PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 24100
Electricity & Optics
Lecture 2

Fall 2012 Semester

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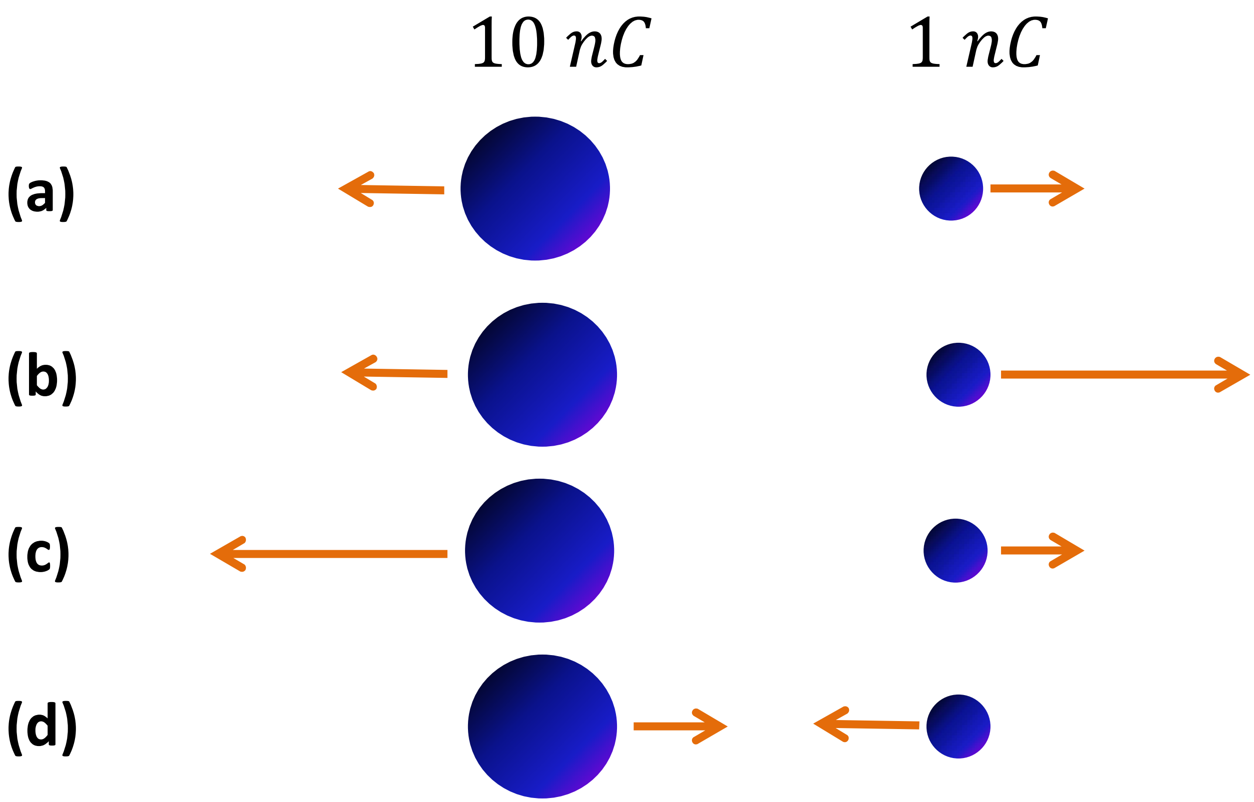
Tuesday's Clicker Question

- Which diagram most accurately shows the forces acting on the charges:

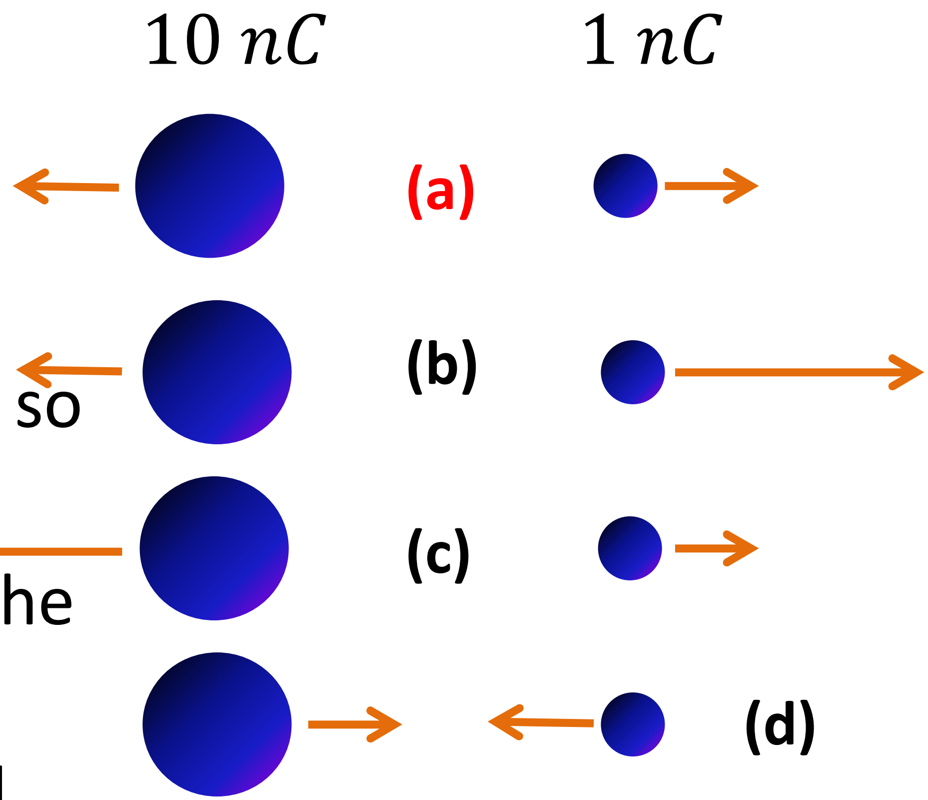


Tuesday's Clicker Question

- Both charges are positive: the force should be repulsive, so it isn't (d).
- Recall that

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r}$$

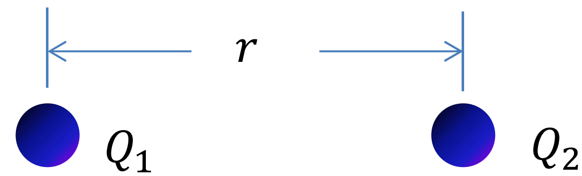
- The magnitudes are equal so the correct answer is (a).
- Don't confuse force with the resulting acceleration!
  - Did you assume the big ball would be more massive?



Lecture 2 – Electric Fields

- Coulomb's law of electric force:

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r}$$



- We had to keep track of which direction this force acted:
  - Force exerted **by** Q_1 **on** Q_2?
  - Force exerted **by** Q_2 **on** Q_1?
- What is the mechanism by which the force is exerted?
  - Voodoo?
  - Action at a distance?
- **Electric field:** an electric charge creates "electric field" that surrounds it and extends over macroscopic distances.
- **Electric force:** a charged particle in an electric field experiences a force.

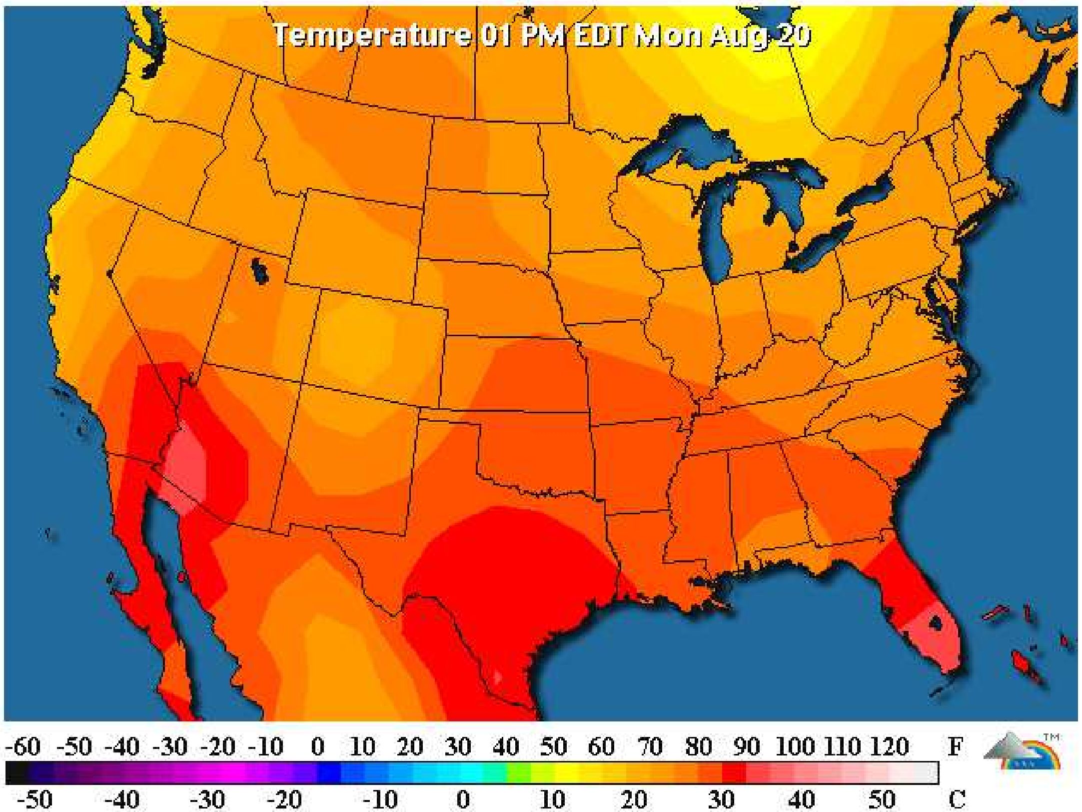
Electric Fields

- Consider the electric force on a *small*, *positive*, "test charge", q_0...
  - q_0 should be so small that it has a negligible effect on the local electric field
  - Electric force, $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_0 Q_1}{r^2}\hat{r}$
  - Electric field, $\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r^2}\hat{r}$
- More formally, the electric field due to a point charge, Q, can be defined:

$$\vec{E} = \lim_{q_0 \to 0}\frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

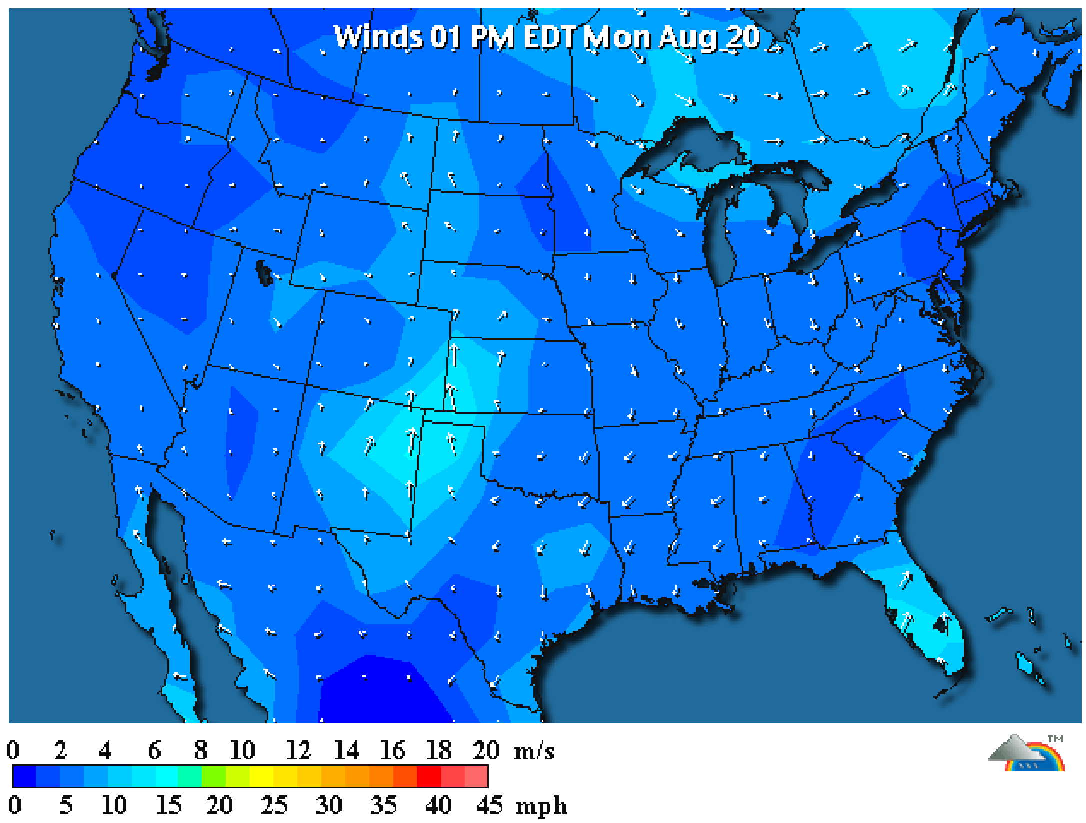
Scalar and Vector Fields

- A field is a function that can be evaluated at each point in space.
- A scalar field has one value at each point.
  - Examples: temperature, air pressure, density



Scalar and Vector Fields

- A field is a function that can be evaluated at each point in space.
- A vector field has a magnitude and direction at each point.
  - Examples: wind speed



Electric Fields

- If a charge, Q_1, is located at a position $\vec{x}_1$ then the force **on** a test charge at position $\vec{x}$ is:

$$\vec{F}(\vec{x}) = \frac{1}{4\pi\epsilon_0}\frac{q_0 Q_1}{|\vec{r}_{10}|^2}\hat{r}_{10}$$

  where $\vec{r}_{10} = \vec{x} - \vec{x}_1$.

- The electric field at position $\vec{x}$ due to the charge Q_1 can be written:

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{|\vec{x} - \vec{x}_1|^3}(\vec{x} - \vec{x}_1)$$

  – It is a vector valued function of $\vec{x}$

Principle of Superposition

- Net electric force is the vector sum of forces from several point charges.
- The net electric field is also the vector sum:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ \dots$$

By definition,

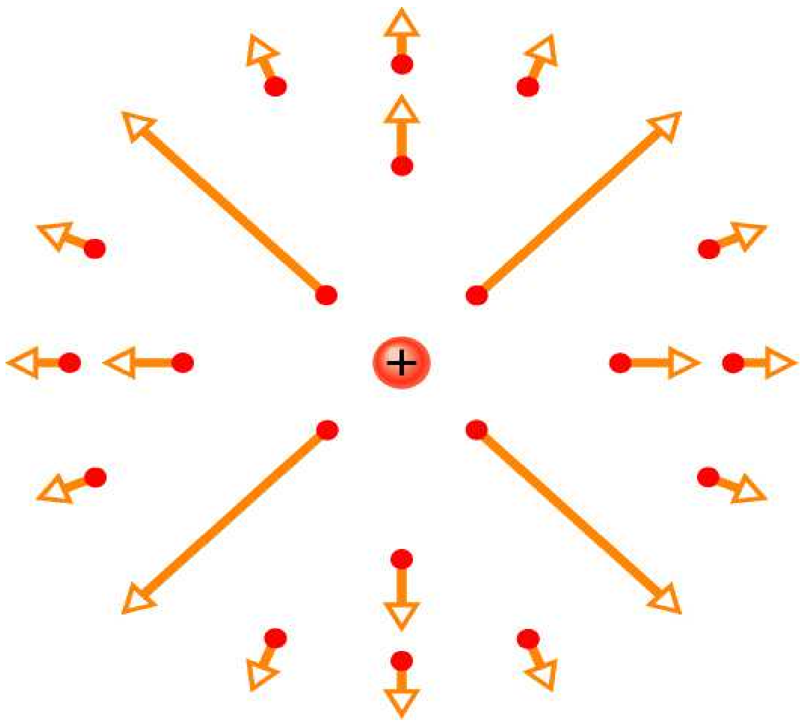
$$\vec{E} = \frac{\vec{F}_1}{q_0} + \frac{\vec{F}_2}{q_0} + \frac{\vec{F}_3}{q_0} \dots$$

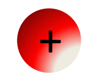
Therefore,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ \dots$$

- Electric field has SI units of *Newtons per Coulomb*

Electric Field Lines

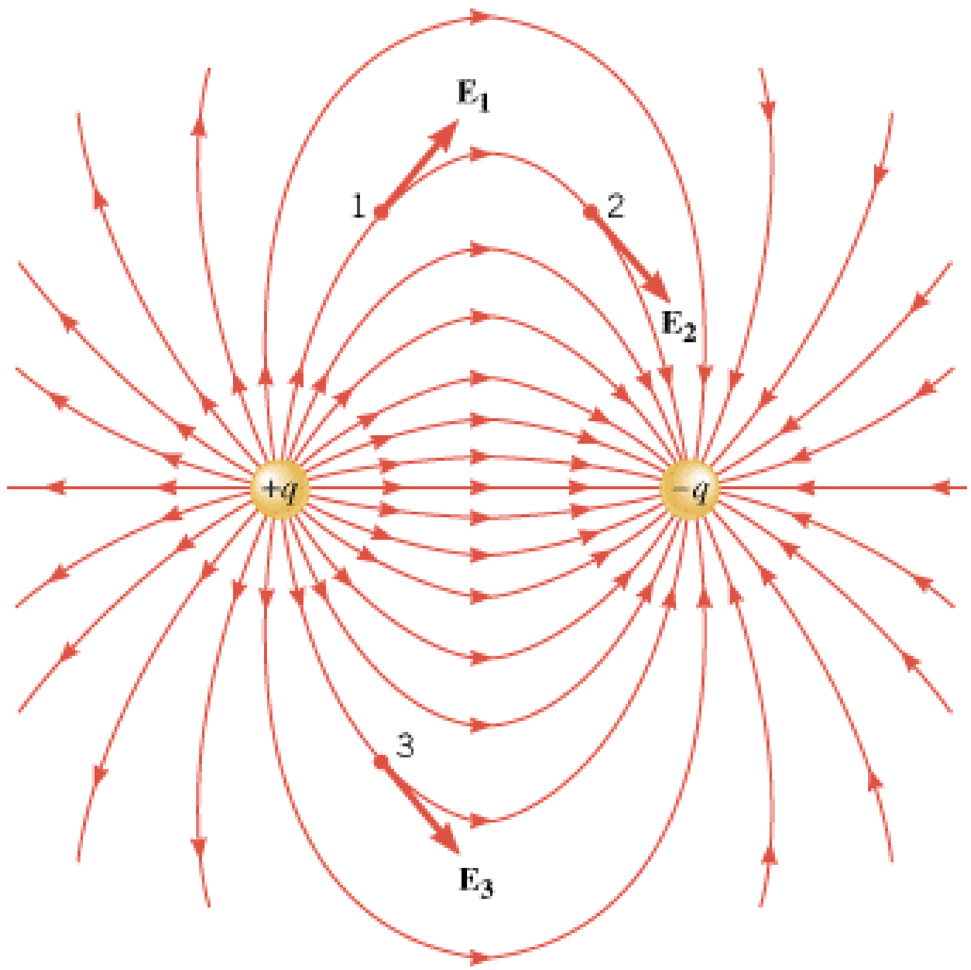


- You can draw the electric field as a bunch of little vectors.
- A positive charge is indicated by 
- Electric field vectors are indicated by arrows.

- Their length indicates the magnitude of the electric field.

There is a better way…

Electric Field Lines

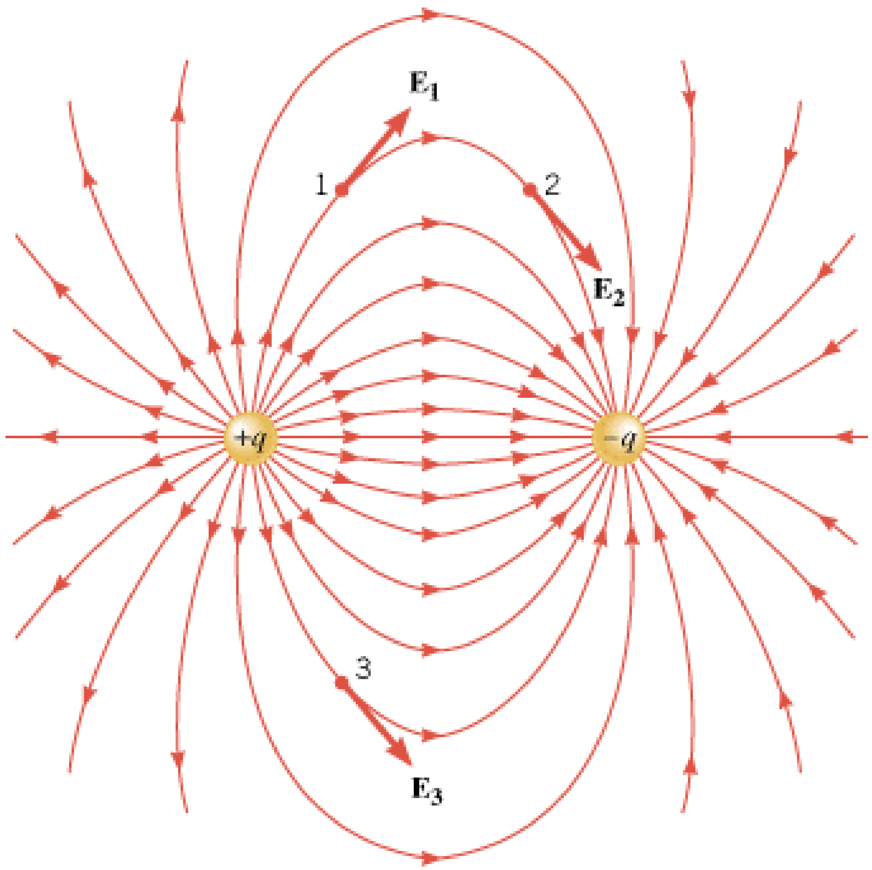


- A positive charge has more lines ***coming out*** than going in.
- A negative charge has more lines ***going in*** than coming out.
- The ***direction*** of the electric field is ***tangent*** to the electric field lines at any point.
- The ***magnitude*** of the electric field is proportional to the ***density*** of electric field lines.

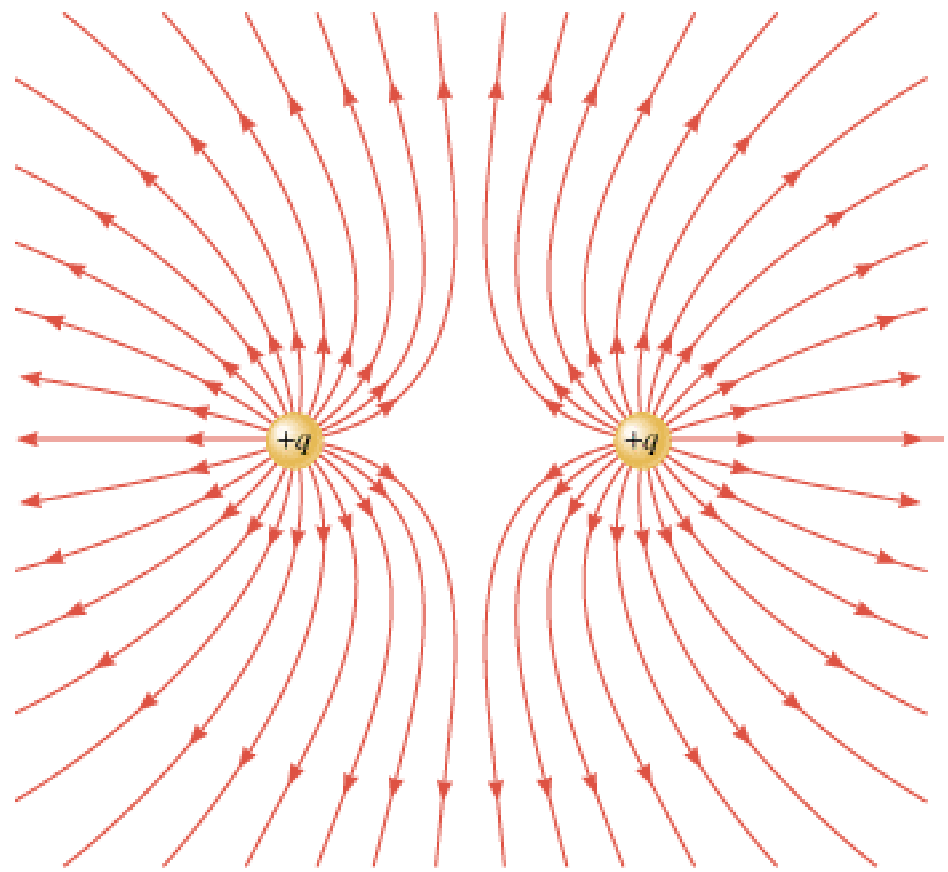
A test charge placed in the electric field will feel a force, $\vec{F} = q_0\vec{E}$.

Electric Field Lines

Electric Dipole: opposite signs but equal magnitude

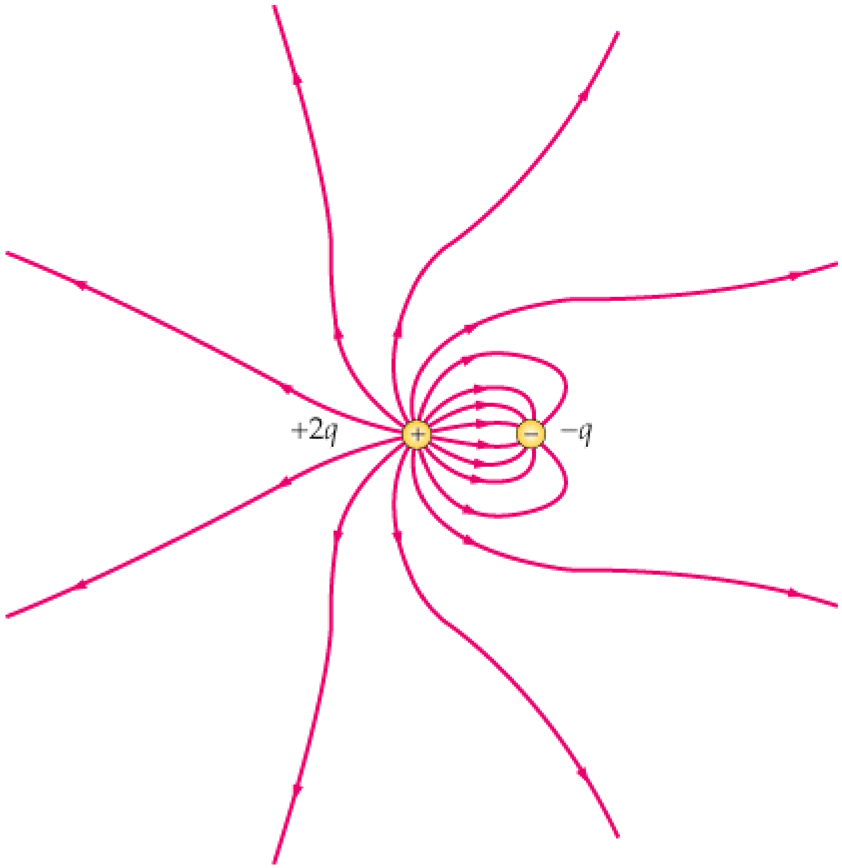


Two Positive Charges: with equal magnitude



Electric Field Lines

Opposite charges with unequal magnitudes:

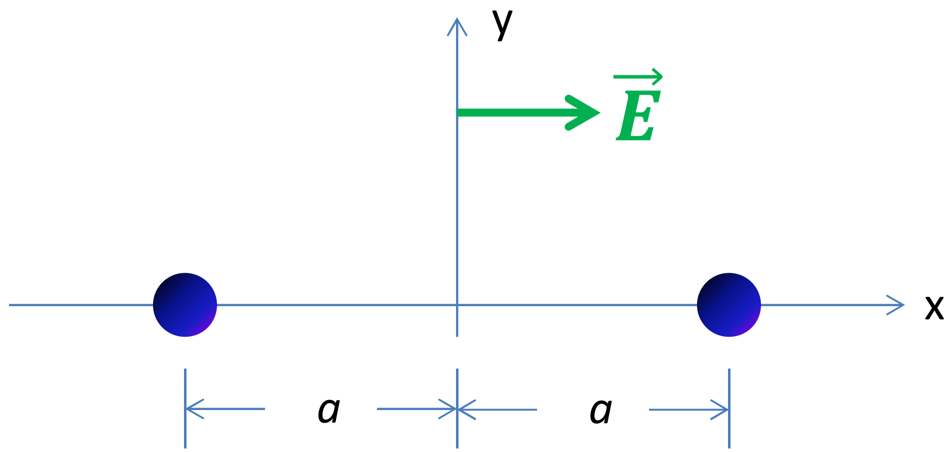


At very large distances, the electric field is the same as one produced by a single point charge with magnitude $Q = +2q - q = +q$.

Density of lines is proportional to the magnitude of the electric field.

Question:

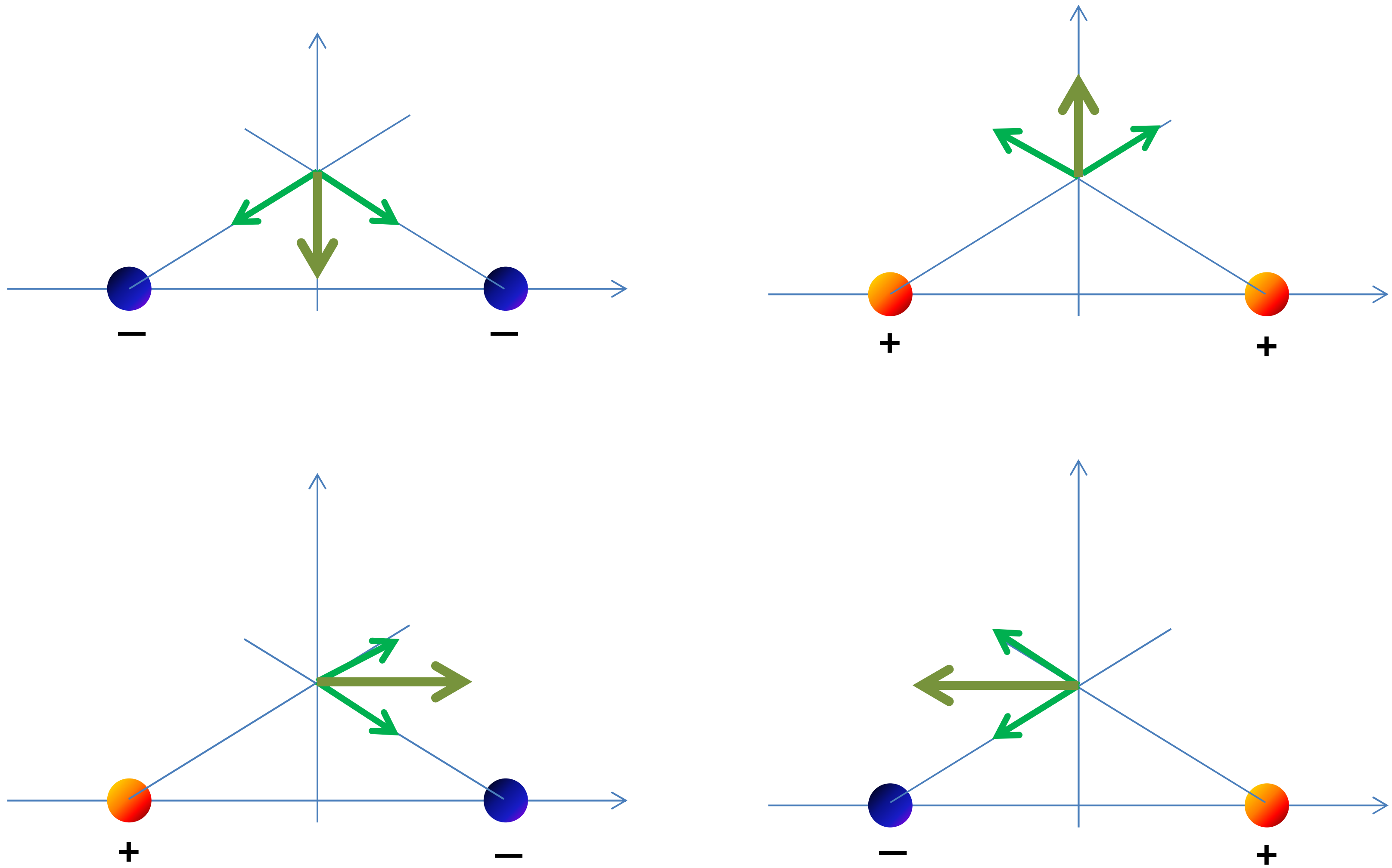
- Two charges are located on the x-axis at positions $\vec{x}_1 = -a\hat{\imath}$ and $\vec{x}_2 = a\hat{\imath}$ as shown:



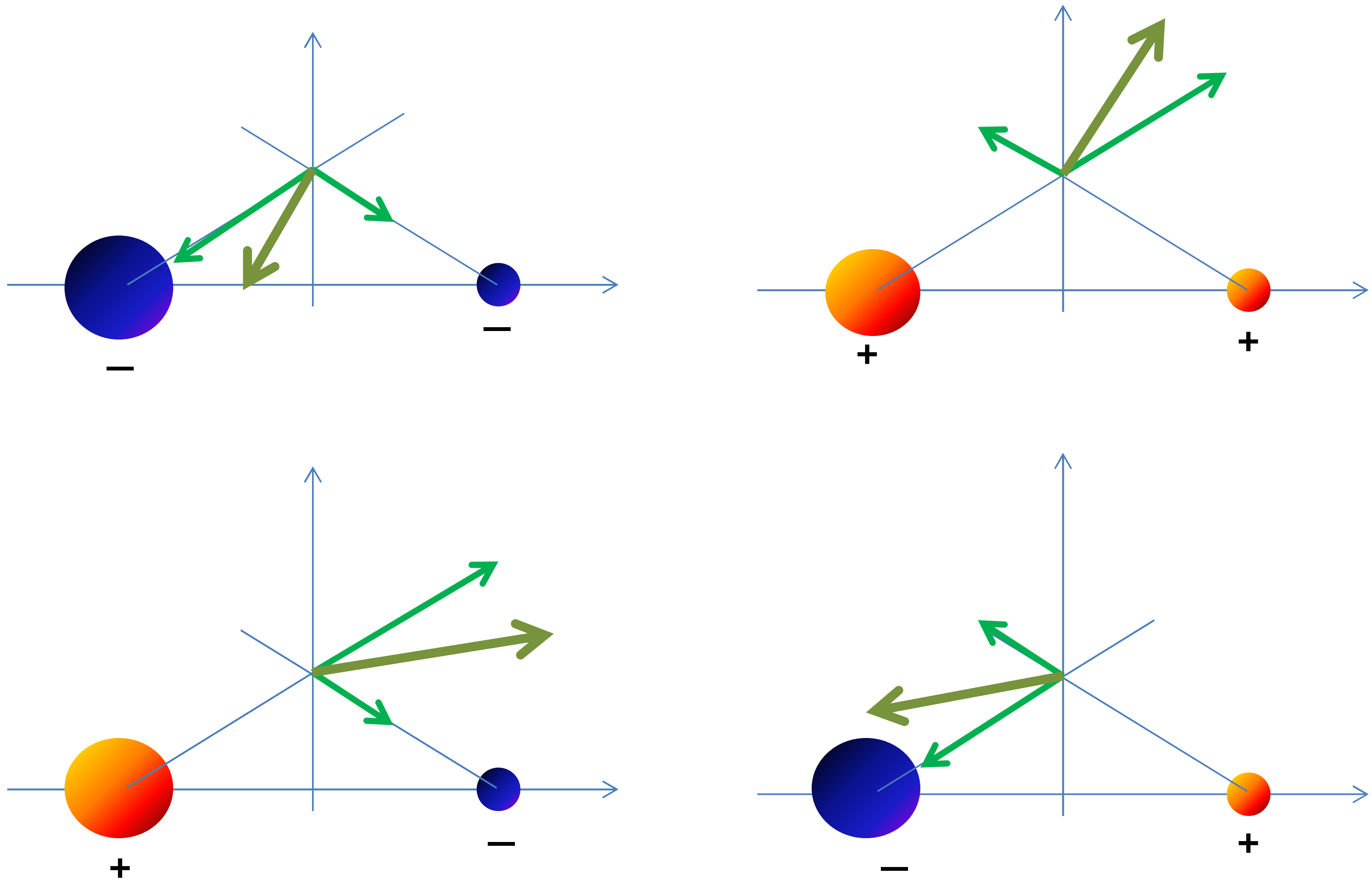
- The electric field at a point on the y-axis points in the $\hat{\imath}$-direction.
- Which statement is true:

(a) Both charges are positive
(b) Both charges are negative
(c) The charges are equal in magnitude but opposite in sign
(d) The charge on the right is positive

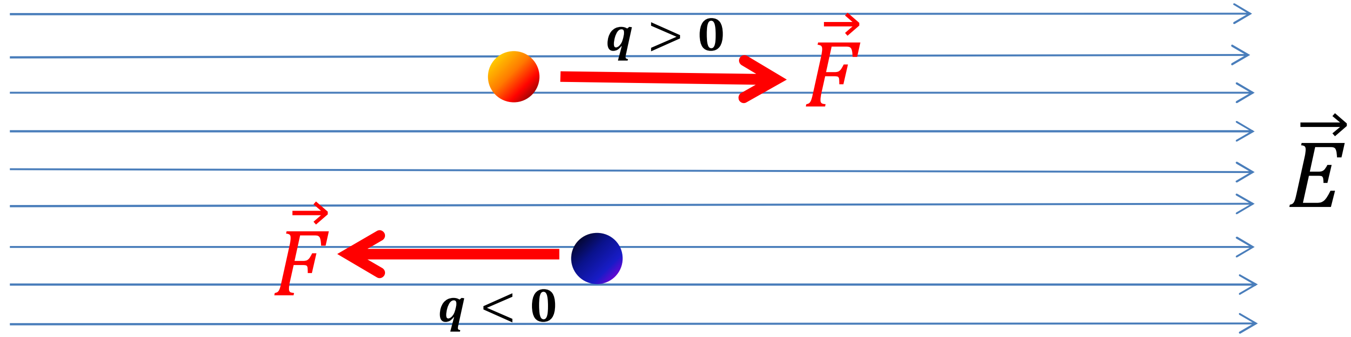
Electric Field from Two Charges



Electric Field from Two Charges



Charge in a Uniform Electric Field



- Force is constant: $\vec{F} = q\,\vec{E}$
- From Newton's second law: $\vec{F} = m\vec{a} = m\frac{d^2\vec{x}}{dt^2}$
- Solve by integration (twice): $\frac{d^2\vec{x}}{dt^2} = \frac{q\vec{E}}{m}$

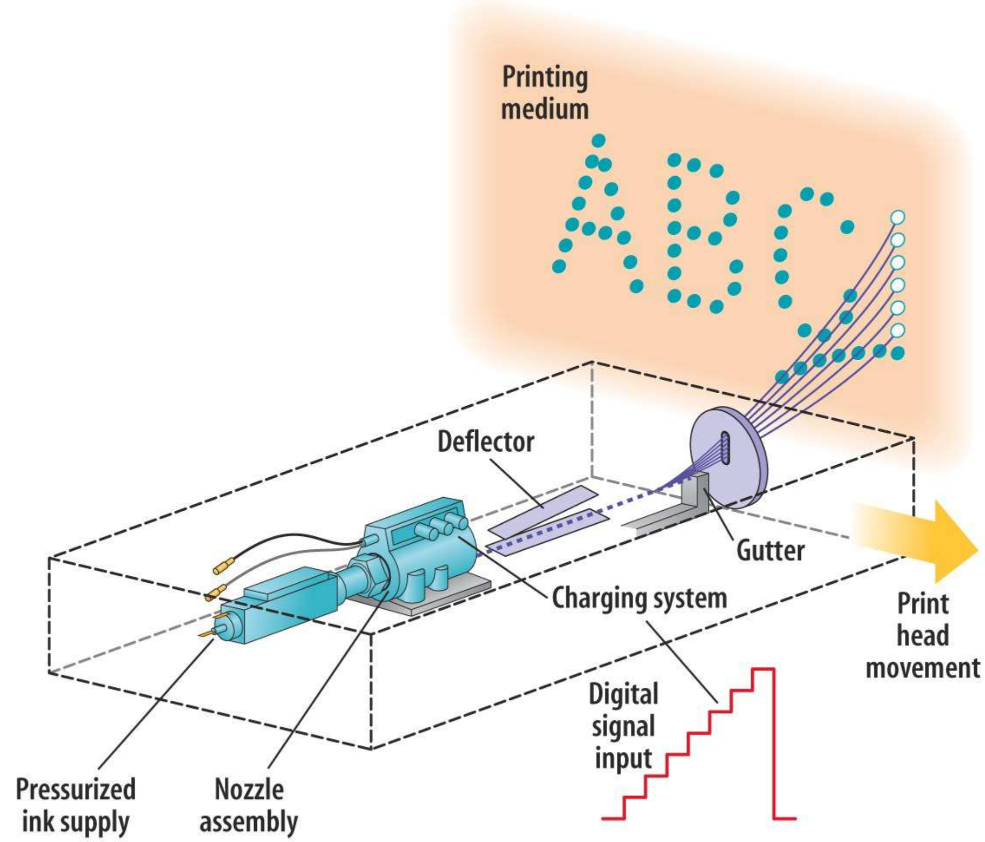
$$\frac{d\vec{x}}{dt} = \int \frac{q\vec{E}}{m}dt = \vec{v}_0 + \frac{q\vec{E}}{m}t$$

Constants of integration!

$$\vec{x}(t) = \int \left(\vec{v}_0 + \frac{q\vec{E}}{m}t\right)dt = \vec{x}_0 + \vec{v}_0 t + \frac{q\vec{E}}{2m}t^2$$

Example Application – CIJ Printer

- Droplets are shot out of the nozzle assembly with a velocity of $50\ m/s$.
- A deflector is $1\ mm$ long and has a constant electric field of $10^5\ N \cdot C^{-1}$.
- The mass of a droplet is $10^{-10}\ kg$ if the diameter is about $70\ \mu m$.
- What charge is needed to deflect a droplet by an angle $\theta = 5.73°\ (0.1\ rad)$?



Work done by Electric Fields

- After the ink drop has been deflected, its velocity has changed by

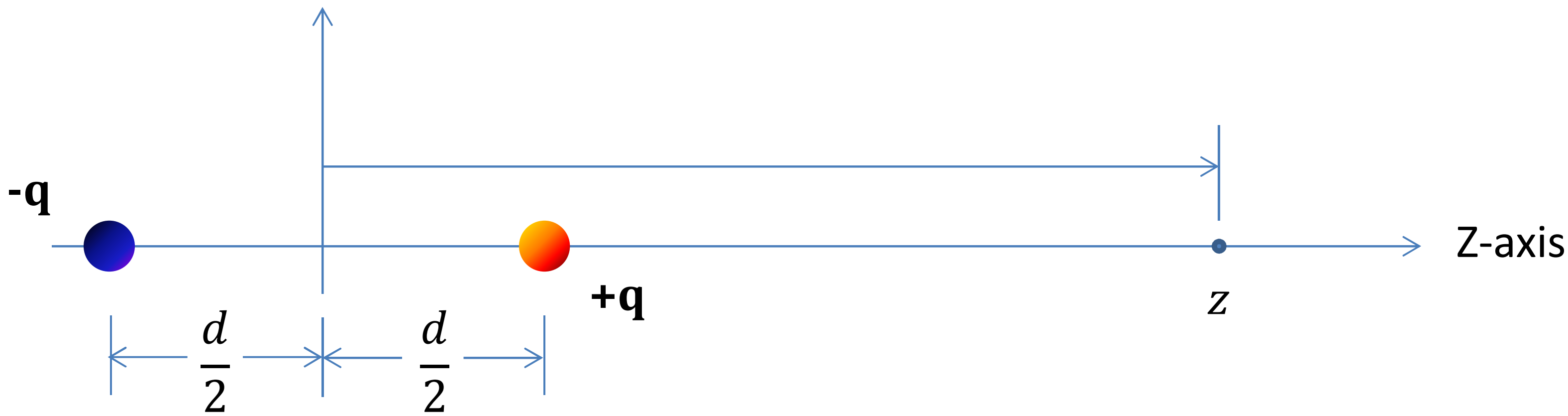
$$\Delta v_y = \frac{qEd}{mv_x}$$

and its kinetic energy changed by

$$\Delta T = \frac{1}{2}m\left(\Delta v_y\right)^2 = \frac{q^2E^2d^2}{2m(\Delta v_x)^2}$$

- How did it get this extra energy?
- The electric field did ***work*** on the charge.
  - Energy is conserved…
  - The electric field must have lost energy.
  - *Electric fields must store some form of energy*.

Dipole Electric Field



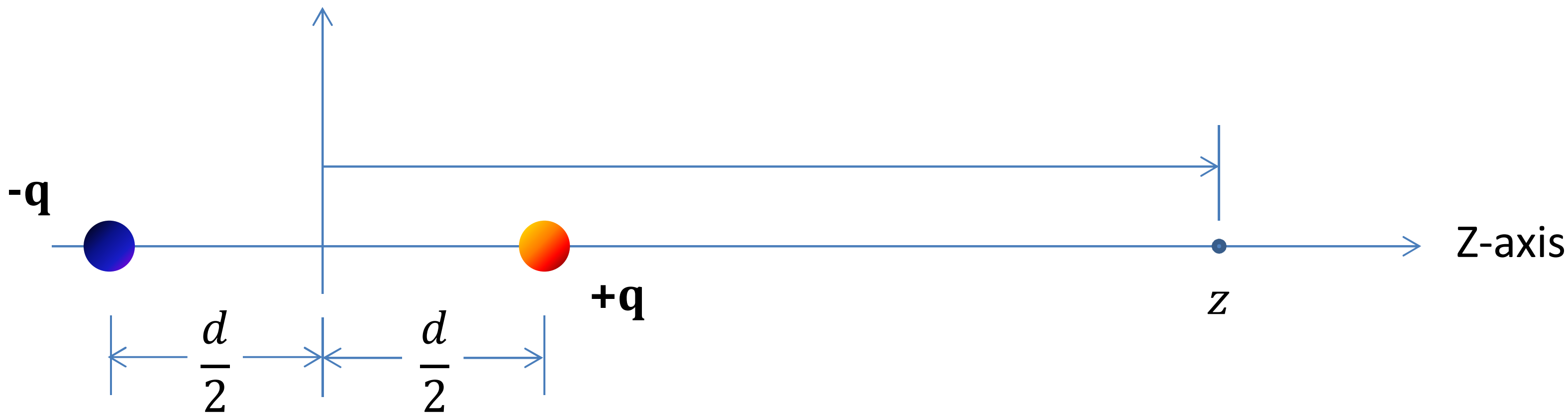
- What is the electric field at a point on the z-axis?

$$\vec{E}_{+q}(z) = \frac{1}{4\pi\epsilon_0}\frac{q}{(z-d/2)^2}\hat{k}$$

$$\vec{E}_{-q}(z) = \frac{1}{4\pi\epsilon_0}\frac{(-q)}{(z+d/2)^2}\hat{k}$$

$$\vec{E}(z) = \vec{E}_{+q}(z) + \vec{E}_{-q}(z)$$

Dipole Electric Field



- What is the electric field at a point on the z-axis?

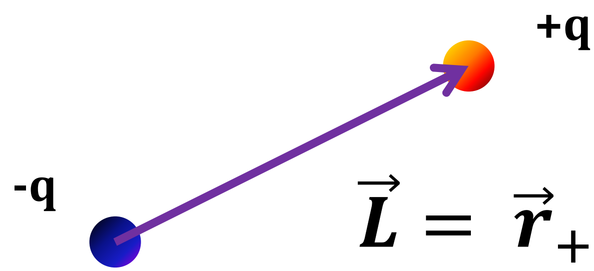
$$\vec{E}_{+q}(z) = \frac{1}{4\pi\epsilon_0}\frac{q}{(z-d/2)^2}\hat{k} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{z^2}+\frac{d}{z^3}+\cdots\right)\hat{k}$$

$$\vec{E}_{-q}(z) = \frac{1}{4\pi\epsilon_0}\frac{(-q)}{(z+d/2)^2}\hat{k} = \frac{-q}{4\pi\epsilon_0}\left(\frac{1}{z^2}-\frac{d}{z^3}+\cdots\right)\hat{k}$$

$$\vec{E}(z) = \vec{E}_{+q}(z)+\vec{E}_{-q}(z) \approx \frac{q}{4\pi\epsilon_0}\left(\frac{d}{z^3}+\frac{d}{z^3}\right)\hat{k} = \frac{qd}{2\pi\epsilon_0 z^3}\hat{k}$$

Dipole Moments

- A dipole moment is defined for equal and opposite charges as a vector:



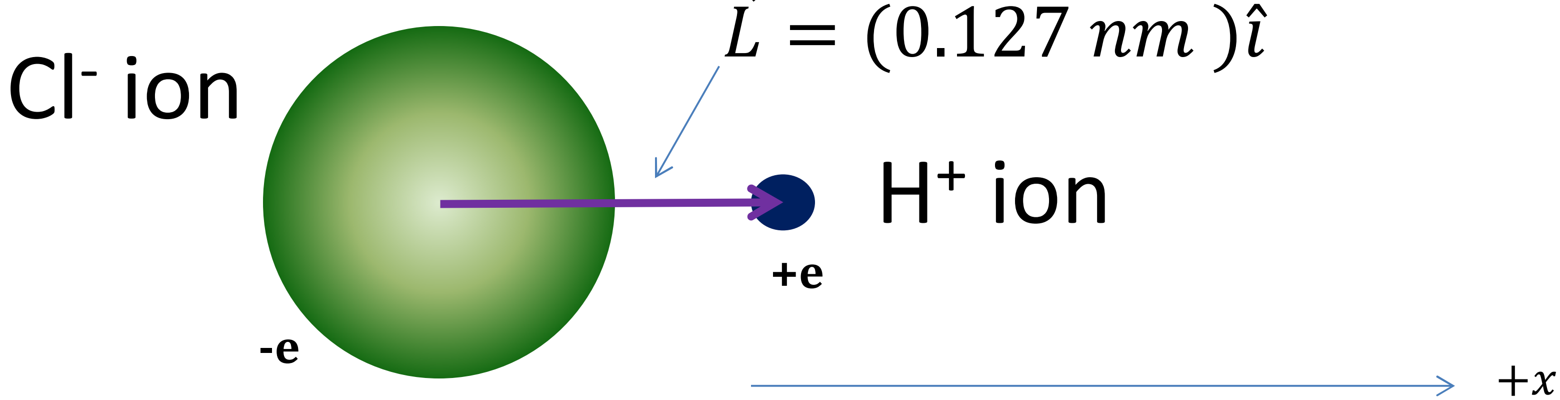
$$\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}}_{+} - \vec{\boldsymbol{r}}_{-}$$

- The vector $\vec{L}$ points from the negative charge to the positive charge.
- The dipole moment is defined: $\vec{p} = q\,\vec{L}$
- Along the axis of the dipole, $E = \frac{1}{4\pi\epsilon_0}\frac{2p}{z^3}$

Example: HCl molecule

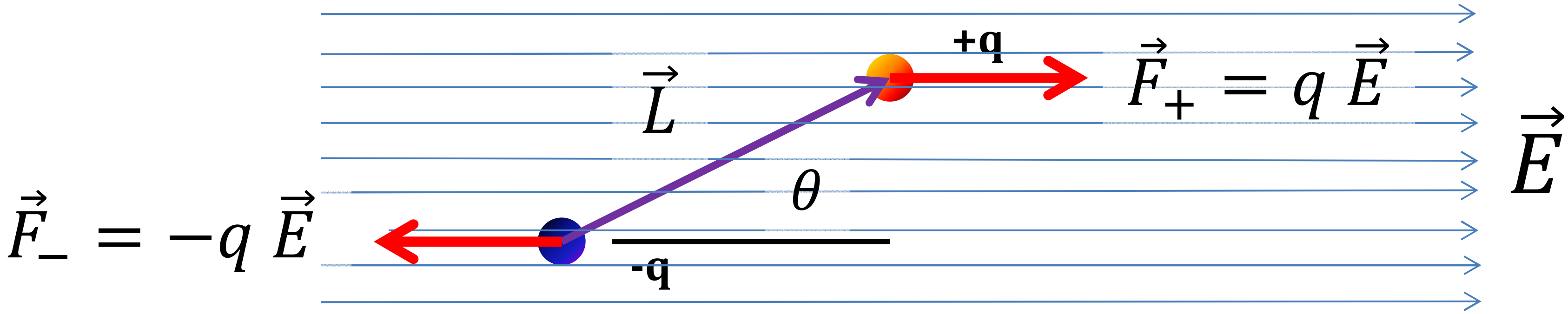
- Chlorine likes to have its outer electron shell completely filled.

$\vec{L} = (0.127\ nm)\hat{\imath}$

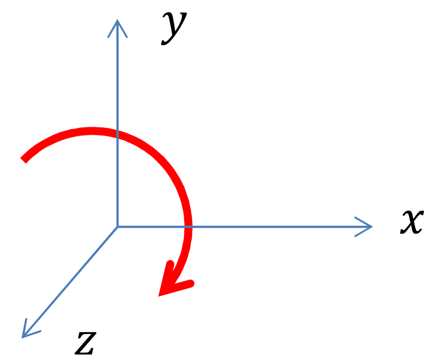


$$\vec{p} = e\,\vec{L} = (1.602 \times 10^{-19}\ C)(0.127\ nm)\hat{\imath}$$
$$= (2.035 \times 10^{-29}\ C\cdot m)\hat{\imath}$$
$$p = |\vec{p}| = 2.035 \times 10^{-29}\ C\cdot m$$

Forces on a Dipole in an Electric Field



- Net force on the dipole is zero
- The forces are equal and opposite but do not act through the same point. ("couple")
- There is a torque:



$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\tau = \vec{\tau} \cdot \hat{k} = -p\,E\,\sin\theta$$

Torque is clockwise!

Potential Energy of a Dipole

- If we turn the dipole in the direction opposite to the torque, (like winding a spring), then we do work on the dipole: its potential energy increases.
- If the dipole turns in the direction of the torque (like unwinding a spring), then it can do work: its potential energy decreases.

$$\Delta U = -\int_{\theta_0}^{\theta} \tau \, d\theta = \int_{\theta_0}^{\theta} p\, E \ \sin\theta \ d\theta$$

$$= -p\, E \ \cos\theta \ - \ U_0$$

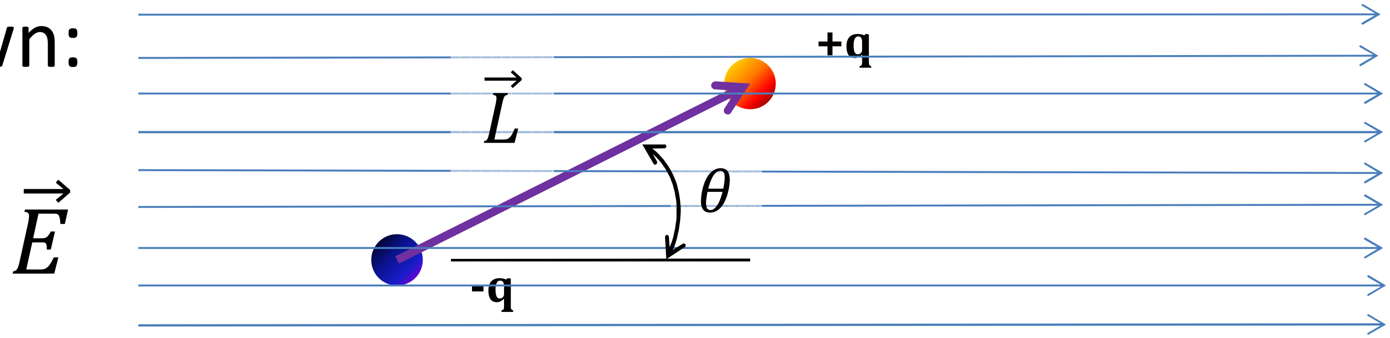
Constant of integration!

- We can *define* $U = 0$ when $\theta = 90°$
- Then, the potential energy of the dipole is:

$$U = -p\, E \ \cos\theta = -\vec{p} \cdot \vec{E}$$

Clicker Question for Credit

- An electric dipole is placed in an electric field as shown:



- If someone rotates the dipole from this orientation to one where $\theta = 90°$ then…

(a) Work is done ***on*** the electric field
(b) Work is done ***by*** the electric field
(c) The net force is zero, so no work is done
(d) The potential energy of the dipole decreases