PURDUE UNIVERSITY DEPARTMENT OF PHYSICS
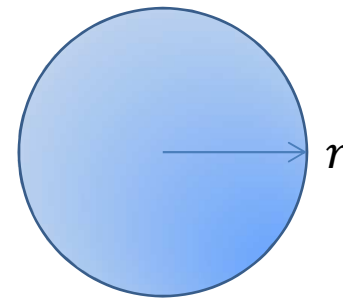
Physics 24100

Electricity & Optics

Lecture 5 – Chapter 22 sec. 4-5

Fall 2012 Semester

Matthew Jones

Thursday's Question:

- Consider a spherical *shell* of charge with radius R and surface charge density, σ.

- The total charge is $Q = \sigma A = 4\pi R^2 \sigma$.

- What is $\vec{E}$ at a point $\vec{r}$ located outside the sphere? $(r > R)$

r

(a) $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{R^2}\hat{r}$ (b) $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}\hat{r}$ (c) $\vec{E} = \dfrac{1}{2\pi\epsilon_0}\dfrac{Q}{r}\hat{r}$

Thursday's Question:

- Gauss's Law: $\oint_S \hat{n} \cdot \vec{E}\, dA = \dfrac{Q_{inside}}{\epsilon_0}$

- Spherical symmetry: $\vec{E} = E_r \hat{r}, \quad \hat{n} = \hat{r}$

- $\oint_S \hat{n} \cdot \vec{E}\, dA = E_r \oint_S dA = E_r A = \dfrac{Q_{inside}}{\epsilon_0}$

  - A is the surface area of a sphere of radius r.
  - Independent of how the charge is distribute (provided it remains spherically symmetric)

- We had previously worked this our for a sphere with uniform charge density ρ and the result is the same:

(a) $\vec{E} = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q}{R^2} \hat{r}$ (b) $\vec{E} = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q}{r^2} \hat{r}$ (c) $\vec{E} = \dfrac{1}{2\pi\epsilon_0} \dfrac{Q}{r} \hat{r}$

Gauss's Law

Closed surface

$$\oint_S \hat{n} \cdot \vec{E} \, dA = \frac{Q_{inside}}{\epsilon_0}$$
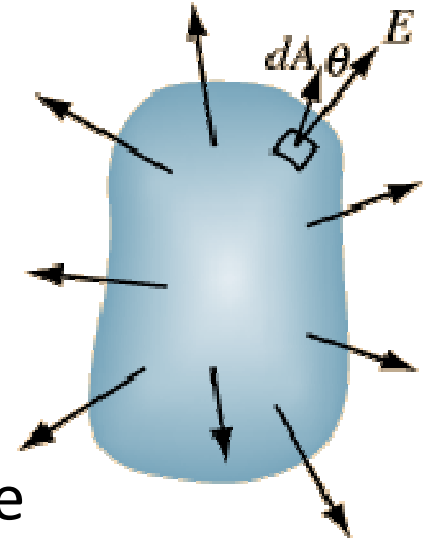
- Special cases:
  - $\hat{n} \cdot \vec{E}$ is constant everywhere on a surface
    - Then $\int_S \hat{n} \cdot \vec{E} \, dA = \hat{n} \cdot \vec{E} \int_S dA = \hat{n} \cdot \vec{E} \, A$

      Area of the surface
  - $\hat{n} \cdot \vec{E} = 0$ everywhere on a surface
    - Then $\int_S \hat{n} \cdot \vec{E} \, dA = 0$
  - $\hat{n} \cdot \vec{E}$ is unknown but the surface area goes to zero
    - Then $\int_S \hat{n} \cdot \vec{E} \, dA = 0$
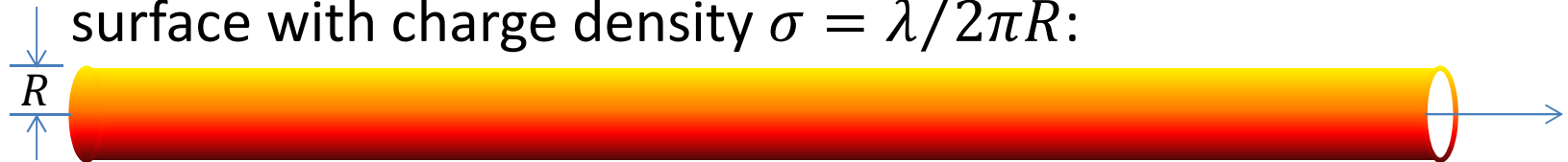- You are free to pick the surface to satisfy either or both of the special cases.

Question

Consider the electric field at a distance $r > R$ from the z-axis on which is located:

(a) An infinite line of charge with linear charge density λ:

(b) An infinite cylinder of radius R with charge located only on its surface with charge density $\sigma = \lambda/2\pi R$:
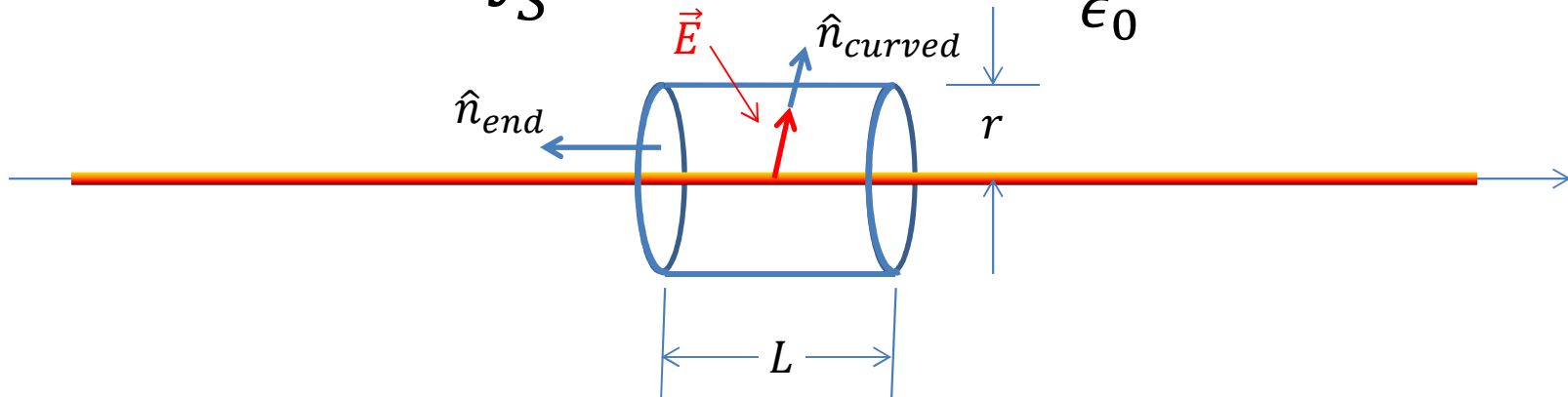
R

(c) An infinite cylinder of radius R with uniform volume charge density $\rho = \lambda/\pi R^2$:

Which produces the greatest electric field?   Enter (d) if you think they will all be equal.
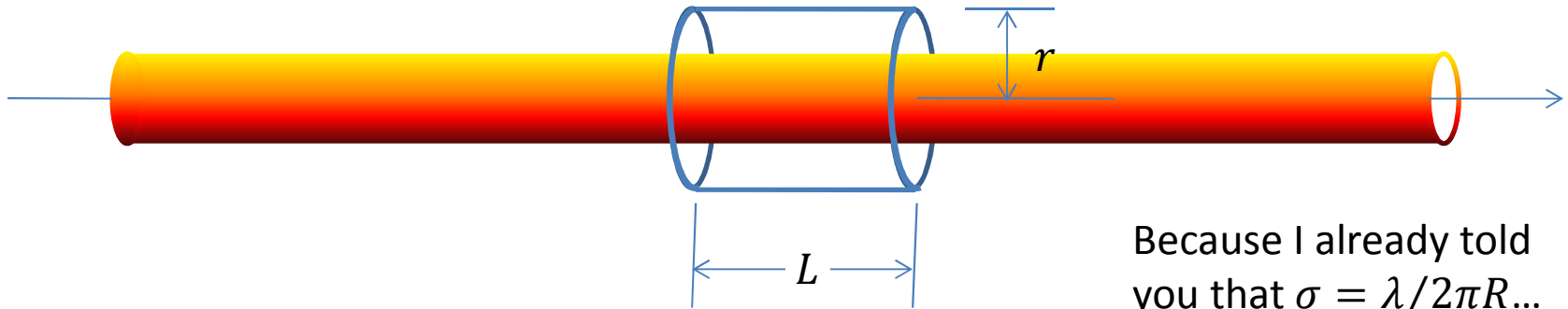
When $r > R$...

- Gauss's Law: $\oint_S \hat{n} \cdot \vec{E}\, dA = \dfrac{Q_{inside}}{\epsilon_0}$

- Radial symmetry, $\hat{n} \cdot \vec{E} = E_r$ on the curved surface, $\hat{n} \cdot \vec{E} = 0$ on the ends.

- $\oint_S \hat{n} \cdot \vec{E}\, dA = 2\pi r L E_r$ and $Q_{inside} = \lambda L$

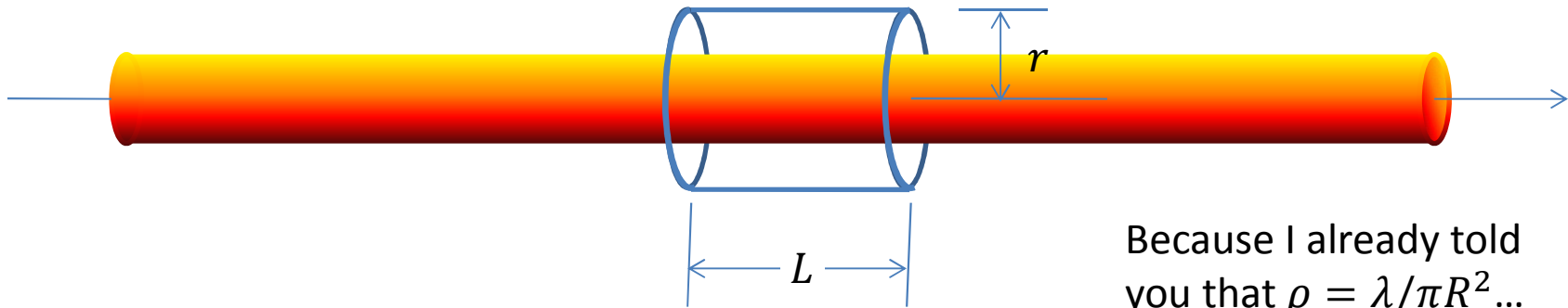- Electric field: $\vec{E} = \lambda / 2\pi\epsilon_0 r$

When $r > R$…

- Same cylindrical symmetry:

$$\oint_S \hat{n} \cdot \vec{E}\, dA = 2\pi r L E_r$$

- This time, $Q_{inside} = 2\pi R L \sigma = \lambda L$
- Electric field: $\vec{E} = \lambda/2\pi \epsilon_0 r$
- This is the same as for the infinite line of charge.

When $r > R...$

Because I already told you that $\rho = \lambda/\pi R^2 ...$

- Same cylindrical symmetry:

$$\oint_S \hat{n} \cdot \vec{E} \, dA = 2\pi r L E_r$$

- This time, $Q_{inside} = \pi R^2 L \rho = \lambda L$
- Electric field: $\vec{E} = \lambda/2\pi\epsilon_0 r$
- This is the same as for the infinite line of charge and for the cylinder. The correct answer is (d).
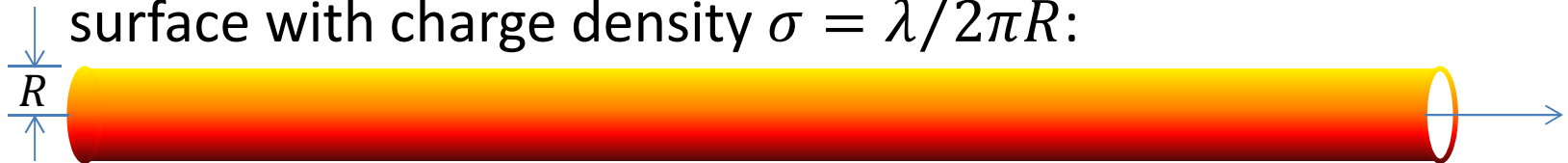
Question

Now consider the electric field at a distance $r < R$ from the z-axis on which is located:

(a) An infinite line of charge with linear charge density λ:

(b) An infinite cylinder of radius R with charge located only on its surface with charge density $\sigma = \lambda/2\pi R$:

R

(c) An infinite cylinder of radius R with uniform volume charge density $\rho = \lambda/\pi R^2$:

Which produces the greatest electric field?   Enter (d) if you think they will all be equal.

When $r < R$…

- Gauss's Law: $\oint_S \hat{n} \cdot \vec{E} \, dA = \frac{Q_{inside}}{\epsilon_0}$

- For the line, all charge is located on the z-axis, so $Q_{inside} = \lambda L$

- For the cylindrical surface, all charge is located at radius $R > r$ so $Q_{inside} = 0$.

- For the solid cylinder, some charge is located at radius $R > r$, so $Q_{inside} < \lambda L$.

- Correct answer is (a).

Electric Field at a Surface

- Consider an sheet with surface charge density σ. What is the electric field at the surface?

- In the absence of any external electric field, we expect that $\vec{E}$ is perpendicular to the surface.
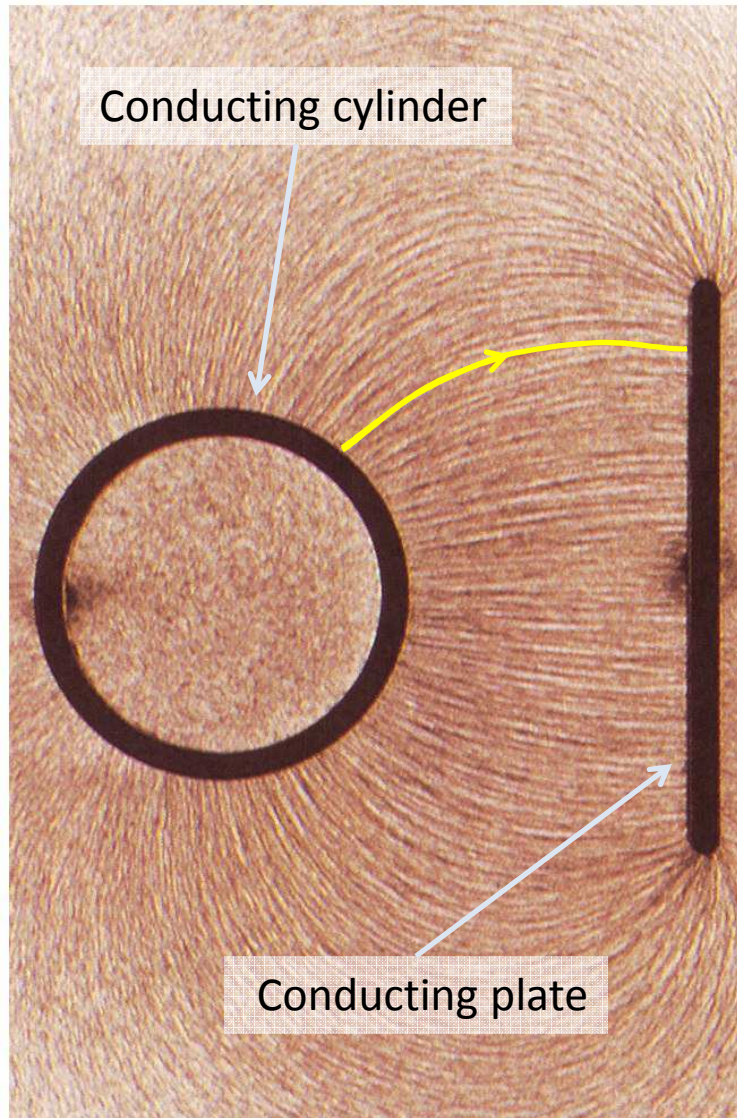

$$\left|\vec{E}_+\right| = \left|\vec{E}_-\right| \equiv E_n$$
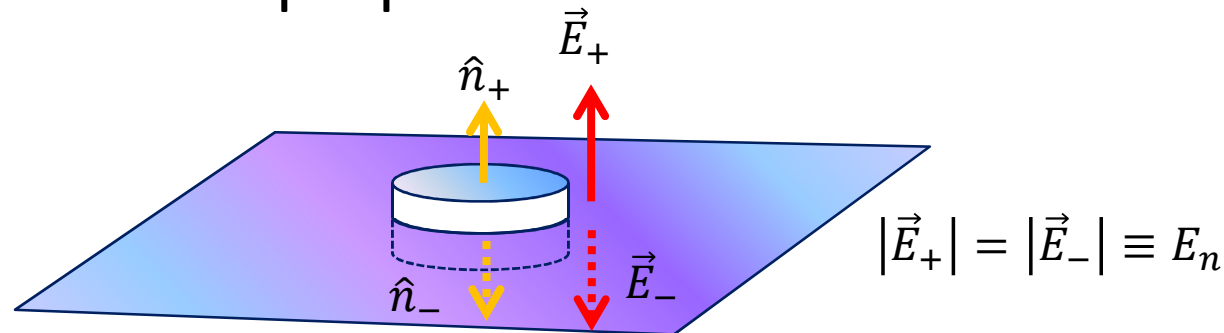
- Gauss's Law: $\oint_S \hat{n} \cdot \vec{E}\, dA = \dfrac{Q_{inside}}{\epsilon_0}$

$$\left(\hat{n}_+ \cdot \vec{E}_+\right)A + \left(\hat{n}_- \cdot \vec{E}_-\right)A = 2E_n A = {\sigma A}/{\epsilon_0}$$

- Normal component of E-field: $E_n = \dfrac{\sigma}{2\epsilon_0}$

Electric Field in a Conductor

- Here's the argument:
  - An external electric field exerts a force on charge carriers which causes them to move
  - They will continue to move until their electric field negates the external field
  - The motion stops when the *net* electric field is zero
  - This is called "electrostatic equilibrium".
- Main point to remember:

    Inside a conductor, the electric field is always zero.

   **(Provided it is in a state of electrostatic equilibrium.)**

Electric Field in a Conductor

Conducting cylinder

Conducting plate

- Tiny pieces of dust line up along the electric field lines.
- Electric field lines are always perpendicular to the conducting surfaces.
- There is no electric field inside the conducting cylinder.

Surface Charge on a Conductor

- An electric field will attract or repel the charge carriers in a conductor, and they accumulate on the surface.

- What is the surface charge density of a conductor in an electric field?
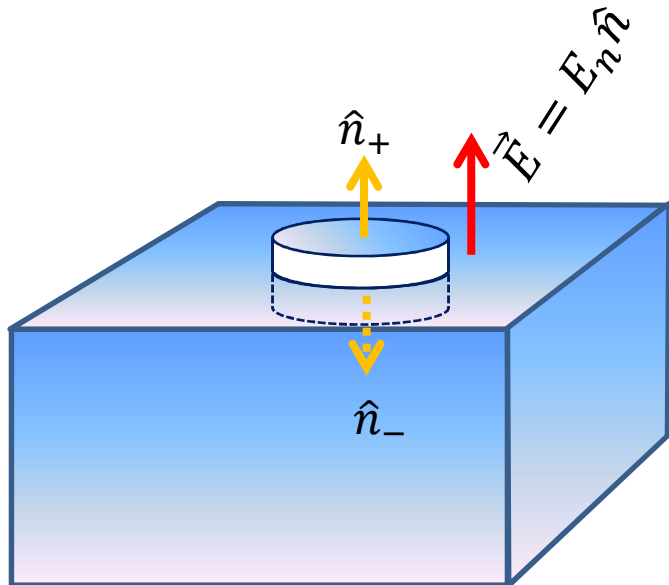
  - Gauss's Law: $\oint_S \hat{n} \cdot \vec{E} \, dA = \dfrac{Q_{inside}}{\epsilon_0}$

  - $\oint_S \hat{n} \cdot \vec{E} \, dA = \int_{top} \hat{n} \cdot \vec{E} \, dA = E_n A$

  - $\dfrac{Q_{inside}}{\epsilon_0} = \dfrac{\sigma A}{\epsilon_0}$

$$E_n = \frac{\sigma}{\epsilon_0}$$

$\hat{n}_+$

$\vec{E} = E_n \hat{n}$

$\hat{n}_-$

Surface Charges: Summary

- The normal component of the electric field on each side of a sheet with surface charge σ:
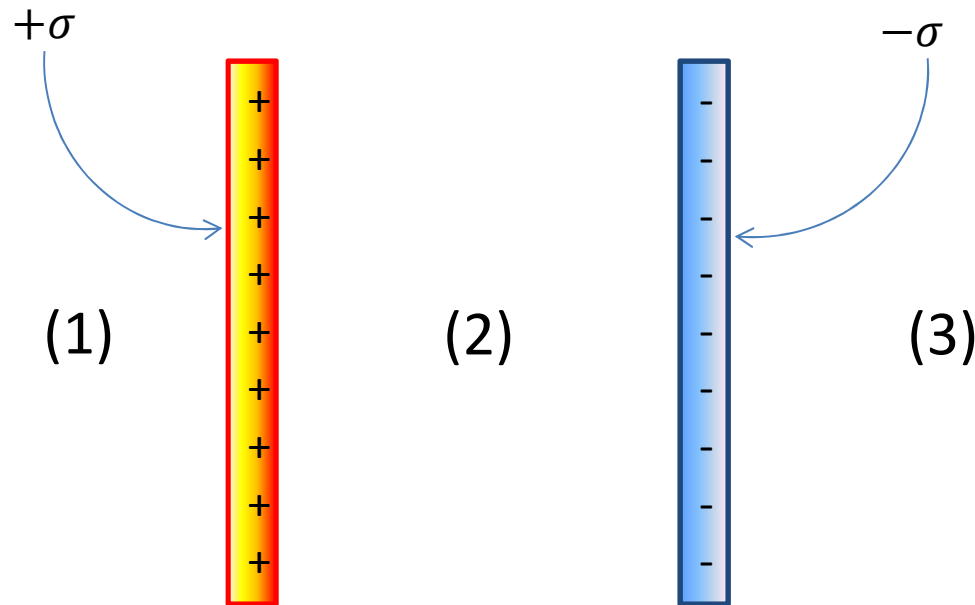
$$E_n = \frac{\sigma}{2\epsilon_0}$$

- The normal component of the electric field outside a conductor in electrostatic equilibrium:

$$E_n = \frac{\sigma}{\epsilon_0}$$

Example:

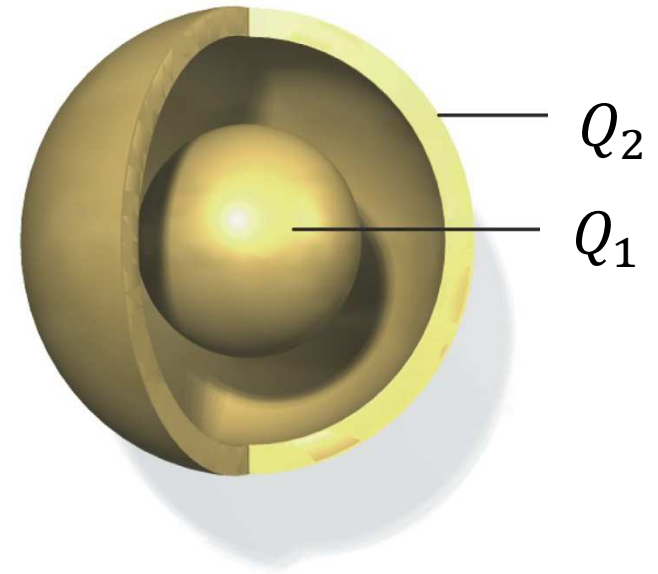- Consider two infinite sheets with surface charge densities $\pm\sigma$:

$+\sigma$

$-\sigma$

(1)   (2)   (3)

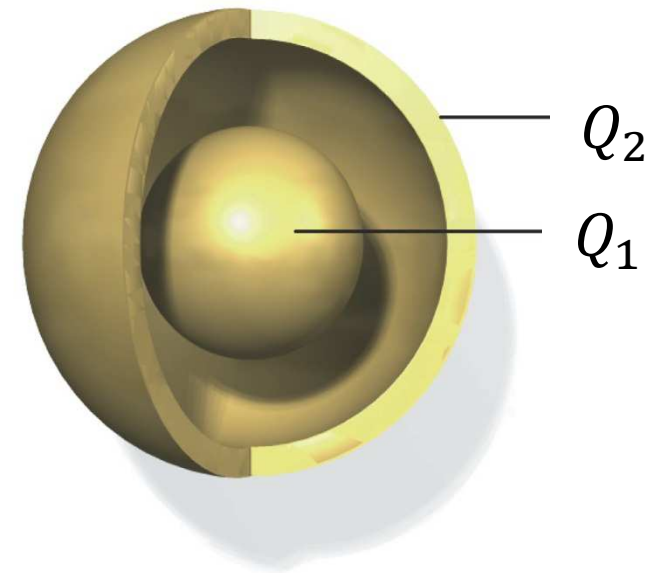- What are $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$ in regions (1), (2) and (3)?

Example:

- A conducting sphere of radius R_1 is concentric with a thin, conducting shell of radius R_2:
  - Inner sphere has charge Q_1
  - Outer sphere has charge Q_2

Q_2

Q_1

- **How is the charge distributed on the sphere?**

(a) There is no charge on the sphere.

(b) The charge is uniformly distributed on the outer surface of the sphere

(c) The charge is uniformly distributed throughout the sphere.

Example:

- A conducting sphere of radius R_1 is concentric with a thin, conducting shell of radius R_2:
  - Inner sphere has charge Q_1
  - Outer sphere has charge Q_2

Q_2

Q_1

- **How is the charge distributed on the sphere?**

We just said it had charge Q_1...

(a) There is no charge on the sphere.

(b) The charge is uniformly distributed on the outer surface of the sphere

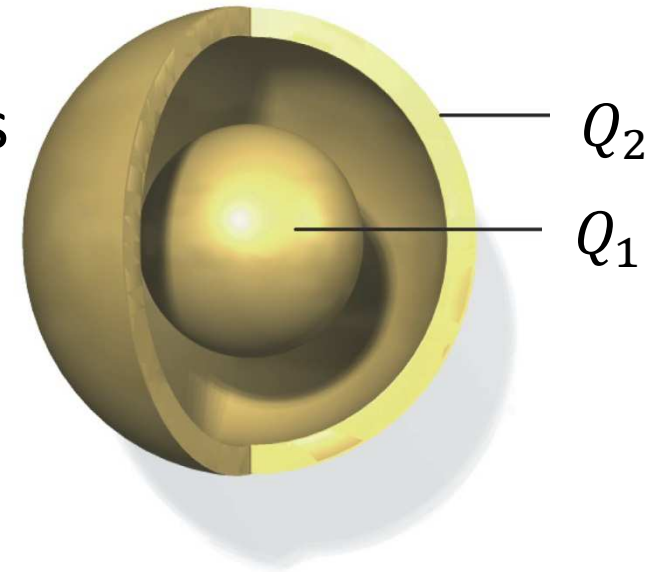(c) The charge is uniformly distributed throughout the sphere.

But then $Q_{inside} \neq 0$ and therefore $\vec{E} \neq 0$ which can't happen in a conductor.

Example:

- A conducting sphere of radius R_1 is concentric with a thin, conducting shell of outer radius R_2:
  - Inner sphere has charge Q_1
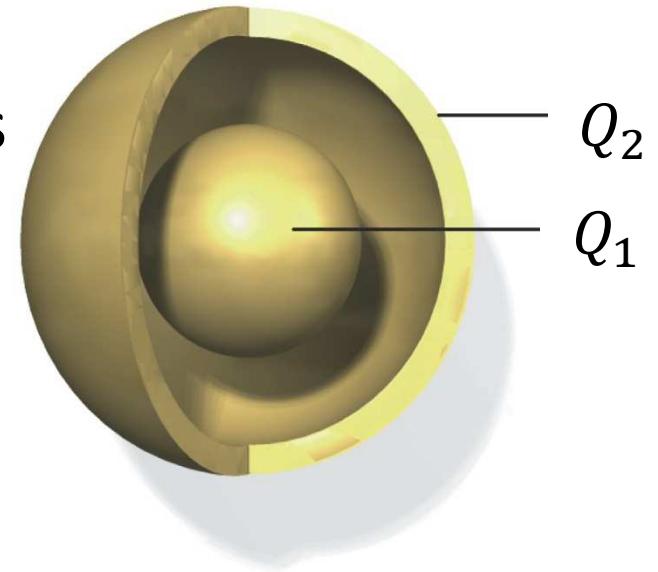 - Outer sphere has charge Q_2

- **What is the electric field at…**

(a) $r < R_1$? ⟹ $\vec{E} = 0$ because it's in a conductor.
(in electrostatic equilibrium)

(b) $R_1 < r < R_2$? ⟹ $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{r^2}\hat{r}$

(c) $r > R_2$? ⟹ $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1 + Q_2}{r^2}\hat{r}$
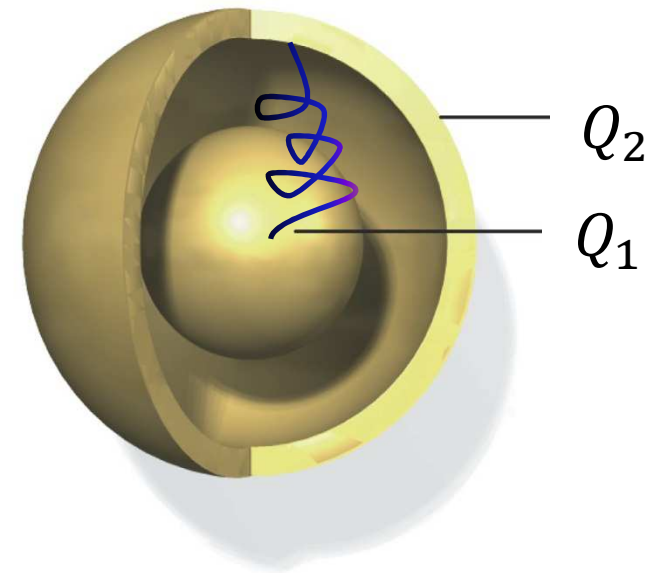
Example:

Q_2

Q_1

- A conducting sphere of radius R_1 is concentric with a thin, conducting shell of outer radius R_2:
  - Inner sphere has charge Q_1
  - Outer sphere has charge Q_2

- **What is the surface charge on the inner surface of the shell?**
  - Electric field inside the shell must be zero (it's a conductor)
  - Gauss's law says that $Q_{inside} = 0$.
 - There must be a net charge of $-Q_1$ on the inner surface of the shell.

Example:

Q_2
Q_1

- A conducting sphere of radius R_1 is concentric with a thin, conducting shell of outer radius R_2:
  - Inner sphere has charge Q_1
  - Outer sphere has charge Q_2
- **What if the inner and outer conductors were connected with a wire?**

(a) There is no charge on the shell.

(b) All the charge is uniformly distributed only on the outer surface of the spherical shell

(c) The charge is uniformly distributed on the outer surface of the sphere and the shell.

Example:

- A conducting sphere of radius R_1 is concentric with a thin, conducting shell of outer radius R_2:
  - Inner sphere has charge Q_1
  - Outer sphere has charge Q_2
- **What if the inner and outer conductors were connected with a wire?**
  - There can be no surface charge inside the shell. If there was, it would create an electric field, charges would move and the system would not be in electrostatic equilibrium.

Q_2

Q_1
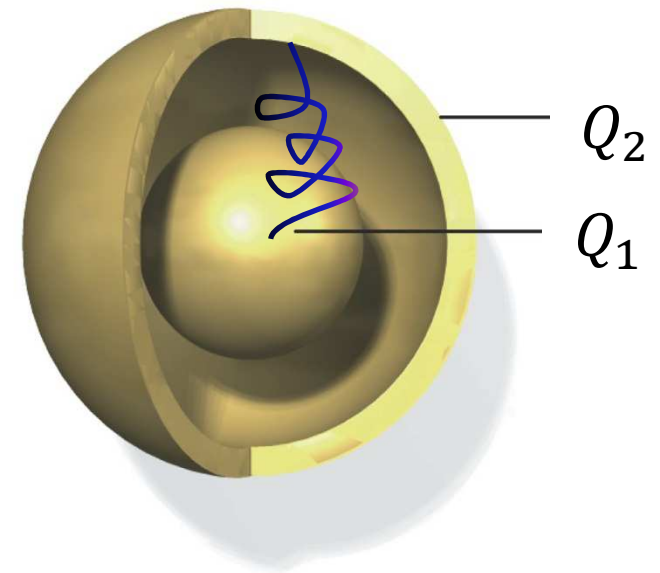
The total charge must be distributed on the outer surface of the shell.

Question

- An isolated conductor of arbitrary shape has a net charge of +10 Coulombs.
- Inside the conductor is a cavity that contains a point charge of -3 Coulombs.
- What is the net charge on the inner surface of the cavity?

(a) -13 Coulombs

(b) -3 Coulombs

(c) 0 Coulombs

(d) +3 Coulombs

(e) +13 Coulombs