Clicker Question

• Treat a lightning bolt like a long, straight wire.
• If the current in a lightning bolt is 100 kA, how would the magnetic field 1 km away compare with the Earth’s magnetic field \((5 \times 10^{-5} \, T)\)?

\[
|\vec{B}| = \frac{\mu_0}{4\pi} \frac{2I}{R} = \frac{2 \times 10^{-7} \times 10^5}{10^3} \sim 10^{-5} \, T
\]

(a) Much less
(b) Much greater
(c) About the same

\((\mu_0 = 4\pi \times 10^{-7} \, T \cdot m/A)\)
Physics Help Center Survey

*PHYS Building, Rms. 11-12*

How often do you use the Help Center on average?

A. Never
B. Once or at most twice a semester
C. Several times in a semester
D. Around once a week
E. More than once a week
Physics Help Center Survey

*PHYS Building, Rms. 11-12*

How useful was your Help Center visit?

A. I did not use the Help Center  
B. Useless  
C. Not very useful  
D. Useful  
E. Very useful
Magnetic Fields

\[ \vec{B} = \frac{\mu_0}{4\pi} \int I \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \int I \frac{d\vec{l} \times \hat{r}}{r^3} \]

\[ |\vec{B}| = \frac{\mu_0}{4\pi} \frac{2I}{R} \]

\[ |\vec{B}| = \frac{\mu_0}{2} \frac{IR^2}{(R^2 + z^2)^{3/2}} \]

\[ |\vec{B}| = \mu_0 n I \]
Forces on Current Carrying Wires

• Two wires carrying currents $I_1$ and $I_2$ will exert forces on each other:

  – Magnetic field from $I_1$ is $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I_1 \, d\vec{l}_1 \times \hat{r}}{r^2}$

  – Force on $I_2$ is $d\vec{F}_{12} = I_2 \, d\vec{l}_2 \times \vec{B}$

• Conversely

  – Magnetic field from $I_2$ is $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I_2 \, d\vec{l}_2 \times \hat{r}}{r^2}$

  – Force on $I_1$ is $d\vec{F}_{21} = I_1 \, d\vec{l}_1 \times \vec{B}$
Forces on Parallel Wires

\[ |\vec{B}| = \frac{\mu_0}{4\pi} \frac{2I_1}{R} \]

\[ d\vec{F}_{12} = I_2 \, d\vec{\ell}_2 \times \vec{B} \]

Force per unit length:

\[ \frac{dF_{12}}{d\ell_2} = \frac{\mu_0}{4\pi} \frac{2I_1 \, I_2}{R} \]
Force on Parallel Wires

$I_1$  $I_2$
attraction

$I_1$  $I_2$
repulsion

Will this loop expand or contract?
Magnetic “Pressure”

Magnets need to withstand about 5 tons of internal forces without distortion.

Normal dipole magnets

\[ B \sim 1 \, T \]

Superconducting dipole magnets

\[ B \sim 5 \, T \]
Remember Gauss’s Law?

• Electric field:

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\hat{r}}{r^2} dQ \]

• Gauss’s Law:

\[ \int_S \hat{n} \cdot \vec{E} \, dA = \frac{Q_{\text{inside}}}{\varepsilon_0} \]

• If \( \vec{E} \) is constant over the surface then we can bring it outside the integral
  – The integral is just the surface area
  – This works only when there is sufficient symmetry
Gauss’s Law Applied Magnetism

- In magnetism we can have dipoles or currents but *no magnetic monopoles*

- Gauss’s law:
\[
\oint_S \hat{n} \cdot \vec{B} \, dA = \frac{Q_{\text{inside}}}{\varepsilon_0} = 0
\]

- One of Maxwell’s Equations:
\[
\nabla \cdot \vec{B} = 0
\]
Ampere’s Law

• But can we do something similar to calculate the magnetic field in cases with lots of symmetry?

• Yes:

\[ \oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_C \]
Example

• What is the magnetic field around a long, straight wire?
• From symmetry, we expect that the magnetic field is always azimuthal: \( \vec{B} = B \hat{\phi} \)

• The path length element is also azimuthal: \( d\vec{\ell} = d\ell \hat{\phi} \)

\[
\int_C \vec{B} \cdot d\vec{\ell} = B \int_C d\ell = \mu_0 I_C \\
2\pi BR = \mu_0 I_C \quad \Rightarrow \quad B = \frac{\mu_0 I_C}{2\pi R}
\]
Magnetic Field Inside a Long Straight Wire

\[ I_C = I \frac{r^2}{R^2} \]

\[ B(r) = \frac{\mu_0 I_C}{2\pi R} \]

\[ B = \frac{\mu_0 I r}{2\pi R^2} \quad \text{(Inside)} \]

\[ B = \frac{\mu_0 I}{2\pi R} \quad \text{(Outside)} \]
Magnetic Field Inside a Solenoid

• Symmetry principles:
  – The magnetic field always points along the axis of the solenoid: $\vec{B} = B\hat{k}$
  – It is independent of $z$, except at the ends.

• Outside the solenoid, we expect $\vec{B} \to 0$ as $r \to \infty$

• Inside the solenoid, does $\vec{B}$ depend on $r$?
Magnetic Field Inside a Solenoid

\[ \oint_C \mathbf{B} \cdot d\bar{\ell} = \int_a^b \mathbf{B} \cdot d\bar{\ell} + \int_b^c \mathbf{B} \cdot d\bar{\ell} + \int_c^d \mathbf{B} \cdot d\bar{\ell} + \int_d^a \mathbf{B} \cdot d\bar{\ell} = \mu_0 I_C \]

These will all be zero!
Magnetic Field Inside a Solenoid

- Enclosed current: \( I_C = n I h \)

\[
\int_C \vec{B} \cdot d\vec{\ell} = B h = \mu_0 I_C
\]

\[
B = \mu_0 n I
\]

\( n \) is the number of turns per unit length.

Make the path \( cd \) very far away, where \( \vec{B} \approx 0 \).

Independent of \( r \) inside the solenoid.
Magnetic Field Inside a Toroid
When Ampere’s Law doesn’t Help

- $B$ can’t be factored out of the integral.

- Finite length current segment is (unphysical)

- Insufficient symmetry

- Current is not continuous (time dependent)
Magnetic Properties of Materials

- Atoms in many materials act like magnetic dipoles.
- Magnetization is the net dipole moment per unit volume:
  \[
  \vec{M} = \frac{d\vec{\mu}}{dV}
  \]
- In the presence of an external magnetic field, these dipoles can start to line up with the field:

Net current inside the material is zero. We are left with a surface current and therefore a magnetic moment.
Magnetization and “Bound Current”

Magnetic dipole for a current loop: \( \vec{\mu} = A I \hat{n} \)

Magnetic moment per unit length:
\[
d\mu = A \, d\ell
\]

Magnetization:
\[
M = \frac{d\mu}{dV} = \frac{d\mu}{A \, d\ell} = \frac{d\ell}{d\ell}
\]

This is the “surface current” per unit length.

Magnetic field due to the surface current is the same as in a solenoid:
\[
B = \mu_0 \, n \, I = \mu_0 \, M
\]
Magnetization and Magnetic Susceptibility

• How well do the microscopic magnetic dipoles align with an external applied magnetic field?

• Simplest model: linear dependence on $\vec{B}_{app}$
  
  – Magnetization: $\vec{M} \propto \vec{B}_{app}$
  
  – Magnetic field due to surface current:
    
    $$\vec{B}_m = \mu_0 \vec{M} \equiv \chi_m \vec{B}_{app}$$

  – Magnetic susceptibility: $\chi_m$

• Total magnetic field:

  $$\vec{B} = \vec{B}_{app} + \vec{B}_m = (1 + \chi_m) \vec{B}_{app} \equiv K_m \vec{B}_{app}$$

  – Relative permeability: $K_m$
Magnetic Susceptibility

• Different materials react differently to external magnetic fields:

<table>
<thead>
<tr>
<th>$\chi_m$</th>
<th>Paramagnetism</th>
<th>Diamagnetism</th>
<th>Ferromagnetism</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0 small $\chi_m$</td>
<td>aluminum, tungsten</td>
<td>bismuth, copper, silver</td>
<td>iron, cobalt, nickel</td>
</tr>
<tr>
<td>&lt; 0 small $</td>
<td>\chi_m</td>
<td>$</td>
<td></td>
</tr>
<tr>
<td>&gt; 0 large $\chi_m$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• Dipoles in paramagnetic materials align with $\vec{B}_{app}$
• Dipoles in diamagnetic materials align opposite $\vec{B}_{app}$
• Ferromagnetic materials align strongly even in weak $\vec{B}_{app}$
## Magnetic Susceptibility

<table>
<thead>
<tr>
<th>Material</th>
<th>$\chi_m$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi</td>
<td>$-1.66 \times 10^{-5}$</td>
<td>diamagnetic</td>
</tr>
<tr>
<td>Ag</td>
<td>$-2.6 \times 10^{-5}$</td>
<td>diamagnetic</td>
</tr>
<tr>
<td>Al</td>
<td>$2.3 \times 10^{-5}$</td>
<td>paramagnetic</td>
</tr>
<tr>
<td>Fe (annealed)</td>
<td>5,500</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>Permalloy</td>
<td>25,000</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>mu-metal</td>
<td>100,000</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>superconductor</td>
<td>$-1$</td>
<td>diamagnetic (perfect)</td>
</tr>
</tbody>
</table>
Clicker Question

• Rank the current loops in order of increasing force:

A. $1 < 2 < 3$
B. $2 < 3 < 1$
C. $3 < 1 < 2$
D. $3 < 2 < 1$
E. $3 = 2 = 1$