Question

- A rectangular conductor carries current $I$ in a magnetic field as shown:

- If the charge carriers are electrons, on which surface will a negative charge accumulate?
  
  (a) Top  (b) Bottom  (c) Left  (d) Right
• The force on the charge carriers will be perpendicular to both $\vec{B}$ and $\vec{v}$ so it must be either top or bottom.
• Electrons move in the direction opposite the current.
• The right-hand-rule tells you that $\vec{v} \times \vec{B}$ points down.
• But $q$ is negative, so the negative charge accumulates on the top.
The Hall Effect

- The Lorentz force is balanced by the electrostatic force:
  \[ qv_d B = qE_H = q \frac{V_H}{w} \]
  \[ V_H = v_d Bw \]
- This tells us the direction and magnitude of the drift velocity of the charge carriers.
The Hall Effect

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- This tells us the **direction** and **magnitude** of the drift velocity of the charge carriers.
Torque on a Current Loop

- Consider a rectangular loop of wire carrying current $I$ in a magnetic field.
- The orientation of the loop is given by the unit vector $\hat{n}$ perpendicular to the plane of the loop.
Torque on a Current Loop

- Magnitude of torque is $\tau = IabB \sin \theta$
- Direction is perpendicular to $\vec{B}$ and $\hat{n}$:
  $\vec{t} = \vec{\mu} \times \vec{B}$
  $\vec{\mu} = Iab\hat{n}$
Torque on a Current Loop

- In general, the torque does not depend on the shape, just the area.
- With $N$ turns of wire in the loop, multiply by $N$.

\[ \vec{\mu} = NIA \hat{n} \]

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
Potential Energy

- Work done by the magnetic field:
  \[ dW = -\tau \, d\theta \]

- Loss of potential energy:
  \[ dU = -dW = \mu B \sin \theta \, d\theta \]

- Total change in potential energy:
  \[ \Delta U = \int_{\theta_0}^{\theta} \mu B \sin \theta \, d\theta = -\mu B \cos \theta \]

- Potential energy of a dipole in a magnetic field:
  \[ U = -\vec{\mu} \cdot \vec{B} \]

- Minimal potential energy when \( \vec{\mu} \) and \( \vec{B} \) are aligned.
Magnetic Field

• Electrostatics:
  – An electric field exerts a force on a charge
  – An charge produces an electric field
    \[ \vec{E} = \frac{Q}{4\pi\varepsilon_0 \ r^2} \hat{r} \]

• Magnetism:
  – A magnetic field exerts a force on a moving charge
  – A moving charge produces a magnetic field
    \[ \vec{B} = \frac{\mu_0 q\vec{v} \times \hat{r}}{4\pi r^2} \]
    \[ \mu_0 = 4\pi \times 10^{-7} \ T \cdot m/A \]
Magnetic Field

• Moving charge:

\[ \vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2} \]

• Current flowing in a wire:

\[ d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2} \]

(Biot-Savart Law)
Observations

\[ d\vec{B} = \mu_0 \frac{I d\ell \times \hat{r}}{4\pi r^2} \]
Question

- What is the direction of the magnetic field at the center of the current loop:

(a) $\hat{k}$  
(b) $-\hat{k}$  
(c) $\hat{i}$  
(d) $\hat{j}$


**Question**

- What is the direction of the magnetic field at the center of the current loop:

(a) \( \hat{k} \)  
(b) \( -\hat{k} \)  
(c) \( \hat{i} \)  
(d) \( \hat{j} \)
Current Carrying Wires

• Use the principle of superposition:

\[ \vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{\ell} \times \hat{r}}{r^3} \]

• Very similar to the way we calculated \( \vec{E} \):
  1) Pick a coordinate system
  2) Label source and field points
  3) Pick variables to express their components
  4) Express \( d\vec{\ell}, \hat{r} \) and \( r \) using these variables
  5) Evaluate \( d\vec{\ell} \times \hat{r} \)
  6) Write out the integral for each component
  7) Evaluate the integrals one way or another.
Example

- Magnetic field around a long, straight wire:

  Without loss of generality, pick the field point at $R$ on the $y$-axis and the source point on the $z$-axis.

  The magnetic field will be in the $-\hat{i}$ direction.

  We could also use a cylindrical coordinate system with unit vectors $\hat{\rho}, \hat{\phi}, \hat{z}$.

  $$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{2I}{R}$$
Example

• Magnetic field at the center of a current loop

\[ |\vec{B}| = \frac{\mu_0 I}{2R} \]
Example

• Magnetic field on the axis of a current loop

\[ |\vec{B}| = \frac{\mu_0}{2} \frac{IR^2}{(R^2 + z^2)^{3/2}} \]
Example

- A solenoid is like a bunch of current loops with $n = N/L$ loops per unit length.

$$dB = \frac{\mu_0}{2} \frac{di \ R^2}{\left(R^2 + z^2\right)^{3/2}}$$

where $di = n \ I \ dz$.

- Inside a long solenoid, $L \gg R$:

$$B = \mu_0 nI$$
Clicker Question

• Treat a lightning bolt like a long, straight wire.
• If the current in a lightning bolt is 100 kA, how would the magnetic field 1 km away compare with the Earth’s magnetic field ($5 \times 10^{-5}\ T$)?

(a) Much less
(b) Much greater
(c) About the same

($\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$)