Physics 24100

Electricity & Optics

Lecture 9 – Chapter 24 sec. 3-5

Fall 2012 Semester

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Thursday’s Clicker Question

• To double the capacitance of a parallel plate capacitor, you should:

(a) Double the area of the plates
(b) Half the distance between the plates
(c) Both (a) and (b)
(d) Either (a) or (b)
Thursday’s Clicker Question

• To double the capacitance of a parallel plate capacitor, you could *either* double the area or half the distance:

\[ C = \frac{\varepsilon_0 A}{d} \]

• If you did both you would *quadruple* the capacitance.
Parallel Plate Capacitor

\[ C = \frac{\varepsilon_0 A}{d} \]
Two Parallel Plate Capacitors

\[ C_1 = \frac{\epsilon_0 A_1}{d} \]

\[ C_2 = \frac{\epsilon_0 A_2}{d} \]
One Big Parallel Plate Capacitor

\[ C = \frac{\varepsilon_0 (A_1 + A_2)}{d} \]
Two Parallel Plate Capacitors

- Remember: *conductors in electrical contact have the same electric potential.*
- We could measure the potential difference between points A and B if they were connected to the plates with wires.

\[
Q = CV
\]

\[
C = \frac{\varepsilon_0 (A_1 + A_2)}{d}
\]
Capacitors In Parallel

• Symbol for a capacitor: $\cap$

• Capacitors in parallel:

• Equivalent capacitance:

$$C = C_1 + C_2$$
Capacitors in Parallel

• Consider two capacitors, \( C_1 \) and \( C_2 \), with the same potential difference, \( V \), across them.

• The charge on each one is

\[
Q_1 = C_1 V \\
Q_2 = C_2 V
\]

• If they are connected in parallel then the potential difference doesn’t change but now we can write:

\[
Q = Q_1 + Q_2 = (C_1 + C_2)V = C \cdot V
\]

where

\[
C = C_1 + C_2
\]
Capacitors in Series

- Consider two capacitors, $C$, each with potential difference, $V$, and charge $Q = CV$:

- If they are brought into electrical contact the potential difference is $2V$ but the charge is still $Q$.

- The effective capacitance is $C'$ where
  \[ C' = \frac{Q}{2V} = \frac{C}{2} \]

- Equivalent series capacitance is reduced.
Capacitors in Series

- Consider two parallel plate capacitors:

\[ C_1 = \frac{\varepsilon_0 A}{d_1} \]

\[ C_2 = \frac{\varepsilon_0 A}{d_2} \]
Capacitors in Series

• Consider two parallel plate capacitors:

\[ C_1 = \frac{\varepsilon_0 A}{d_1} \]

\[ C_2 = \frac{\varepsilon_0 A}{d_2} \]
Capacitors in Series

• Consider two parallel plate capacitors:

\[ C = \frac{\epsilon_0 A}{d_1 + d_2} \]
Capacitance in Series

\[ C = \frac{\epsilon_0 A}{d_1 + d_2} \]

\[ \frac{1}{C} = \frac{d_1 + d_2}{\epsilon_0 A} = \frac{1}{C_1} + \frac{1}{C_2} \]

• Equivalent series capacitance:

\[ C = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} \]
Example: Equivalent Capacitance

- Coaxial cylinders have capacitance

\[ C = \frac{2\pi \epsilon_0 L}{\log(R_2/R_1)} \]

- Suppose three identical coaxial cylinders were connected together in a particular way...

- What would be the equivalent capacitance?
Example: Equivalent Capacitance

- There are two ways to connect them:
  - In series (outside to inside):
  - In parallel (inside to inside, outside to outside):
Example: Equivalent Capacitance

- What is the equivalent capacitance when connected in the following way?
Example: Equivalent Capacitance

- What is the equivalent capacitance when connected in the following way?

Combine capacitors in parallel: \( C = C_1 + C_2 \)
Example: Equivalent Capacitance

- What is the equivalent capacitance when connected in the following way?

\[
C = \left( \frac{1}{C} + \frac{1}{2C} \right)^{-1} = \frac{2}{3}C
\]

Combine capacitors in series: \( C = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} \)
Question

• What is the value of the equivalent capacitance:

(a) $C_{equiv} = \frac{2}{3}C$
(b) $C_{equiv} = \frac{3}{2}C$
(c) $C_{equiv} = 3C$
(d) $C_{equiv} = \frac{1}{3}C$
Question

• What is the value of the equivalent capacitance:

\[ C' = \left( \frac{1}{C} + \frac{1}{C} \right)^{-1} = \frac{C}{2} \]

\[ C_{equiv} = C + \frac{C}{2} = \frac{3}{2}C \]

(a) \( C_{equiv} = \frac{2}{3}C \)  
(b) \( C_{equiv} = \frac{3}{2}C \)  
(c) \( C_{equiv} = 3C \)  
(d) \( C_{equiv} = \frac{1}{3}C \)
Another Question

• How would the capacitance of a parallel plate capacitor change if we replaced half the space with a conductor?

(a) It would double  
(b) It would be half  
(c) No change
Adding Conducting Material

- The capacitance would double.

\[ C' = \frac{\varepsilon_0 A}{(d/4)} = 4C \]

\[ C_{equiv} = \left( \frac{1}{C'} + \frac{1}{C'} \right)^{-1} = \frac{1}{2} \times 4C = 2C \]

- The capacitance would double.
Adding an Insulating Material (Dielectric)

- Empirical observation: inserting a dielectric material reduces the electric potential difference.
- If $Q$ is constant, the capacitance must increase.
- The “dielectric constant”, $\kappa$, of the material is the ratio of the capacitance when the space between the conductors is filled with the dielectric material, to the original capacitance.

$$\kappa = \frac{C'}{C} > 1$$

$$C = \frac{\kappa \varepsilon_0 A}{d}$$
Permittivity

• The ratio $\kappa$ is called the dielectric constant.
• We can define

$$\varepsilon = \kappa \varepsilon_0$$

which is called the permittivity of the material with dielectric constant $\kappa$.
• We can replace $\varepsilon_0$ with $\varepsilon$ in formulas when the space between conductors is not vacuum.
• For example, the parallel plate capacitor...

$$C = \frac{\varepsilon_0 A}{d} \quad \rightarrow \quad C = \frac{\varepsilon A}{d}$$
Why?

• Remember the electric dipole? Insulators in an electric field are composed of electric dipoles:

The dipoles line up with the electric field lines.

The surface charge on the dielectric partially cancels the charge on the plates of the capacitor.

The effective $Q$ is reduced.

\[ C = \frac{Q}{V} \text{ decreases.} \]
Dielectric Breakdown

• If the electric field is too strong, it can ionize the dielectric (rip electrons off its molecules).
• Free electrons = conductor... (sparks).
• Can result in damage to the dielectric.
• **DIELECTRIC STRENGTH**: maximum *electric field* a dielectric can tolerate before breaking down.

• **BREAKDOWN POTENTIAL**: maximum *electric potential* a device can tolerate before breaking down.
# Dielectric Constants

<table>
<thead>
<tr>
<th>material</th>
<th>$\kappa$</th>
<th>dielectric strength (kV/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>air (1 atm)</td>
<td>1.00054</td>
<td>3</td>
</tr>
<tr>
<td>paraffin</td>
<td>2.1–2.5</td>
<td>10</td>
</tr>
<tr>
<td>glass (Pyrex)</td>
<td>5.6</td>
<td>14</td>
</tr>
<tr>
<td>mica</td>
<td>5.4</td>
<td>10–100</td>
</tr>
<tr>
<td>polystyrene</td>
<td>2.55</td>
<td>24</td>
</tr>
<tr>
<td>$\text{H}_2\text{O (20}^\circ\text{C)}$</td>
<td>80</td>
<td>?</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>240</td>
<td>8</td>
</tr>
<tr>
<td>Sulfur hexafluoride</td>
<td>1.002026</td>
<td>9</td>
</tr>
</tbody>
</table>
Preventing Electrical Discharge

• The PRIME lab in the basement uses a tandem Van de Graaff generator to accelerate atomic fragments.
• It is full of Sulfur Hexafluoride to increase the breakdown potential.

• For more fun with Sulfur Hexafluoride, check out http://www.youtube.com/watch?v=u19QfJWl1oQ
Dielectric Combinations

\[ \frac{d}{2} \]

\[ \frac{d}{2} \]

\[ \kappa_1 \]

\[ \kappa_2 \]

\[ \kappa_1 \]

\[ \kappa_2 \]

\[ \frac{A}{2} \]

\[ \frac{A}{2} \]

\[ d \]

\[ d \]
Summary

- Capacitors in parallel:
  \[ C = C_1 + C_2 \]

- Capacitors in series:
  \[ C = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} \]

- Dielectrics increase capacitance: \( \kappa > 1 \)
Question:

- Three circuits, consisting of two capacitors and a switch, are initially charged as indicated.
- After the switches are closed, in which circuit will the charge on the left increase?

(a) \(6q\) \(2C\) \(3q\) C
(b) \(6q\) \(3C\) \(3q\) C
(c) \(6q\) \(2C\) \(3q\) 2C
(d) None of them