Purdue University Department of Physics

Physics 24100

Electricity & Optics

Lecture 15 – Chapter 28 sec. 4-5

Fall 2012 Semester

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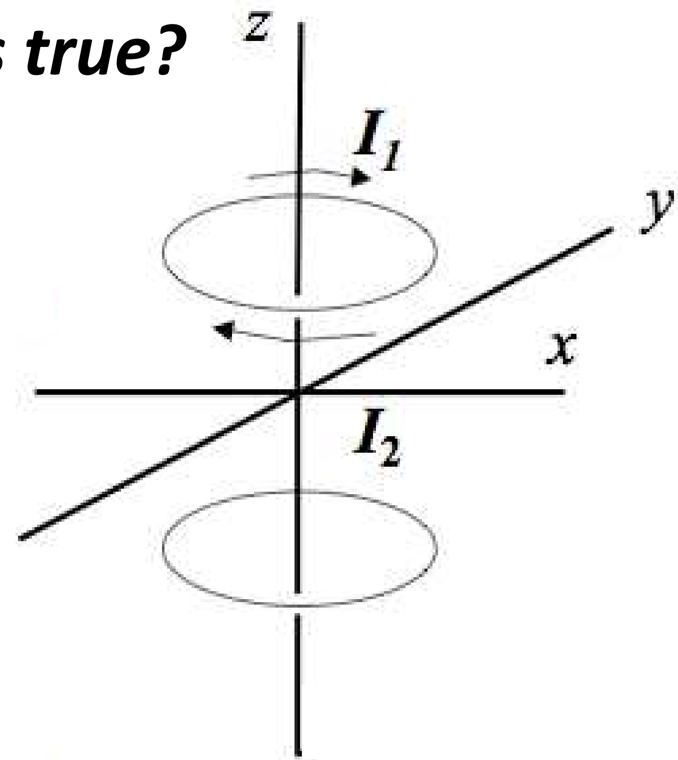
Last Thursday's Clicker Question

Two current loops are perpendicular to the z axis and are centered on the this axis.

- Current I_1 is clockwise.
- I_2 is the induced current in the bottom loop.

If I_2 is clockwise, which statement is true?



A. I_1 is decreasing in magnitude

B. I_1 is constant

C. I_1 is increasing in magnitude

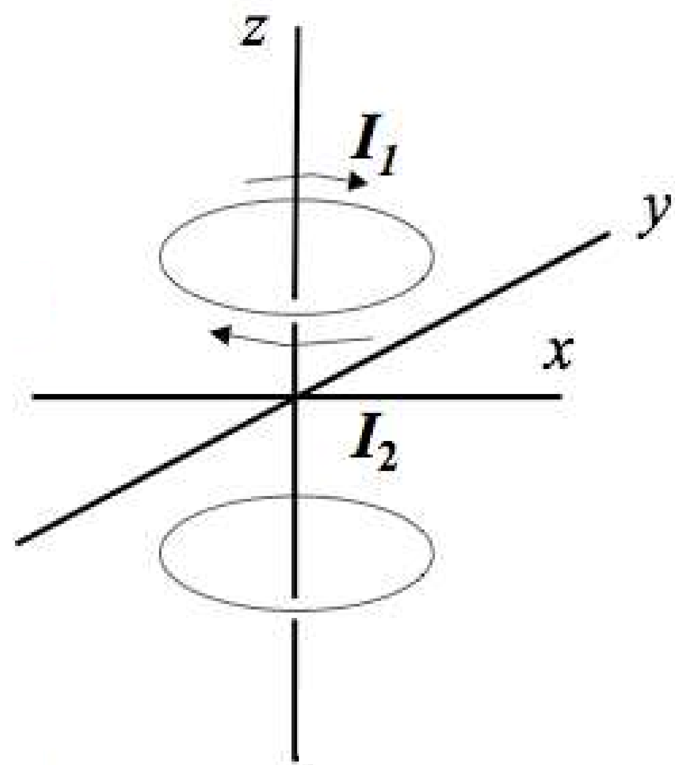
Lenz's Law

"The induced current will create a magnetic field that ***opposes*** the change in flux."

The clockwise current I_1 creates a field pointing down in the middle of the loop.

If the current I_2 is clockwise, then it also creates a magnetic field pointing down.

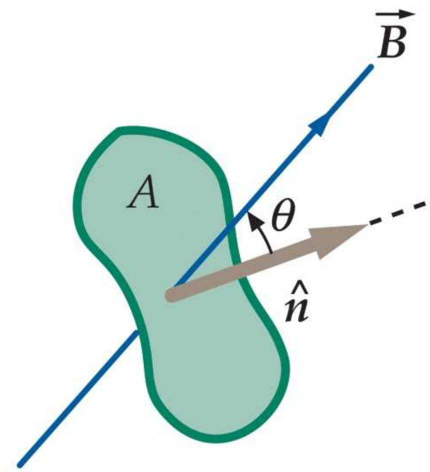
Therefore, I_1 must be decreasing.



Changing Magnetic Flux

$$\phi_m = \int_S \vec{B} \cdot \hat{n}\, dA = NBA\cos\theta$$

(at least it is when $\vec{B} \cdot \hat{n}$ is constant)

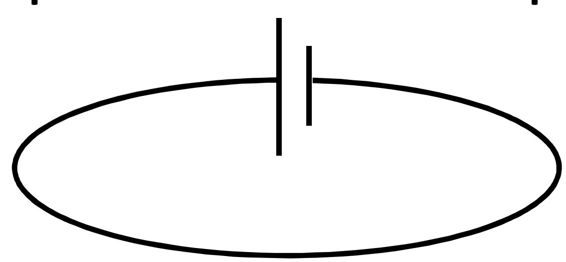


1. $|\vec{B}|$ changes: $\dfrac{d\phi_m}{dt} = \dfrac{dB}{dt} NA\cos\theta$
2. A changes: $\dfrac{d\phi_m}{dt} = \dfrac{dA}{dt} NB\cos\theta$
3. θ changes: $\dfrac{d\phi_m}{dt} = NBA\dfrac{d(\cos\theta)}{dt}$
4. Somehow, N changes: $\dfrac{d\phi_m}{dt} = \dfrac{dN}{dt} BA\cos\theta$

Faraday's Law

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

- A changing magnetic flux through a loop induces an "electromotive force"...
  - This is not quite like an ideal voltage source which changes the potential at one point in the wire



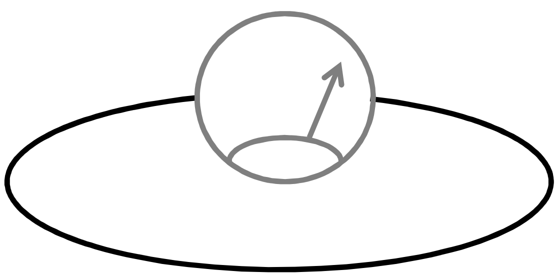
???

This is the wrong way to look at it… the potential can't be the same everywhere in the wire and yet have a potential difference across the voltage source.

Faraday's Law

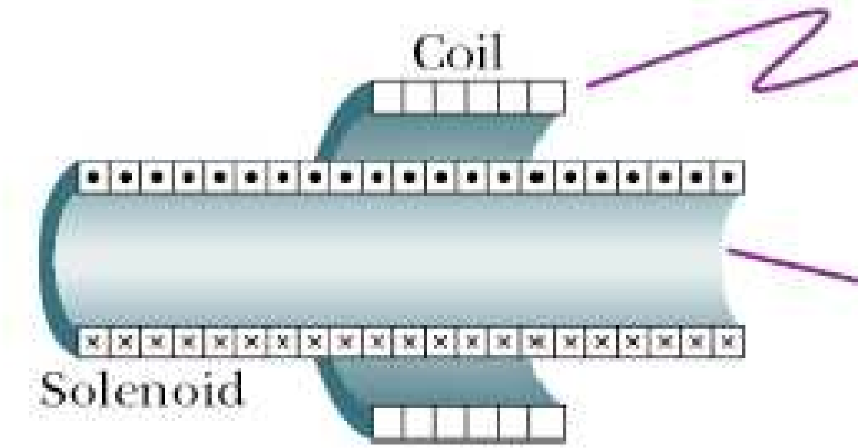
$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

- A changing magnetic flux through a loop induces an "electromotive force"...
  - This is the voltage we would measure if we cut the wire and inserted a volt meter...



If we know the resistance of the wire, then we can calculate the current in the loop.

Example:



Coil: Resistance, $R = 5\ \Omega$

Radius, $r = 3\ cm$

Number of turns, $N = 100$

Solenoid:

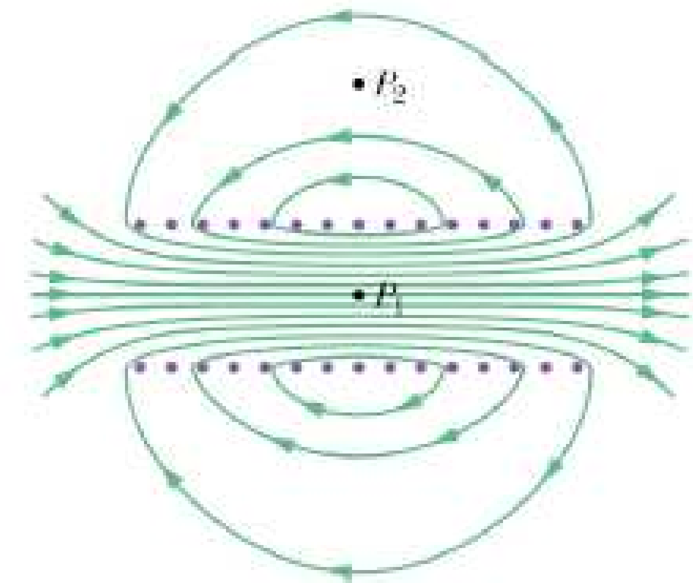
Radius, $r = 2\ cm$

Number of turns per cm,

$n = 200\ /cm$

If the initial current in the solenoid is $I = 1.5\ A$ and it is reduced to zero over $25\ ms$, what current is induced in the coil?

Example:



Coil: Resistance, $R = 5\ \Omega$

Radius, $r = 3\ cm$

Number of turns, $N = 100$

Solenoid:

Radius, $r = 2\ cm$

Number of turns,

$n = 200/cm$

Step 1: What is $\vec{B}$ inside the solenoid?

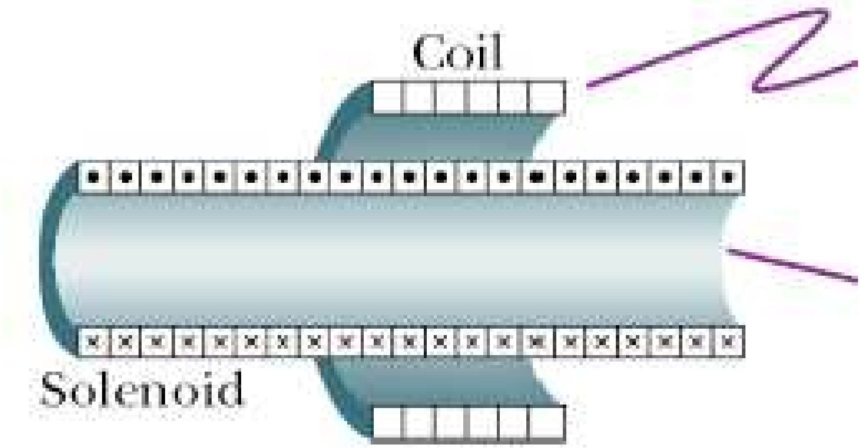
Step 2: What is $\vec{B}$ outside the solenoid?

Step 3: What is ϕ_m through the coil?

Step 4: Calculate $\mathcal{E} = -{}^{d\phi_m}/_{dt}$

Step 5: Calculate $I = \mathcal{E}/R$

Clicker Question



Coil: Resistance, $R = 5\ \Omega$

Radius, $r = 3\ cm$

Number of turns, $N = 100$

Solenoid:

Radius, $r = 2\ cm$

Number of turns,

$n = 200\ /cm$

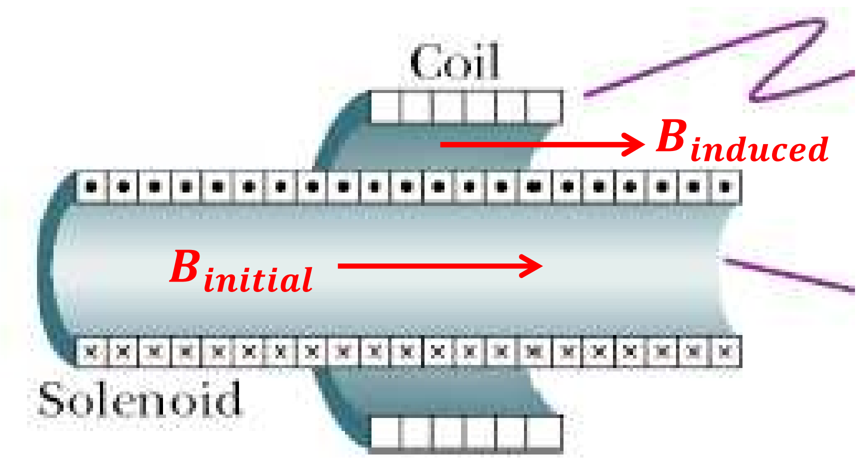
In what direction is the current in the coil?

(a) Same direction as the solenoid

(b) Opposite direction as the solenoid

(Remember Lenz's Law…)

Clicker Question



Coil: Resistance, $R = 5\ \Omega$

Radius, $r = 3\ cm$

Number of turns, $N = 100$

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In what direction is the current in the coil?

(a) Same direction as the solenoid

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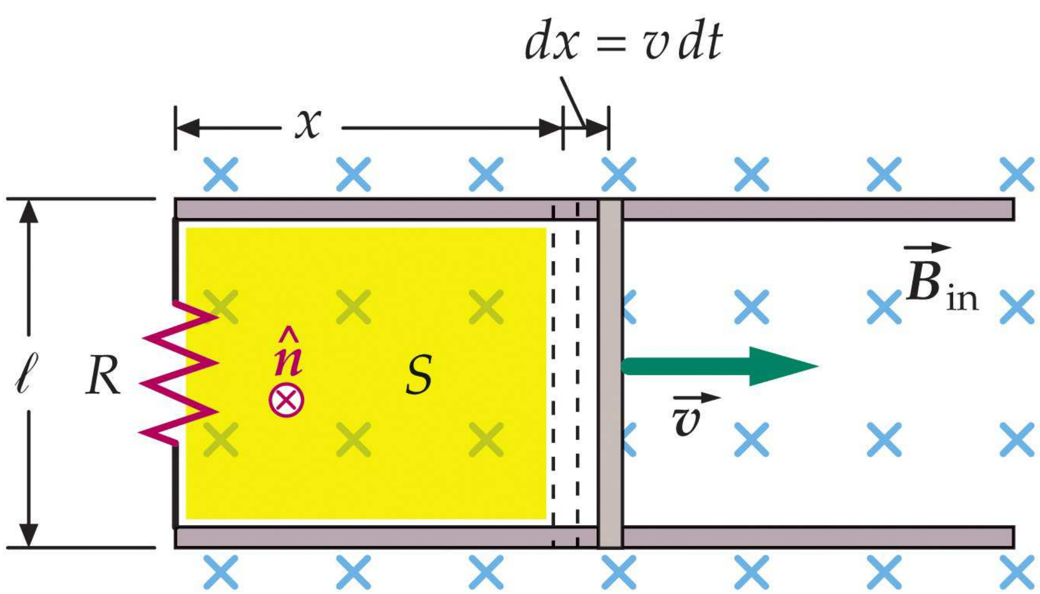
(Remember Lenz's Law…)

Motional EMF

Motional EMF is the electromotive force induced by the motion of a conductor in a magnetic field.

$$\mathcal{E} = -\frac{d\phi_m}{dt} = -B\frac{dA}{dt}$$

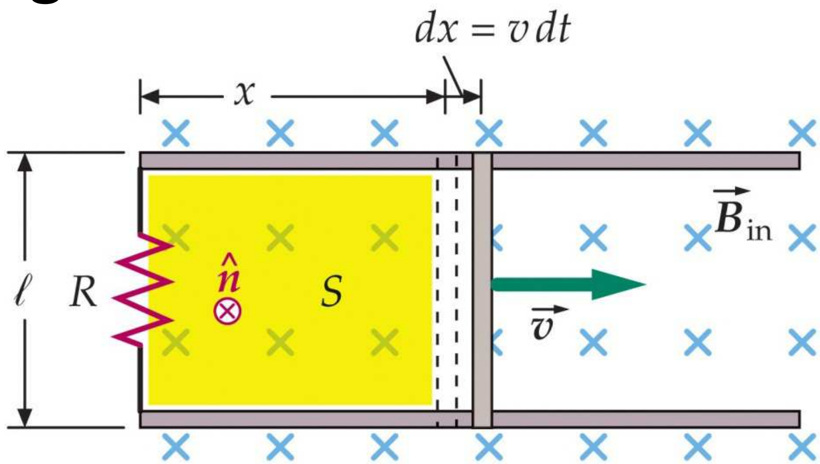


$$A = \ell x$$

$$\frac{dA}{dt} = \ell\frac{dx}{dt} = \ell v$$

Example

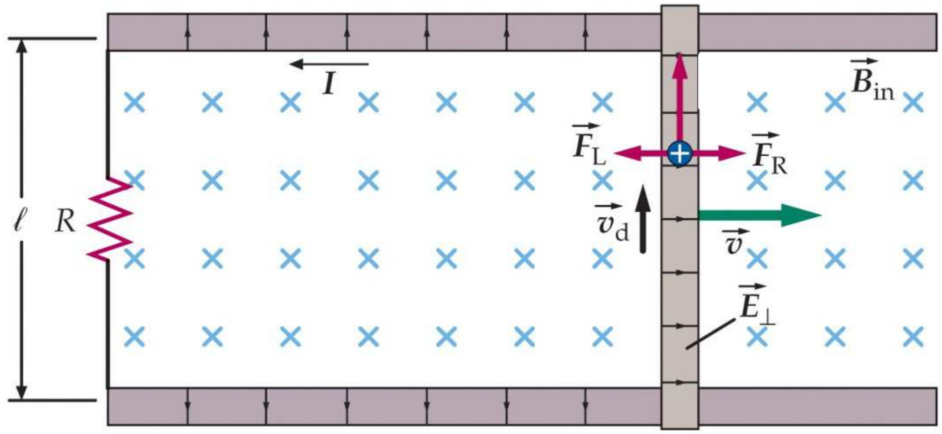
- What is the magnitude and direction of the induced EMF?



$$\mathcal{E} = -\frac{d\phi_m}{dt} = -B\frac{dA}{dt} = -B\ell\frac{dx}{dt} = -B\ell v$$

- Lenz's law: Induced $\vec{B}$ field will oppose any change in ϕ_m.
  - Out the page… The right-hand rule says that I is counterclockwise
  - Opposite direction of fingers when thumb points along $\hat{n}$

Energy Conservation



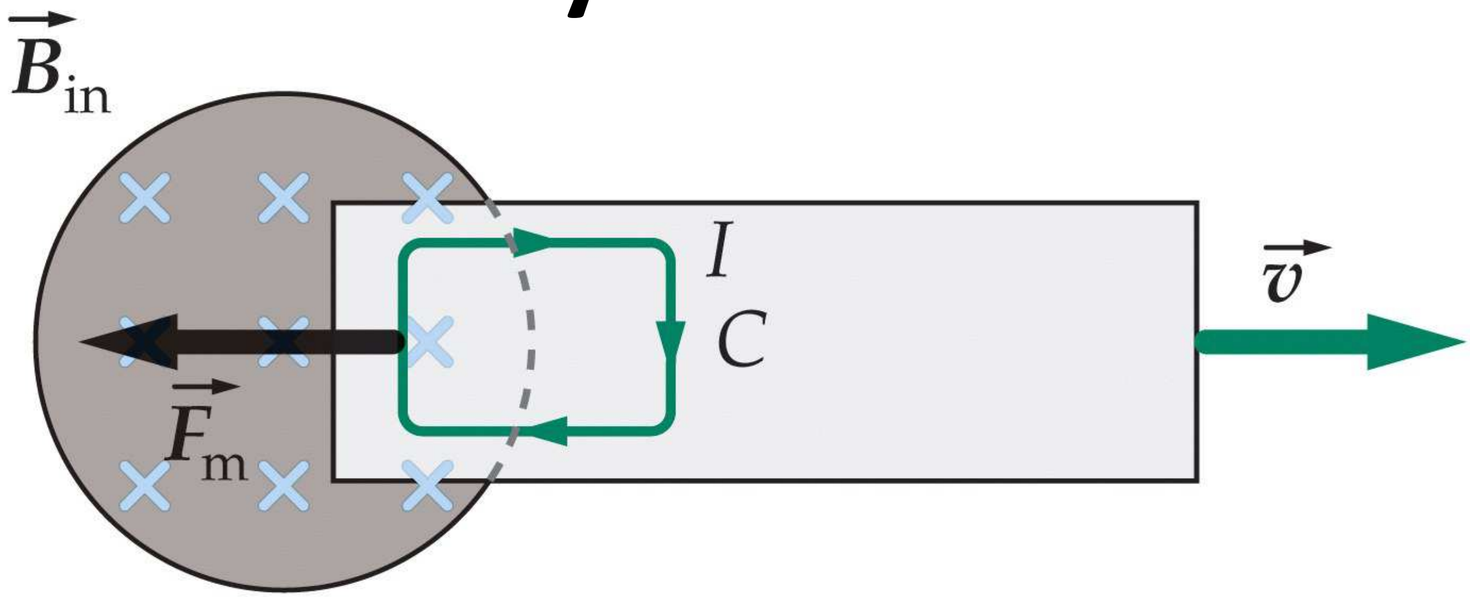
- Power dissipated in the resistor:

$$P = I\mathcal{E} = \mathcal{E}^2/R$$
$$= \frac{B^2\ell^2v^2}{R}$$

- But this is also $P = \dfrac{dW}{dt} = Fv$
- Force on the wire is $F = \dfrac{B^2\ell^2 v}{R}$

This is the force applied to the wire that moves it at a constant velocity.

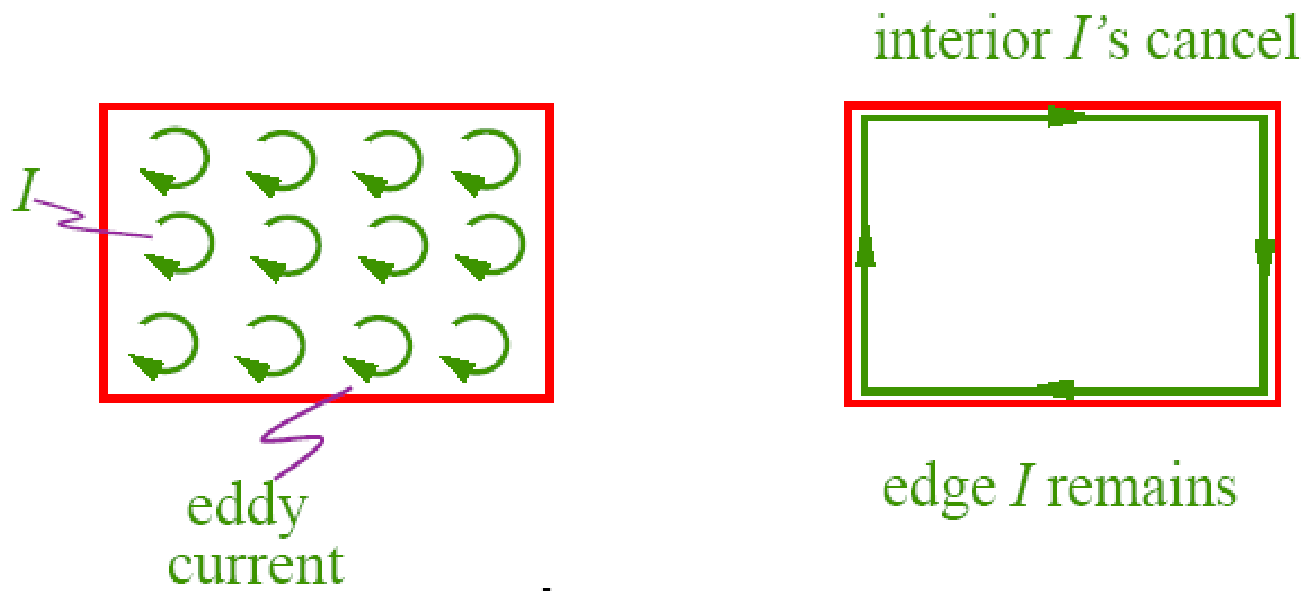
Eddy Currents



Relative motion between magnetic field and conductor induces a current in the conductor.

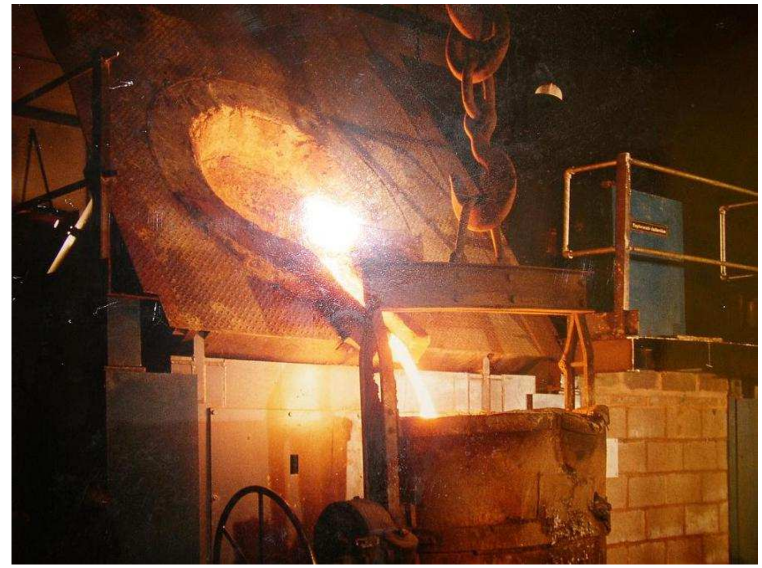
The current induces a force in the magnetic field which opposes the motion.

Eddy Currents



- The energy is dissipated by heating the metal.
- Conducting material near magnetic fields will heat up, sometimes a lot.

Induction Heating

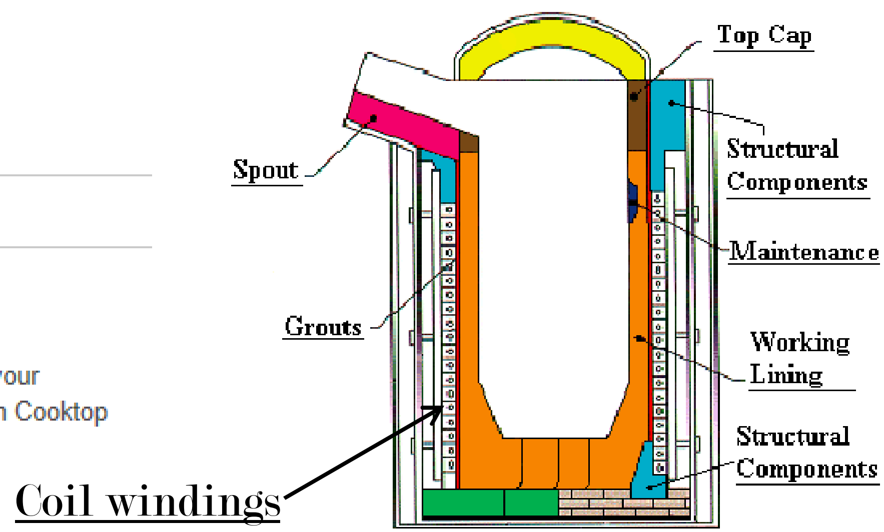


Rosewill RHIC-11001 Induction cooker with Stainless steel pot

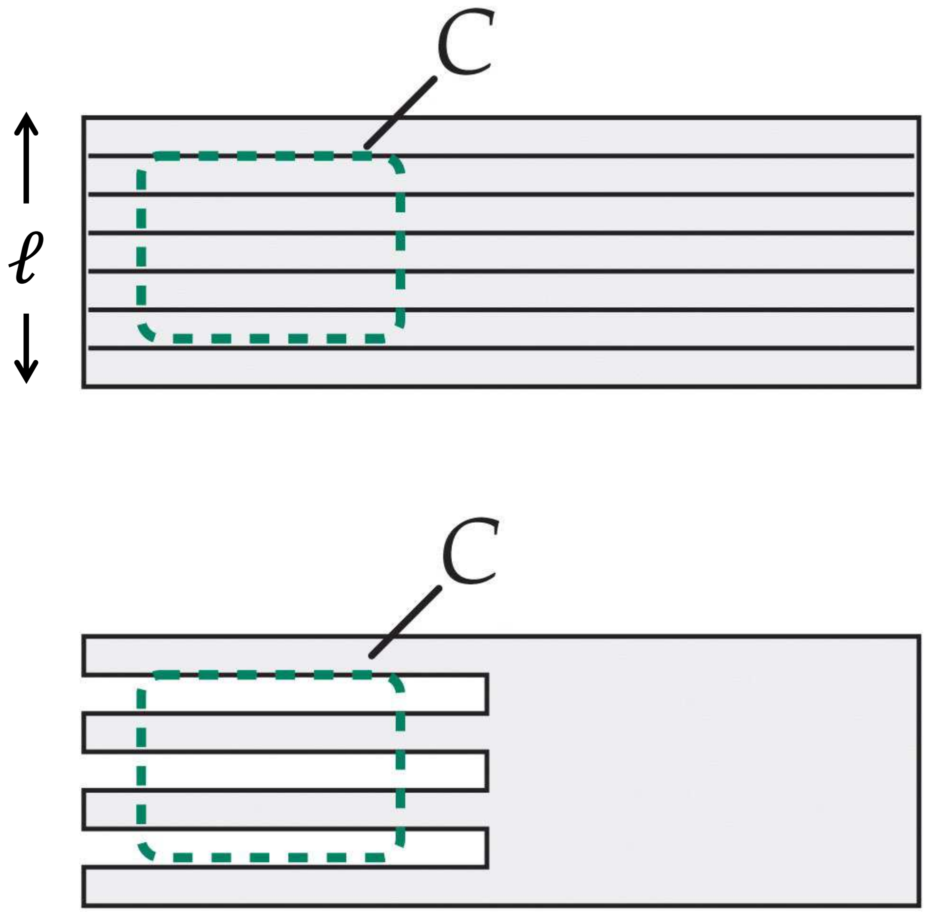
(32) | Write a Review

In stock. Limit 5 per customer.

- 1500W
- Induction Cooktop
- Cook with the power of a magnetic field. If you can stick a magnet to your cookware you can cook with our Induction Cooktop. With the Induction Cooktop you can braise, deep



Reducing Eddy Currents



$$P = \frac{B^2 \ell^2 v^2}{R}$$

Slots cut into metal break the loop into smaller pieces…

Inductance

- Remember capacitance? $Q \propto V$

$$Q = C\,V$$

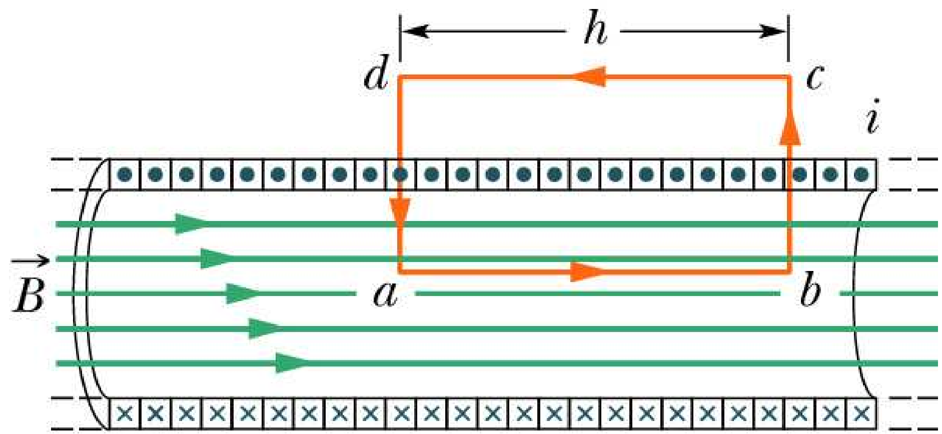
- Magnetic flux is proportional to current:

$$\phi_m = L\,I$$

- The inductance, L, depends on the geometry of the conductor.
- Symbol: 
- Units: Henry = Webers/Ampere = $T \cdot m^2/A$

Inductance

- Example: A solenoid…



$$B = \mu_0 n\, I = \mu_0 I \,\frac{N}{\ell}$$

$$\phi_m = NBA = \boxed{\frac{\mu_0 N^2 A}{\ell}} I = \boxed{L}\, I$$

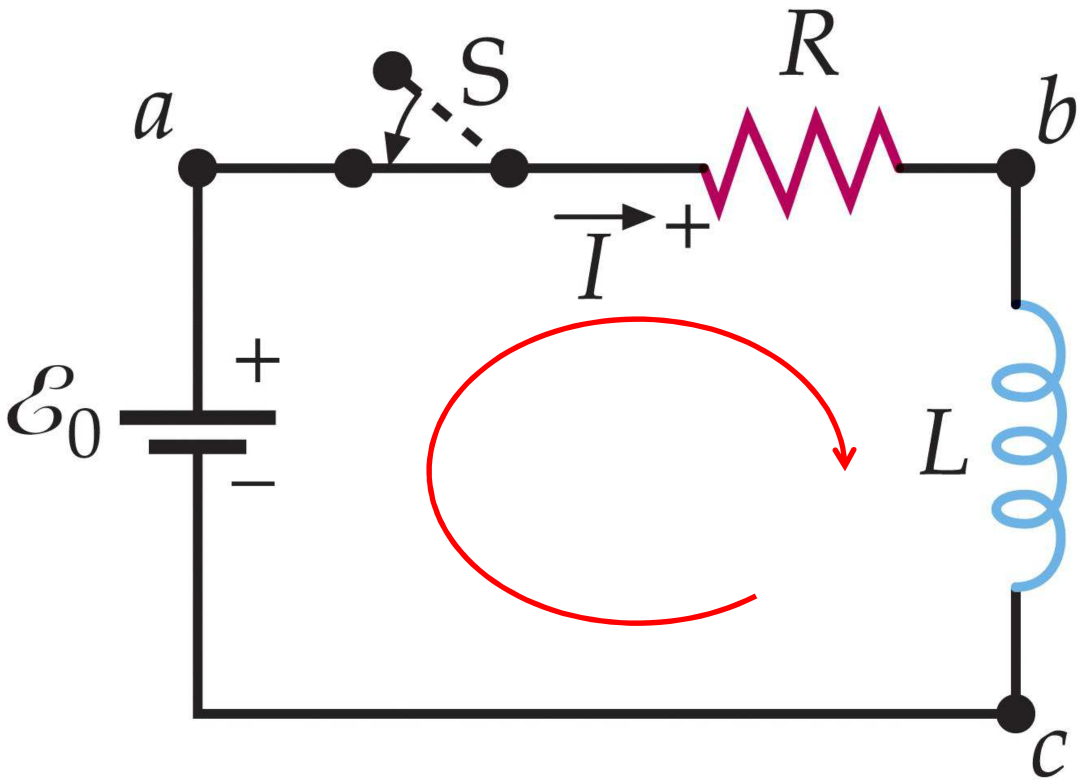
Inductors

- Magnetic flux: $\phi_m = L\,I$
- Changing current:

$$\frac{d\phi_m}{dt} = L\frac{dI}{dt} = -\mathcal{E}$$

- Changing the current through an inductor induces an opposing voltage across the inductor.
- In a circuit it acts like a voltage source with $\Delta V = -\mathcal{E}$

Inductors in Circuits



The voltage induced across the inductor opposes any change in current.

$$\mathcal{E}_0 - I\,R - L\frac{dI}{dt} = 0$$

Kirchhoff's Loop Rule!

Energy Stored in Magnetic Field

- Power delivered to the inductor:

$$I\mathcal{E}_0 = I^2 R + LI\frac{dI}{dt}$$

- Energy stored:

$$U_m = \int P dt = L \int I\, dI = \frac{1}{2} L I^2$$

- Compare this with the energy stored in a capacitor:

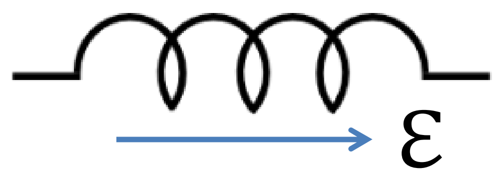
$$U_e = \frac{1}{2} C V^2$$

Energy Stored in a Magnetic Field

- In a solenoid, $B = \mu_0 n\, I = \mu_0 I \frac{N}{\ell}$
- Current expressed in terms of B is $I = \frac{B\,\ell}{\mu_0 N}$
- Inductance: $L = \frac{\mu_0 N^2 A}{\ell}$
- Stored energy: $U_m = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 B^2\, A\ell$
- Magnetic energy density: $u_m = \frac{1}{2\mu_0} B^2$
- Electric energy density: $u_e = \frac{1}{2} \varepsilon_0 E^2$

Clicker Question

- An inductor in a circuit produces an induced EMF that is positive in the orientation shown:



- Which statement could describe the current?

(a) Increasing current to the right

(b) Decreasing current to the right

(c) Increasing current to the left

(d) Either (b) or (c)

(e) Either (a) or (c)