Q when $d = 15$ in, $v = 75$ MPH, and $\Delta v = 5$ MPH.

2. Suppose the force on an object of mass m is a function of its position, x, and has the form
$$F(x) = -\frac{a}{x^2} + \frac{b}{x^3}$$
   (a) Find an expression $V(x)$ that represents the potential energy function corresponding to this force.
   (b) Calculate the separation, x_0, that minimizes the potential energy function.
   (c) Express the potential energy function as a power series in $(x - x_0)$, explicitly showing terms up to order $(x - x_0)^3$.
   (d) For small oscillations about x_0, calculate the effective spring constant, k that approximates this force.

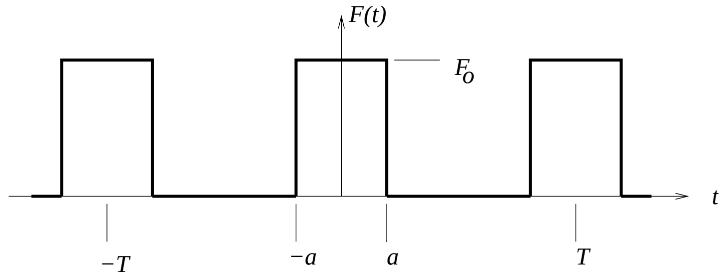
3. *(Fowles and Casssiday, problem 3.11)*
   A mass m moves along the x-axis subject to an attractive force given by $17\beta^2 mx/2$ and a retarding force given by $3\beta m\dot{x}$, where x is its distance from the origin and β is a constant. A driving force given by $mA\cos\omega t$, where A is a constant, is applied to the particle along the x-axis.
   (a) What value of ω results in steady-state oscillations about the origin with maximum amplitude?
   (b) What is the maximum amplitude?

4. *(Fowles and Casssiday, problem 3.19)*
   A simple pendulum of length ℓ oscillates with an amplitude of 45°.
   (a) What is the period?
   (b) If this pendulum is used as a laboratory experiment to determine the value of g, find the error included in the use of the elementary formula $T_0 = 2\pi\sqrt{\ell/g}$.
   (c) Find the approximate amount of third-harmonic content in the oscillation of the pendulum.

5. A harmonic oscillator is described by the differential equation

$$m\ddot{x} + c\dot{x} + kx = F(t) \Longrightarrow \ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = F(t)/m$$

where $F(t)$ is a periodic force of the form:



which can be expressed:

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t)$$

where

$$\omega = 2\pi/T$$

and

$$a_n = \frac{T}{2}\int_{-T/2}^{T/2} F(t)\cos(n\omega t)$$

Calculate the amplitude of oscillations when $T = 2\pi\sqrt{m/k}$, $4\pi\sqrt{m/k}$ and $6\pi\sqrt{m/k}$ for the case where $\gamma << \omega_0$.

6. Using the method of Laplace transforms, find the solution to an underdamped harmonic oscillator problem

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

with initial conditions $x(0) = \dot{x}(0) = 0$ where

$$F(t) = \begin{cases} 0 & \text{for } t < 0 \\ F_0 e^{-at} & \text{for } t > 0 \end{cases}$$

(a) First, evaluate the Laplace transform of both sides of the differential equation, writing the Laplace transform of the solution, $X(s) = \mathcal{L}\{x(t)\}$.
(b) Solve the algebraic equation for $X(s)$.
(c) Use the identity:

$$\mathcal{L}^{-1}\{f(s)g(s)\} = \int_0^t F(u)G(t-u)du$$

to evaluate $x(t)$.

7. Use the 4^{th} order Runge-Kutta integration formulas:

$$
\begin{aligned}
k_1 &= x'(t_i, x_i, y_i) \\
j_1 &= y'(t_i, x_i, y_i) \\
k_2 &= x'(t_i + \tfrac{1}{2}h, x_i + \tfrac{1}{2}hk_1, y_i + \tfrac{1}{2}hj_1) \\
j_2 &= y'(t_i + \tfrac{1}{2}h, x_i + \tfrac{1}{2}hk_1, y_i + \tfrac{1}{2}hj_1) \\
k_3 &= x'(t_i + \tfrac{1}{2}h, x_i + \tfrac{1}{2}hk_2, y_i + \tfrac{1}{2}hj_2) \\
j_3 &= y'(t_i + \tfrac{1}{2}h, x_i + \tfrac{1}{2}hk_2, y_i + \tfrac{1}{2}hj_2) \\
k_4 &= x'(t_i + h, x_i + hk_3, y_i + hj_3) \\
j_4 &= y'(t_i + h, x_i + hk_3, y_i + hj_3) \\
t_{i+1} &= t_i + h \\
x_{i+1} &= x_i + \tfrac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \\
y_{i+1} &= y_i + \tfrac{1}{6}h(j_1 + 2j_2 + 2j_3 + j_4)
\end{aligned}
$$

to calculate the period of oscillation of a simple pendulum with $\ell = 1$ m, $g = 9.81$ m/s^2 and initial displacements of $\theta_{\text{max}} = 2°$, $10°$, $45°$ and $90°$. To do this, use $h = 10^{-4}$ sec and find the time at which θ returns to θ_{max} to within $\pm h$.

Compare the answers with the linear approximation, and the first order non-linear approximation (question 4).

Print out our program and hand it in with your assignment.