Physics 310 - First Midterm - October 12, 2006

Instructions

• This exam has three pages including this cover sheet.
• There are four questions on this exam.
• The questions are to be answered in the blue exam booklets.
• Write your name on every exam booklet you use.
• No notes, text books, crib sheets, etc. are to be used when writing this exam.
• You will not need a calculator for this exam.
• The exam begins at 9:30 am and must be turned in by 11:30 am.
• No guinea-pigs were harmed in the making of this exam.
1. A particle travels on a spiral path that is described in polar coordinates by the variables:

\[ r(t) = be^{kt} \]
\[ \theta(t) = ct \]

where \( b \), \( k \) and \( c \) are constants.

(a) Calculate the velocity \( \mathbf{v} = \frac{dr}{dt} \).
(b) Calculate the magnitude of the velocity, \( v = |\mathbf{v}| \).
(c) Calculate the acceleration \( \mathbf{a} = \frac{d\mathbf{v}}{dt} \).
(d) Calculate the magnitude of the acceleration, \( a = |\mathbf{a}| \).

2. A particle of mass \( m \) is shot horizontally into a liquid that exerts a force that is proportional to the velocity of the particle: \( F_{\text{drag}} = -cv \).

(a) If the particle has initial velocity \( v_0 \) when it enters the liquid at \( t = 0 \), calculate its horizontal distance, \( x(t) \), as a function of time.
(b) What is the maximum distance that it can reach in the liquid?

3. Show that any “central force” is conservative. That is, show that a force \( \mathbf{F}(\mathbf{r}) \) is conservative when it is of the form

\[ \mathbf{F}(\mathbf{r}) = f(r)\mathbf{\hat{r}} \]

where \( r = \sqrt{x^2 + y^2 + z^2} \) is the distance from the origin of the coordinate system. Remember that you can write \( \mathbf{\hat{r}} = \mathbf{r}/r \).
4. A merry-go-round rotates with constant angular velocity $\omega = \omega \hat{k}$. “Chipper”, a grade 4 class guinea-pig at the Miller Elementary School in Lafayette, starts at the center at $t = 0$ and crawls with a constant speed in the radial direction so that its position is given by $r(t) = vt\hat{r}$. Here, $\hat{r}$ is the radial unit vector that rotates with the merry-go-round so that it is constant when viewed in this non-inertial reference frame.

(a) If the mass of the guinea-pig is $m$, calculate the inertial, or “fictitious” forces that appear to act on the animal when viewed from the rotating reference frame of the merry-go-round.

(b) Calculate the magnitude of the sum of the inertial forces as a function of time.

(c) If the coefficient of static friction between Chipper and the merry-go-round is $\mu$, how far will Chipper crawl before he slips?

Remember that

$$a' = a - \hat{\omega} \times r' - 2\omega \times v' - \omega \times (\omega \times r').$$

where $a$ is the acceleration in an inertial reference frame and the primed quantities are measured in a reference frame that rotates with angular velocity $\omega$. Also,

$$\hat{r} \times \hat{\theta} = \hat{k},$$
$$\hat{\theta} \times \hat{k} = \hat{r},$$
$$\hat{k} \times \hat{r} = \hat{\theta}.$$