Clicker Question

• Rank the current loops in order of increasing force:

\[ \frac{dF_{12}}{d\ell_2} = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{R} \]

A. \( 1 < 2 < 3 \)
B. \( 2 < 3 < 1 \)
C. \( 3 < 1 < 2 \)
D. \( 3 < 2 < 1 \)
E. \( 3 = 2 = 1 \)
Clicker Question

Forces on the sides parallel to $I_1$ are opposite.

The force on the near side is greater when it is longer:
  - Therefore $(2) < (1)$

The force is smaller when the side is farther from $I_1$:
  - Therefore $(3) < (2)$

\[
\frac{dF_{12}}{d\ell_2} = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{R}
\]

A. $1 < 2 < 3$
B. $2 < 3 < 1$
C. $3 < 1 < 2$
D. $3 < 2 < 1$
E. $3 = 2 = 1$
General Review

So far we have covered:

• **Electrostatics**
  – Forces on charge $q$ in an electric field
  – Electric fields produced by $q$

• **Magnetostatics**
  – Forces on charge $q$ moving in a magnetic field
  – Magnetic field produced by motion of $q$

Next we consider **Electrodynamics**:
  – A changing magnetic field produces an electric field
  – A changing electric field produces a magnetic field
Magnetic Flux

• We define magnetic flux in the same way we defined electric flux:

\[ \phi_e = \int_S \hat{n} \cdot \vec{E} \, dA \quad \quad \phi_m = \int_S \hat{n} \cdot \vec{B} \, dA \]
Faraday’s Law of Magnetic Induction

• A change in magnetic flux through a conducting loop induces a current in the loop.

• What causes the charges in the conductor to move? An electromotive force, $\mathcal{E}$...

$$\mathcal{E} = - \frac{d\phi_m}{dt}$$

• Another way to think about it:
  – Changing magnetic flux induces an electric field
  – This changes the electric potential of the charge carriers
Demonstration

- When a bar magnet is moved in the vicinity of a loop of wire, the magnetic flux through the loop will change.
- A current is induced...
- Faster motion induces a larger current.
- No current when the motion stops.
Changes in Magnetic Flux

• The magnetic flux through a loop can change in various ways:
  – The magnetic field could change
  – The source of the magnetic field could move
  – The loop could move in a non-uniform field
  – The orientation of the loop could change
  – Others?

• Examples...
Changes in Magnetic Flux

\[ \phi_m = \int_S \hat{n} \cdot \vec{B} \, dA \]

- Only depends on the component of \( \vec{B} \) perpendicular to the plane of the loop.

Use the right hand rule!

\[
\phi_m > 0 \quad \text{Magnetic flux increases} \quad \mathcal{E} < 0 \quad \phi'_m > \phi_m
\]
Changes in Magnetic Flux

\[ \phi_m = \oint_S \hat{n} \cdot \vec{B} \, dA \]

- Only depends on the component of \( \vec{B} \) perpendicular to the plane of the loop.

Use the right hand rule!

Magnetic flux increases (because it’s negative!)

\( \phi_m < 0 \)

\( \phi'_m > \phi_m \)

\( \mathcal{E} < 0 \)
Changes in Magnetic Flux

$\phi_m > 0$  \hspace{2cm}  $\varepsilon < 0$  \hspace{2cm}  $\phi'_m > \phi_m$

Magnetic flux increases
Changes in Magnetic Flux

\[ \phi'_m < \phi_m \]

\[ \phi_m > 0 \]  \hspace{1cm} \text{Magnetic flux decreases} \hspace{1cm} \varepsilon > 0
Changes in Magnetic Flux

$\phi_m > 0$  
Magnetic flux decreases  
$\phi'_m = 0$  
$\varepsilon > 0$
Calculating Magnetic Flux

- If the surface is a plane over which the magnetic field is constant, then

\[
\phi_m = \int_S \hat{n} \cdot \vec{B} \, dA = BA \cos \theta
\]

Units of magnetic flux: Webers... \(1 \, \text{Wb} = 1 \, T \cdot m^2\)
Clicker Question

- A loop of wire in the Earth’s magnetic field has its normal vector pointing north.
- It is rotated so that its normal vector is pointing south.
- How did the magnetic flux through the loop change?
  (a) $\phi_{\text{final}} > \phi_{\text{initial}}$
  (b) $\phi_{\text{final}} < \phi_{\text{initial}}$
  (c) $\phi_{\text{final}} = \phi_{\text{initial}}$
Clicker Question

- Initial magnetic flux is positive
- Final magnetic flux is negative
- The flux decreased: $\phi_{final} < \phi_{initial}$
Lenz’s Law

• Changing magnetic flux induces a current in a loop of wire.

• The induced current will create its own magnetic field.

• Lenz’s Law: *The induced magnetic field will oppose changes to the original magnetic field.*
  - If the flux is decreasing, the induced field will increase the flux will add to the applied field
  - If the flux is increasing the induced field will be opposite the applied field
Lenz’s Law

\[ B = \frac{\mu_0 I_c}{2\pi R} \]

1. Pick an orientation for the loop
   • eg, counter-clockwise: \( \hat{n} \) points out
2. Magnetic field is pointing in
   • Magnetic flux is negative
3. \( |\vec{B}| \) increases as the loop gets closer
   • Magnetic flux is decreasing
4. **Induced field will point out**
   (in the direction of \( \hat{n} \))
Lenz’s Law

\[ B = \frac{\mu_0 I_c}{2\pi R} \]
Lenz’s Law

1. Pick an orientation for the loop
   • eg, counter-clockwise: \( \hat{n} \) points out

2. Magnetic field is pointing in
   • Magnetic flux is negative

3. \( |\mathbf{B}| \) decreases as the loop moves away
   • Magnetic flux is increasing

4. **Induced field will point in**
   **(opposite the direction of \( \hat{n} \))**
Demonstration: The Electromagnetic Cannon

- An alternating voltage source produces a changing magnetic field in the solenoid.
- When the magnetic field increases, the induced current tries to oppose it.
- The magnetic field induced in the loop will oppose changes in $\vec{B}$.
- The magnetic forces fling the ring.
Clicker Question

Two current loops are perpendicular to the z axis and are centered on the this axis.

- Current $I_1$ is clockwise.
- $I_2$ is the induced current in the bottom loop.

*If $I_2$ is clockwise, which statement is true?*

A. $I_1$ is decreasing in magnitude
B. $I_1$ is constant
C. $I_1$ is increasing in magnitude