Tuesday’s Clicker Question

If \( Q_2 = 1 \, \mu C \) and \( Q_1 = -2 \, \mu C \) which graph most accurately shows \( V(r) \)?

(a) \[\text{Graph (a)}\]
(b) \[\text{Graph (b)}\]
(c) \[\text{Graph (c)}\]
(d) \[\text{Graph (d)}\]
What we know already:

- Net charge is $-1 \mu C$
- Charge on inner surface of outer spherical shell is $+2 \mu C$

**Tuesday’s Clicker Question**

- Electric field is always pointing towards the origin.
  - $V(r)$ will always be negative.
  - A positive charge has less potential energy closer to the sphere.
- Electric potential inside the conductors is constant.
  - $V(r)$ will be flat in the shell and in the sphere.

$Q_2 = 1 \mu C$ and $Q_1 = -2 \mu C$
Storing Energy

Tuesday’s example with the Van de Graaff...

Initial charge: $Q$

Initial electric potential at the surface:

$$V(R) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$$

Work needed to add an additional charge, $\Delta Q$:

$$\Delta U = V(R)\Delta Q = \frac{1}{4\pi\varepsilon_0} \frac{Q\Delta Q}{R}$$

Total work needed to add charge $Q_{total}$:

$$U = \frac{1}{4\pi\varepsilon_0} \frac{1}{R} \int_0^{Q_{total}} Q \, dQ = \frac{1}{4\pi\varepsilon_0} \frac{1}{R} \times \frac{1}{2} (Q_{total})^2$$
Storing Energy

Total work needed to charge sphere:

\[ U = \frac{1}{4\pi\varepsilon_0} \frac{1}{R} \times \frac{1}{2} Q^2 \]

Final voltage of sphere:

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \]

\[ Q = 4\pi\varepsilon_0 RV \]

Stored energy:

\[ E = \frac{1}{2} (4\pi\varepsilon_0 R) V^2 \]

In principle, we can use this energy to do work...
Capacitors

- A *capacitor* is a device that stores electrostatic potential energy.
- These are (almost) always two conductors in close proximity separated by an insulator:
  - Air or vacuum
  - Something that prevents sparks
  - Something that also stores electrostatic potential energy
- Examples:
  - Two parallel wires
  - Two parallel plates
  - Two coaxial cylinders
How Much Energy?

- Consider two conductors with electric potentials $V_1$ and $V_2$...
- We can always pick $V_1 = 0$ and then $V_2$ is just the potential difference between them.
How Much Energy?

• Take a small charge $\Delta Q$ from the blue conductor and move it to the red conductor...
• Work done is $\Delta U = V \Delta Q$
• How much will this change $V$?
  – It turns out that $\Delta V \propto \Delta Q$
Capacitance

\[ \Delta Q \propto \Delta V \]

- The constant of proportionality is the capacitance, \( C \):
  \[ \Delta Q = C \Delta V \]
- If you transfer a total charge \( Q \), then the potential difference will be:
  \[ V = \frac{Q}{C} \]
- The capacitance is defined as:
  \[ C = \frac{Q}{V} \]

Units for Capacitance:

\[ \text{Farad} = \frac{\text{Coulomb}}{\text{Volt}} \]

\[ \epsilon_0 = 8.85 \text{ pF/m} \]
Stored Energy

• How much work, $U$, does it take to charge a capacitor to a final voltage, $V$?

$$\Delta U = V \Delta Q = C V \Delta V$$

$$U = C \int_{0}^{V} V \, dV = \frac{1}{2} C V^2$$

Equally valid:

$$U = \frac{1}{C} \int_{0}^{Q} Q \, dQ = \frac{1}{2C} Q^2$$

• This is the energy stored in the capacitor.
• You don’t have to “move” the same charge from one conductor to the other, just the same amount of charge.
Question

If the charge, \( Q \), is doubled, will the capacitance...

(a) Increase?
(b) Decrease?
(c) Remain the same?
Examples

• General procedure for calculating capacitance:
  
  1. Put charge $\pm Q$ on the two conductors
  2. Calculate $\vec{E}$ between the conductors
  3. Calculate the electric potential difference

\[ V = -\int \vec{E} \cdot d\vec{l} \]

  4. Use $C = Q/V$
First Example

• Parallel plate capacitor:
First Example

• Parallel plate capacitor:

\[ C = \frac{\varepsilon_0 A}{d} \]

This ignores any “edge effects”...
It is a good approximation when \( d \) is small compared with the length and width of a plate.
Coaxial Cylinder

\[ C = \frac{2\pi \epsilon_0 L}{\log(R_2/R_1)} \]
Examples

- Another way to calculate capacitance:
  1. Put charge $\pm Q$ on the two conductors
  2. Calculate electric potential of each conductor
     - Consider each conductor in isolation
     - Use the principle of superposition
  3. Calculate the potential difference, $V$
  4. Use $C = Q/V$
Second Example

• Two long, parallel wires:

We will suppose that $L \gg d \gg a$. 

\[ \begin{align*}
\text{+Q} \\
\vec{E} \\
-\text{Q}
\end{align*} \]
Second Example

- Two long, parallel wires:

\[ C = \frac{\pi \varepsilon_0 L}{\log(d/a)} \]

(assuming \( L \gg d \gg a \).)
Numerical Example

• What is the capacitance of an extension cord, 10 meters long with wires that are 1 mm in diameter, separated by 2 mm?
  o $L = 10 \text{ m}$
  o $d = 2 \text{ mm}$
  o $a = 0.5 \text{ mm}$

$$C \approx \frac{\pi \varepsilon_0 L}{\log \left( \frac{d}{a} \right)} = \frac{\pi (8.85 \text{ pF} \cdot \text{m}^{-1})(10 \text{ m})}{\log \left( \frac{2 \text{ mm}}{0.5 \text{ mm}} \right)} = 200 \text{ pF}$$
Numerical Example

• How much energy is stored if there is a potential difference of 100 volts?

\[
U = \frac{1}{2} C V^2
\]

\[
= \frac{1}{2} (200 \text{ pF})(100 \text{ V})^2
\]

\[
= \frac{1}{2} (200 \text{ pC} \cdot \text{V}^{-1})(100 \text{ V})(100 \text{ J} \cdot \text{C}^{-1})
\]

\[
= 1 \mu\text{J}
\]
Clicker Question

• To double the capacitance of a parallel plate capacitor, you should:

(a) Double the area of the plates
(b) Half the distance between the plates
(c) Both (a) and (b)
(d) Either (a) or (b)