Tuesday’s Question

• Three circuits, consisting of two capacitors and a switch, are initially charged as indicated.
• After the switches are closed, in which circuit will the charge on the left increase?

(a) (b) (c) (d) None of them
Tuesday’s Question

- Charge is conserved, \( Q = 9q \)
- Calculate equivalent capacitance, \( C_{equiv} \)
- Then calculate, \( V = \frac{Q}{C_{equiv}} \)
- Finally, calculate, \( Q_{left} = C_{equiv}V \)
Tuesday’s Question

(a) \[ Q = 9q \]
\[ C_{equiv} = 3C \]
\[ V = \frac{Q}{C_{equiv}} = \frac{3q}{C} \]
\[ Q_{left} = 2C \times \frac{3q}{C} = 6q \]

(b) \[ Q = 9q \]
\[ C_{equiv} = 4C \]
\[ V = \frac{Q}{C_{equiv}} = \frac{9q}{4C} \]
\[ Q_{left} = 3C \times \frac{9q}{4C} = \frac{27}{4}q \]
\[ q > 6q \]

(c) \[ Q = 9q \]
\[ C_{equiv} = 4C \]
\[ V = \frac{Q}{C_{equiv}} = \frac{9q}{4C} \]
\[ Q_{left} = 2C \times \frac{9q}{4C} = \frac{9}{2}q \]
\[ q < 6q \]
Mini-Review

• Lecture 1: \( \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2} \hat{r} \)

• Lecture 2: \( \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r} \) and \( \vec{F} = q\vec{E} \)

• Lecture 3: \( \Delta \vec{E}(\hat{x}_p) = \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q(\hat{x}_s)}{r^2} \hat{r} \rightarrow \vec{E}(\hat{x}_p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\hat{r}}{r^2} dQ \)

• Lecture 4: \( \phi_{net} = \oint_S \hat{n} \cdot \vec{E} \, dA = \frac{Q_{inside}}{\varepsilon_0} \)

• Lecture 5: \( \vec{E} \) near conductors and insulators

• Lecture 6: \( \Delta V = - \int_a^b \vec{E} \cdot d\vec{\ell} \) and \( \vec{E} = -\nabla V \)

\[
V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}
\]
Mini-Review

• Lecture 7: \( V(\hat{x}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{Q_i}{r_i} \rightarrow V(\hat{x}) = \frac{1}{4\pi\varepsilon_0} \int \frac{dQ}{r} \)

• Lecture 8: \( C = \frac{Q}{V} \) and \( U = \frac{1}{2} CV^2 \)

• Lecture 9: \( C_\parallel = C_1 + C_2 \) and \( C_{\text{series}} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} \)

• But wait! There’s more...
  – No \( \vec{E} \) field inside a conductor
  – Principle of superposition
  – Surface charge densities
  – \( \vec{E} \) and \( V \) for example geometries
  – Work and energy
  – Dielectrics and \( \varepsilon = \kappa \varepsilon_0 \)
  – Lots of examples and clicker questions...
Electrostatic Equilibrium

• No net motion of charge
• Insulators:
  – No free charges
• Conductors:
  – Charges are pushed by any electric field until their own electric field cancels the original one
  – The motion stops when charges accumulate at a surface
  – The net electric field in the conductor is zero
• What if charge is added to or removed from a surface as quickly as it accumulates?
  – The charge will continue to flow...
  – Not a state of electrostatic equilibrium
Electric Current

• When the motion of charge carriers are not restricted, they will flow.
• By convention, the direction of an electric current is in the direction that positive charge carriers move:

![Diagram showing the flow of positive charge carriers from a more negative to a more positive region, representing the flow of current, I.](image)
Electric Current

• If they are negatively charged, then the current is opposite their motion:

• In metals, the charge carriers are electrons
• In chemical solutions or ionized gasses, the charge carriers can be both positive and negative.
Electric Current

- Electric current is the net positive charge moving across a surface per unit time:

\[ I = \frac{\Delta Q}{\Delta t} \]

Units: Amperes = \( \frac{\text{Coulombs}}{\text{second}} \)
Drift Velocity

- Motion of individual charges is usually not uniform:

\[ v_D = \frac{\Delta x}{\Delta t} \]

The average distance moved per unit time is the drift velocity:

- With \( \vec{E} \)
- Without \( \vec{E} \)
Drift speed, total charge and current

\[ n = \frac{\text{# of charge carriers}}{\text{unit volume}} \]

\[ q = \text{charge of each carrier} \]

\[ v_D = \text{drift velocity} \]
Drift speed, total charge and current

\[
\begin{align*}
\Delta Q &= n \, q \, \Delta V \\
\Delta V &= A \, \nu_D \Delta t \\
I &= \frac{\Delta Q}{\Delta t} = n \, q \, A \, \nu_D \Delta t \\
&= n \, q \, A \, \nu_D
\end{align*}
\]
Example

• What is the drift velocity in #12 AWG copper wire carrying 1 ampere of current?
  – What’s the diameter of #12 AWG???
    • Google “wire gauge”... it’s roughly 2 mm
  – How many charge carriers?
    • Assume one charge carrier per copper atom
    • How many copper atoms per unit volume?
    • How many copper atoms per unit mass?

Atomic mass: \( m = 63.546 \, g/mol \)
Density of copper: \( \rho = 8.94 \, g/cm^3 \)
Current Density

• The flow of charge might not be uniformly across a surface
  – The magnitude of the local current might change
  – The direction of the drift velocity could change

• Current: \( I = n q \nu_D A \)

• Current density: \( \vec{J} = n q \vec{v}_D \)

• They are related:

\[
I = \int_S \vec{J} \cdot d\vec{A}
\]
Resistance

• Electrons in a metal do not accelerate indefinitely
  – They eventually hit an atom in the metal
  – The collision is inelastic and the electron loses all, or some of its energy

• Instantaneous vs average velocity:

• Resistance is a property of a material related to how rapidly charge carriers lose energy
**Resistance**

- The greater the current, the more energy is transferred to the material through inelastic collisions.
- Electric potential difference: $\Delta V \propto I$

\[
\Delta V = R \ I \quad R = \frac{\Delta V}{I}
\]

Ohms = \frac{Volts}{Ampere}
Ohm’s Law

- Potential difference is proportional to current
  \[ \Delta V = I \cdot R \]
- This is usually a good approximation...

\[ \text{ohmic} \] \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{non-ohmic} \]
Ohm’s Law

• Potential difference is proportional to current
  \[ \Delta V = I \cdot R \]

• This is usually a good approximation...

\[ R(I) = \frac{\Delta V}{\Delta I} \]

ohmic

non-ohmic
Resistance depends on geometry

- Resistance is proportional to $\Delta L$
- Resistance is inversely proportional to $A$
- Resistivity, $\rho$, is independent of geometry

\[ R = \frac{\rho \Delta L}{A} \]
Resistivity Depends on Temperature

• In general, resistivity increases with temperature
  \[ \Delta \rho \propto \Delta T \]

• The temperature coefficient, \( \alpha \), is defined as the fractional change in resistance:
  \[ \alpha = \frac{1}{\rho} \frac{\Delta \rho}{\Delta T} \]

• Resistivity and the temperature coefficient are usually given for a particular reference temperature (for example, 20 °C)
# Resistivities and Temperature Coefficients

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity, $\rho$ ($\Omega \cdot m$)</th>
<th>Temp. coeff., $\alpha$ ($K^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>$1.6 \times 10^{-8}$</td>
<td>$3.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>Cu</td>
<td>$1.7 \times 10^{-8}$</td>
<td>$3.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>W</td>
<td>$5.5 \times 10^{-8}$</td>
<td>$4.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Si</td>
<td>$640$</td>
<td>$-7.5 \times 10^{-2}$</td>
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<tr>
<td>Si, n-type</td>
<td>$8.7 \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>Si, p-type</td>
<td>$2.8 \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>glass</td>
<td>$10^{10}$–$10^{14}$</td>
<td></td>
</tr>
</tbody>
</table>
Temperature Dependence

\[ \rho - \rho_0 = \rho_0 \alpha (T - T_0) \]
\[ \rho(T) = \rho_0 (1 + \alpha (T - T_0)) \]

- Also true for resistance: \( R = \rho L/A \)

\[ R - R_0 = R_0 \alpha (T - T_0) \]
\[ R(T) = R_0 (1 + \alpha (T - T_0)) \]
Example

• What is the resistance of a 10 cm long Tungsten wire with a diameter of 0.2 mm at 20 °C and at 3000 K?

\[ \rho_0 = 5.5 \times 10^{-8} \ \Omega \cdot m \]
\[ \alpha = 4.5 \times 10^{-3} \ K^{-1} \]
Rate of Energy Loss

• Charges moving through a resistor lose energy 
  \[ \Delta U = q \Delta V = q I R \]

• Total energy lost per unit time: 
  \[ P = n q A v_D \times I R \]
  \[ P = I^2 R \]

• Electric potential energy is converted into heat.
Clicker Question

• The resistance across the human body is approximately 2 $k\Omega$
• If it takes only 50 $mA$ of current to kill a human, what voltage could be lethal?

(a) 0.1 Volts  (b) 1 Volt
(c) 10 Volts   (d) 100 Volts
(e) 1000 Volts
Electric Current

- A chemical battery is a *source* of electric potential
  - The chemical reaction creates a potential difference across the poles:

![Image of a lemon with a positive and negative sign](image)

- Positive charges at the + end have a greater electric potential than positive charges at the – end.

- There must be an electric field between the poles.

- If free charges were present, they would be accelerated by the electric field – the field does work on the charges.

- Their potential energy decreases as they move towards the – pole.

- The chemical reaction maintains a constant potential difference.