Purdue University Department of Physics

Physics 24100

Electricity & Optics

Lecture 8 – Chapter 24 sec. 1-2

Fall 2012 Semester

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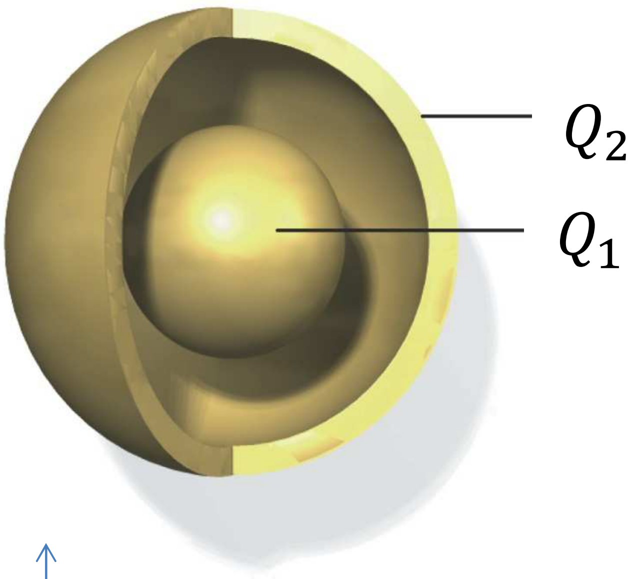
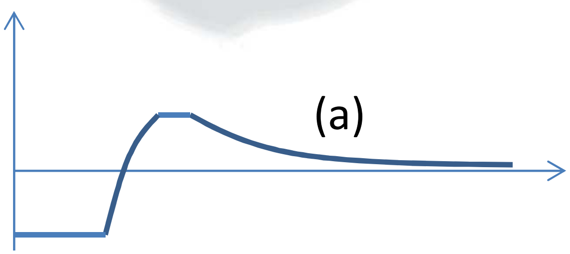
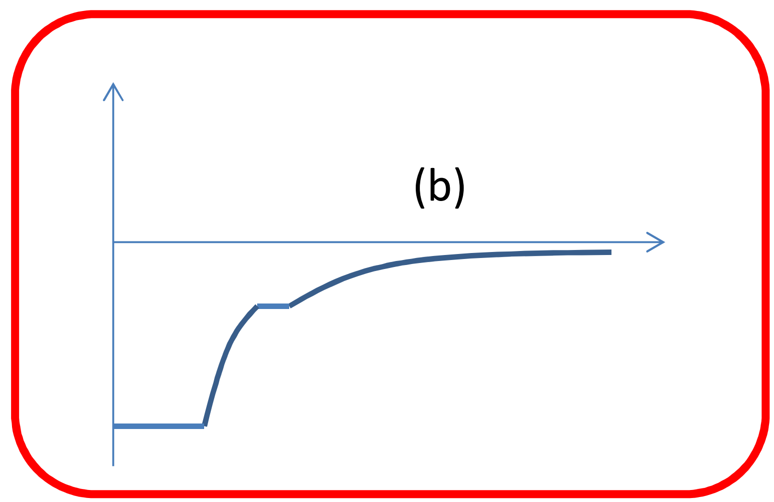
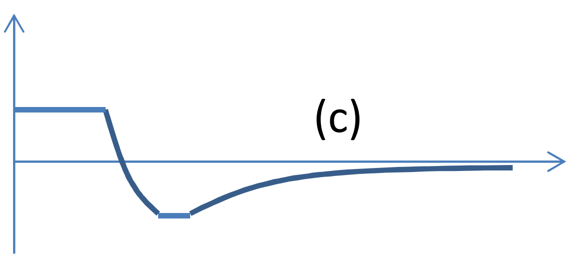
Tuesday's Clicker Question

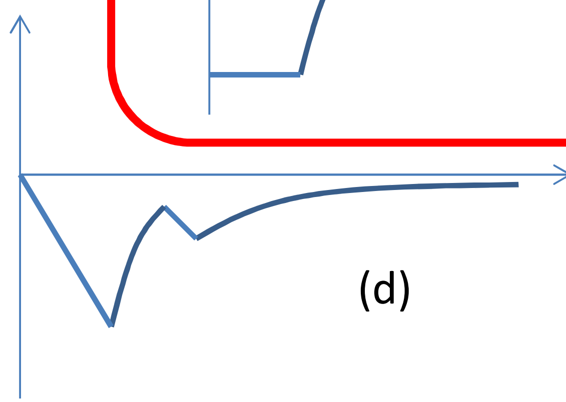
If $Q_2 = 1\ \mu C$ and $Q_1 = -2\ \mu C$ which graph most accurately shows $V(r)$?



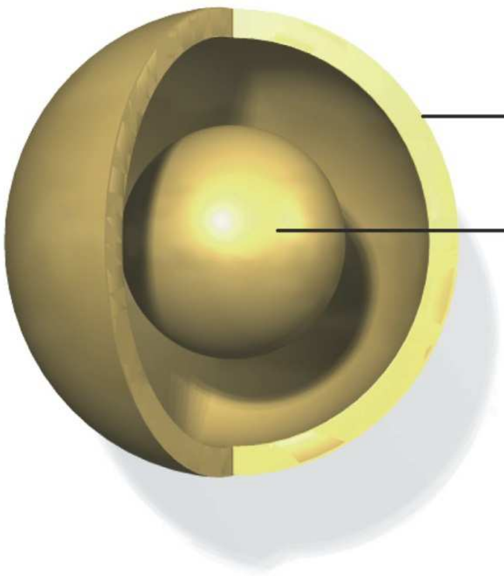








Tuesday's Clicker Question



$Q_2 = 1\ \mu C$ and $Q_1 = -2\ \mu C$

What we know already:

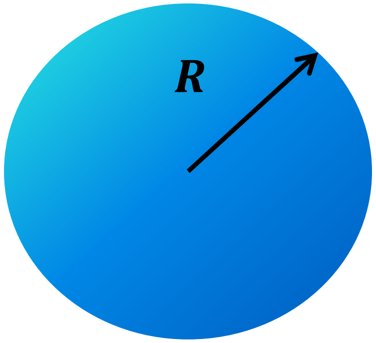
- Net charge is $-1\ \mu C$
- Charge on inner surface of outer spherical shell is $+2\ \mu C$

- Electric field is always pointing towards the origin.
  - $V(r)$ will always be negative.
  - A positive charge has less potential energy closer to the sphere.
- Electric potential inside the conductors is constant.
  - $V(r)$ will be flat in the shell and in the sphere.

Storing Energy

Tuesday's example with the Van de Graaff…

Initial charge: Q



Initial electric potential at the surface:

$$V(R) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

Work needed to add an additional charge, ΔQ:

$$\Delta U = V(R)\Delta Q = \frac{1}{4\pi\epsilon_0}\frac{Q\Delta Q}{R}$$

Total work needed to add charge Q_{total}:

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{R}\int_0^{Q_{total}} Q\, dQ = \frac{1}{4\pi\epsilon_0}\frac{1}{R}\times\frac{1}{2}(Q_{total})^2$$

Storing Energy

Total work needed to charge sphere:

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{R} \times \frac{1}{2} Q^2$$

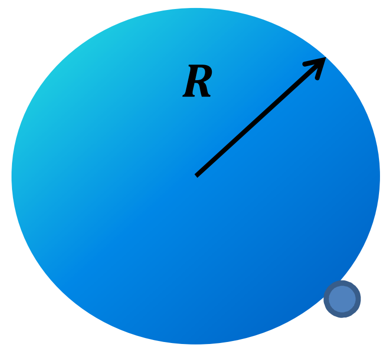
Final voltage of sphere:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

$$Q = 4\pi\epsilon_0 RV$$

Stored energy:

$$E = \frac{1}{2}\,(4\pi\epsilon_0 R)\,V^2$$



In principle, we can use this energy to do work…

Capacitors

- A *capacitor* is a device that stores electrostatic potential energy.
- These are (almost) always two conductors in close proximity separated by an insulator:
  - Air or vacuum
  - Something that prevents sparks
  - Something that also stores electrostatic potential energy
- Examples:
  - Two parallel wires
  - Two parallel plates
  - Two coaxial cylinders

How Much Energy?

- Consider two conductors with electric potentials V_1 and V_2…
- We can always pick $V_1 = 0$ and then V_2 is just the potential difference between them.

How Much Energy?

- Take a small charge ΔQ from the blue conductor and move it to the red conductor…
- Work done is $\Delta U = V\Delta Q$
- How much will this change V?
  - It turns out that $\Delta V \propto \Delta Q$

Capacitance

$$\Delta Q \propto \Delta V$$

- The constant of proportionality is the capacitance, C:

$$\Delta Q = C\,\Delta V$$

- If you transfer a total charge Q, then the potential difference will be:

$$V = \frac{Q}{C}$$

- *The capacitance is defined as:*

$$C = \frac{Q}{V}$$

Units for Capacitance:

$$Farad = \frac{Coulomb}{Volt}$$

$$\epsilon_0 = 8.85\ pF/m$$

Stored Energy

- How much work, U, does it take to charge a capacitor to a final voltage, V?

$$\Delta U = V\,\Delta Q = C\,V\,\Delta V$$

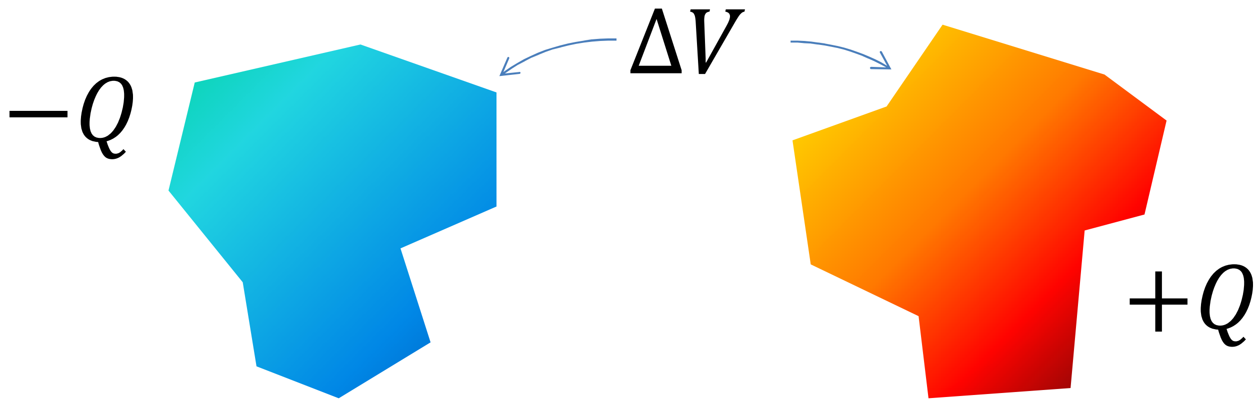
$$U = C\int_0^V V\,dV = \frac{1}{2}C\,V^2$$

Equally valid:

$$U = \frac{1}{C}\int_0^Q Q dQ = \frac{1}{2C}Q^2$$

- This is the energy stored in the capacitor.
- You don’t have to “move” the same charge from one conductor to the other, just the same *amount* of charge.

Question



If the charge, Q, is doubled, will the capacitance…

(a) Increase?

(b) Decrease?

(c) Remain the same?

Examples

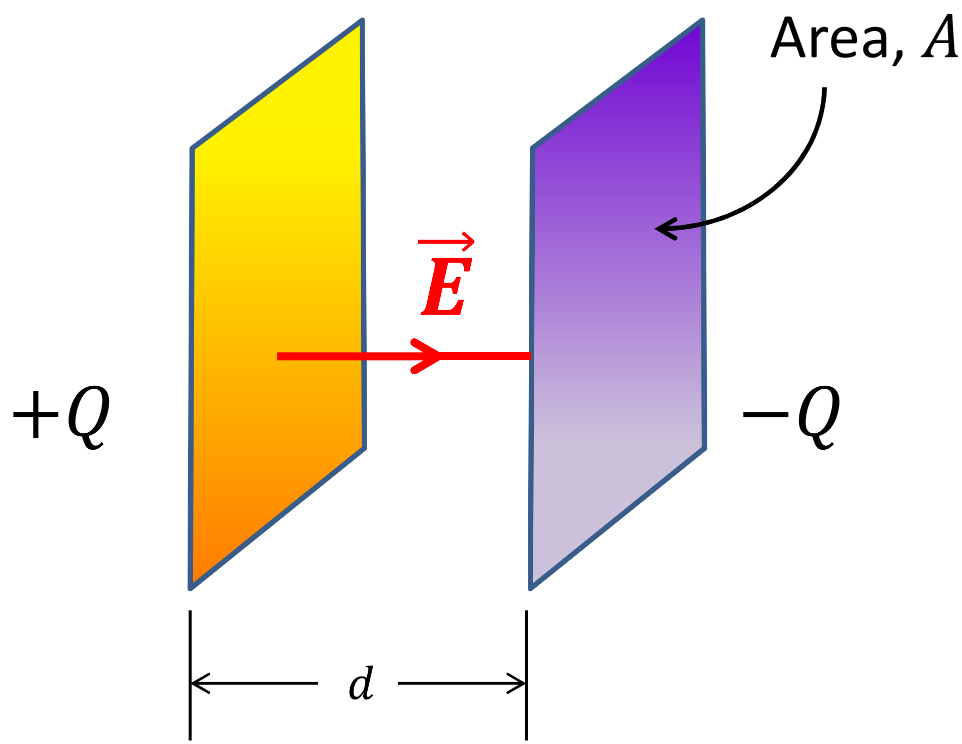
- General procedure for calculating capacitance:
  1. Put charge $\pm Q$ on the two conductors
  2. Calculate $\vec{E}$ between the conductors
  3. Calculate the electric potential difference

$$V = -\int \vec{E} \cdot d\vec{\ell}$$

  4. Use $C = Q/V$

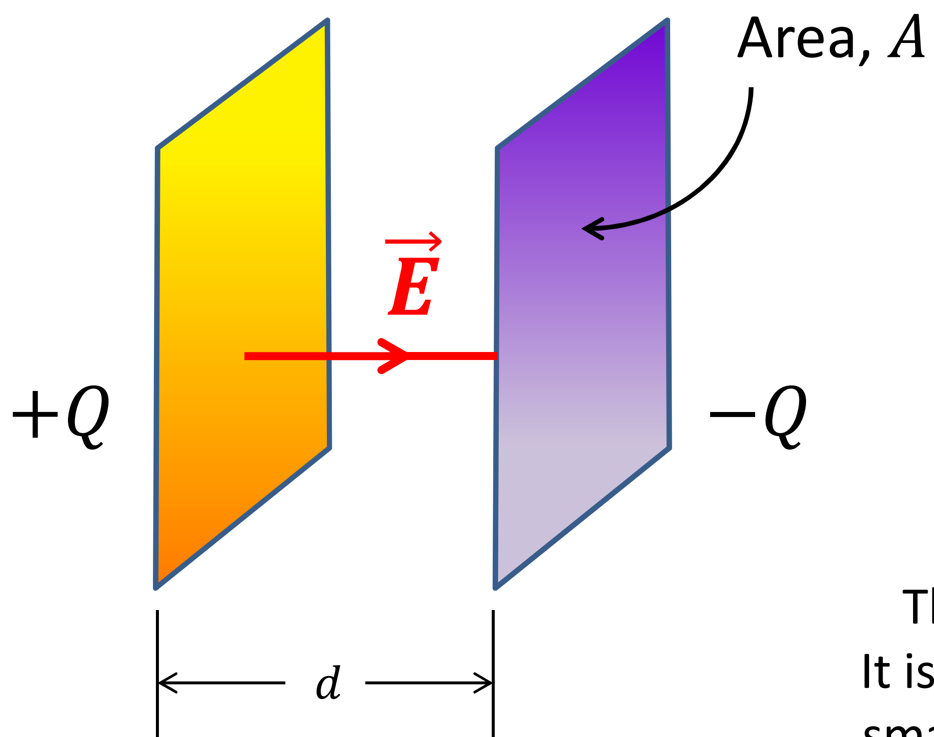
First Example

- Parallel plate capacitor:



First Example

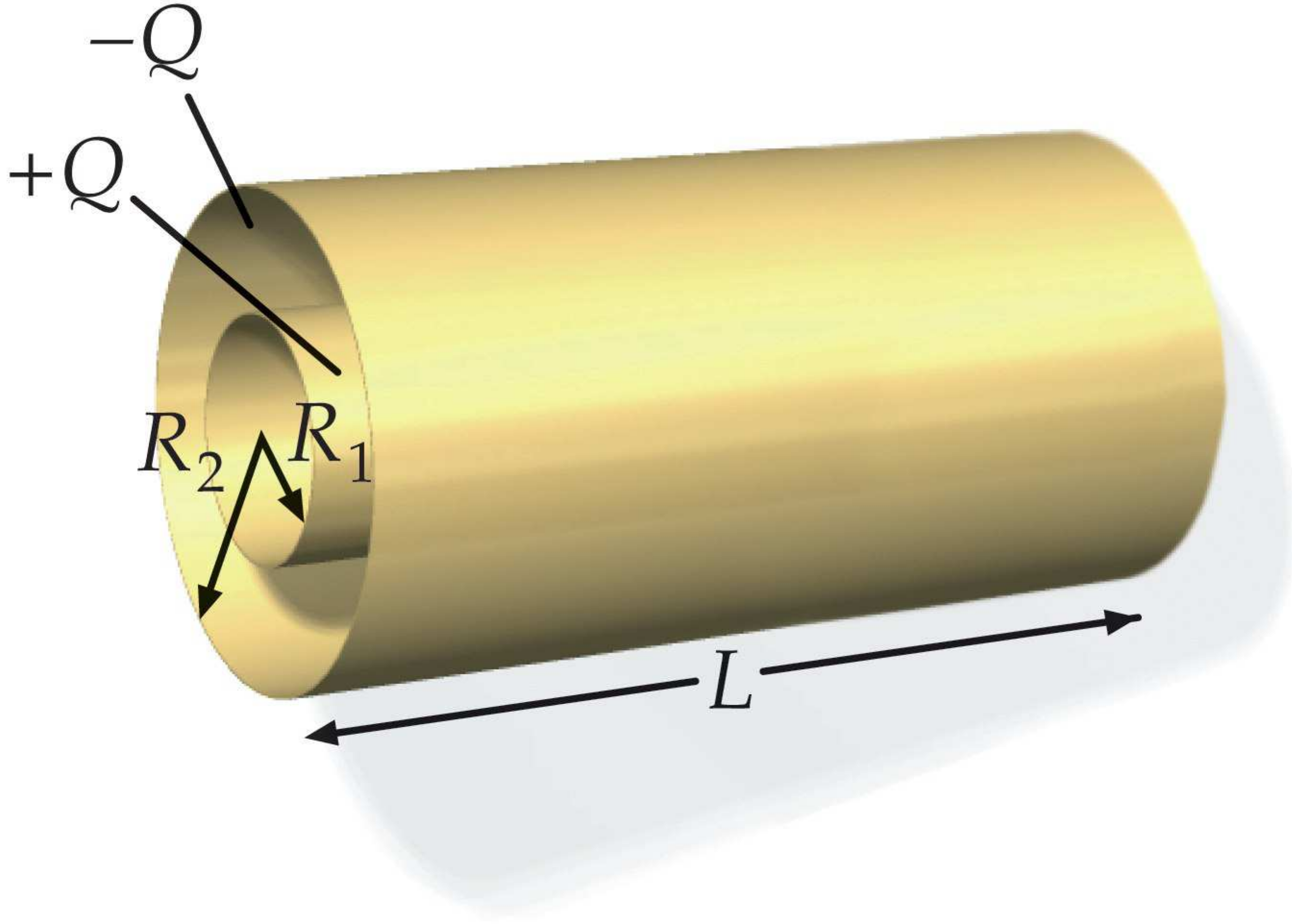
- Parallel plate capacitor:



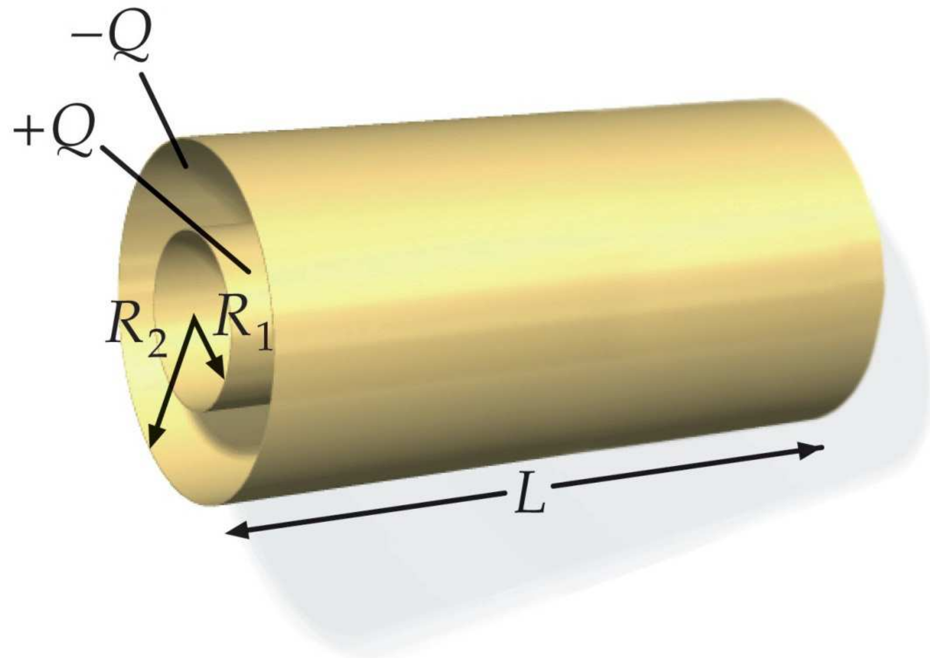
$$C = \frac{\epsilon_0 A}{d}$$

This ignores any "edge effects"…
It is a good approximation when d is small compared with the length and width of a plate.

Coaxial Cylinder



Coaxial Cylinder



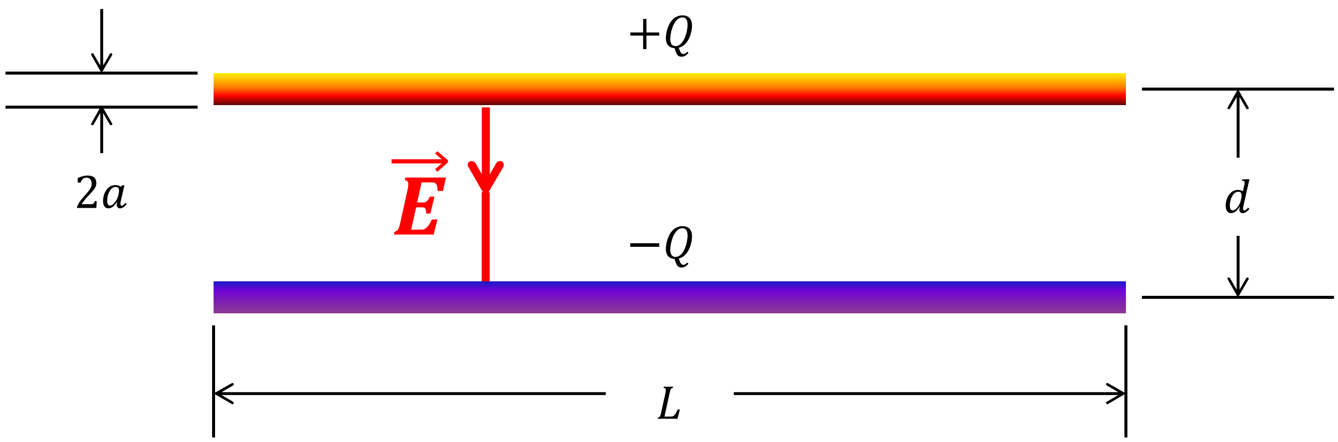
$$C = \frac{2\pi\epsilon_0 L}{\log(R_2/R_1)}$$

Examples

- Another way to calculate capacitance:
  1. Put charge $\pm Q$ on the two conductors
  2. Calculate electric potential of each conductor
     - Consider each conductor in isolation
     - User the principle of superposition
  3. Calculate the potential difference, V
  4. Use $C = Q/V$

Second Example

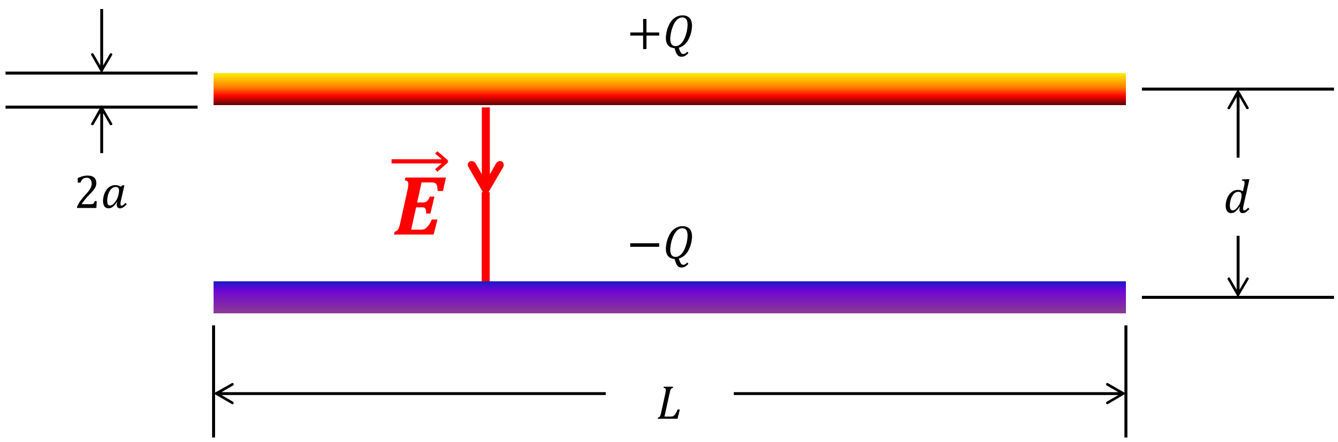
- Two long, parallel wires:



We will suppose that $L \gg d \gg a$.

Second Example

- Two long, parallel wires:



$$C = \frac{\pi \epsilon_0 L}{\log(d/a)}$$

(assuming $L \gg d \gg a$.)

Numerical Example

- What is the capacitance of an extension cord, 10 meters long with wires that are 1 mm in diameter, separated by 2 mm?
  - $L = 10\,m$
  - $d = 2\,mm$
  - $a = 0.5\,mm$

$$C \approx \frac{\pi \epsilon_0 L}{\log\left(\frac{d}{a}\right)} = \frac{\pi(8.85\,pF \cdot m^{-1})(10\,m)}{\log\left(\frac{2\,mm}{0.5\,mm}\right)} = 200\text{ pF}$$

Numerical Example

- How much energy is stored if there is a potential difference of 100 volts?

$$U = \frac{1}{2} C\, V^2$$

$$= \frac{1}{2}(200\ \mathrm{pF})(100\ \mathrm{V})^2$$

$$= \frac{1}{2}(200\ \mathrm{pC} \cdot \mathrm{V}^{-1})(100\ \mathrm{V})(100\ \mathrm{J} \cdot \mathrm{C}^{-1})$$

$$= 1\ \mathrm{\mu J}$$

Clicker Question

- To double the capacitance of a parallel plate capacitor, you should:

(a) Double the area of the plates
(b) Half the distance between the plates
(c) Both (a) and (b)
(d) Either (a) or (b)