PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 22000

# General Physics

*Lecture 7 – Uniform Circular Motion*

Fall 2016 Semester

Prof. Matthew Jones

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SUPPLEMENTAL INSTRUCTION

## Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in **PHYS 220**

| Tuesday and Thursday | 7:30-8:30PM Shreve C113 |
|---|---|

**Office Hour**

**Tuesday 1:30-2:30 4th floor of Krach**

For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

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## Constant Acceleration

- So far we have considered motion when the acceleration is constant in both *magnitude* and *direction*.
- Another situation is where the acceleration is constant in magnitude, but its direction constantly changes with time.
- We expect that Newton's Laws will still apply.

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## Be sure you know how to:

- Find the direction of acceleration using a motion diagram (Section 1.6).
- Draw a force diagram (Section 2.1).
- Use a force diagram to help apply Newton's second law in component form (Sections 3.1 and 3.2).

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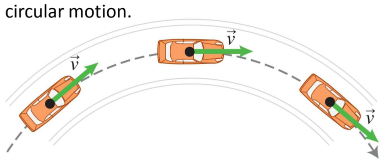
## Forces in More Complex Situations:

- First we addressed constant forces that act along only one axis (Chapter 2).
- Then we addressed constant forces along two dimensions (Chapter 3).
- Most forces are not constant; they can change in both magnitude and direction.
- Now we deal with the simplest case of continually changing forces: circular motion.

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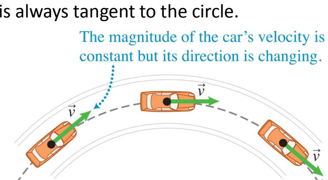
## The Qualitative Changes in Velocity

- At any instant, the instantaneous velocity is tangent to the path along which the object moves.
- In circular motion, the system object travels in a circle and the velocity is always tangent to the circle.




- Even if an object is moving with constant speed around a circle, its velocity changes direction.
- **A change in velocity means there is acceleration.**

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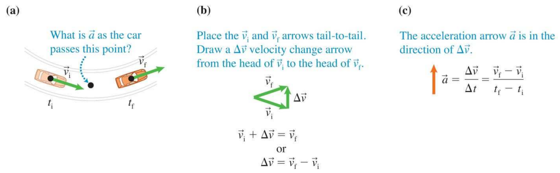
## Estimating the Direction of the Acceleration

- This method is used to estimate the direction of the acceleration of any object during a small time interval $\Delta t = t_f - t_i$.

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## Tips for Estimating the Direction of the Acceleration

- Make sure that you choose initial and final points at the same distance before and after the point at which you are estimating the acceleration direction.
- Draw long velocity arrows so that when you put them tail to tail, you can clearly see the direction of the velocity change arrow.
- Make sure that the velocity change arrow points from the head of the *initial* velocity to the head of the final *velocity* so that $\vec{v}_i + \Delta\vec{v} = \vec{v}_f$.

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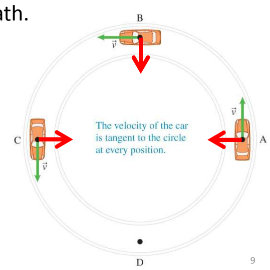
## Conceptual Exercise: Direction of a Racecar’s Acceleration

- Determine the direction of the acceleration of the race car at points A, B, and C as it travels at constant speed around a circular path.

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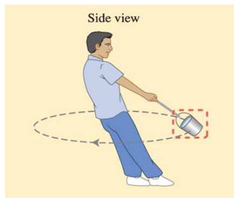
## Testing the Idea: Swing a pail on a rope

- Tie a pail to the end of a rope and swing it around in a circle.
- A rope (or string) can only exert force along the string, not perpendicular to it.
- Force diagram:
  - Vertical force components balance.
  - The horizontal component points towards the center of the circle, as expected.




## Newton's Second Law and Circular Motion

- The sum of the forces exerted on an object moving at constant speed along a circular path points towards the center of that circle in the same direction as the object's acceleration.
- When the object moves at constant speed along a circular path, the net force has no tangential component.

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## Factors that might affect acceleration

- Imagine your experience in a car driving around one of Lafayette's various traffic circles.
  - The faster the car moves around the traffic circle, the greater the risk that the car will skid off the road.
  - For the same speed, there is a greater risk of skidding on the inner lane (smaller radius).
  - **We guess that the acceleration depends on both $v$ and $r$.**

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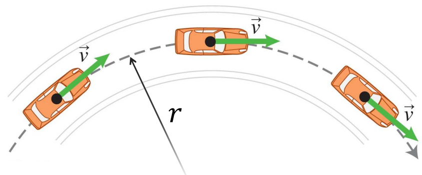
## Factors that might affect acceleration




- Curves on highways are banked, but not in traffic circles.
- Go ahead! Drive up Northwestern Avenue and try it out…
- Just be careful of other motorists and pedestrians.
- And don't blame me if anything unfortunate happens.

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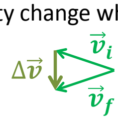
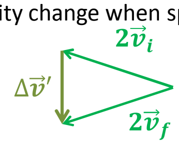
## Dependence of acceleration on speed

- Velocity change when speed is $v$:




- Velocity change when speed is $2v$:




$$\Delta\vec{v}' = 2\Delta\vec{v}$$
$$\Delta t' = \Delta t/2$$

- But the time interval is only half as long, because the object is moving twice as fast

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## Dependence of Acceleration on Speed

$$\Delta\vec{v}' = 2\Delta\vec{v}$$
$$\Delta t' = \Delta t/2$$

- Acceleration is $\vec{a}' = \frac{\Delta\vec{v}'}{\Delta t'} = 4\frac{\Delta\vec{v}}{\Delta t} = 4\vec{a}$
- The magnitude of the acceleration is proportional to the square of the velocity.

$$a_r \propto v^2$$

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## Dependence of Acceleration on Radius

- If $v$ remains constant, how long does it take the object to move in a circle of radius $r$?
- The circumference of a circle is
$$C = 2\pi r$$
- If the radius increases by a factor of 2, then the circumference increases by the same factor.
- The net change in velocity is the same.
- Since the velocity is constant, it will take twice as long to travel around the circle.
$$\Delta t' = 2\,\Delta t$$

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## Dependence of Acceleration on Radius

$$\vec{a}' = \frac{\Delta \vec{v}'}{\Delta t'} = \frac{\Delta \vec{v}}{2\,\Delta t} = \frac{\vec{a}}{2}$$

- The magnitude of the acceleration is inversely proportional to the radius.
$$a_r \propto \frac{1}{r}$$
- In fact, the radial acceleration is:
$$a_r = \frac{v^2}{r}$$

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## Radial Acceleration

- For motion in a circle of radius $r$ with constant speed $v$, the radial acceleration is
$$a_r = \frac{v^2}{r}$$
- The acceleration points towards the center of the circle.
- The SI units for radial acceleration are $m/s^2$
- In the limiting case of a straight line, the radius goes to infinity and the acceleration goes to zero. This kinda makes sense…

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## Period of Circular Motion

- The ***period*** is the time interval it takes for an object to travel once around an entire circular path.
- The period has units of time, so the SI unit is seconds.
- For constant speed, circular motion, we divide the circumference by the velocity to get:

$$T = \frac{C}{v} = \frac{2\pi r}{v}$$

- Do not confuse the symbol $T$ for period with the symbol $T$ for tension in a string.

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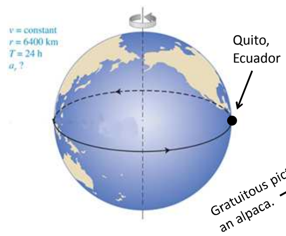
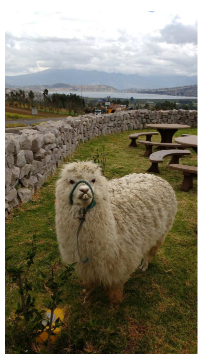
## Example

- What is your radial acceleration when you sleep in a hotel in Quito, Ecuador?




Gratuitous picture of an alpaca.



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## Example

- Remember that the earth turns on its axis once every 24 hours and everything on its surface undergoes constant-speed circular motion with a period of 24 hours.
- The radius of the earth is $r = 6400\ km$

$$a_r = \frac{v^2}{r}$$

- We know $r$, so we need to find $v$.

$$v = \frac{C}{T} = \frac{2\pi r}{T}$$

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## Example

$$v = \frac{C}{T} = \frac{2\pi r}{T}$$

$$a = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(6.4 \times 10^6\ m)}{[(24\ h)(3600\ s/h)]^2} = 0.034\, m/s^2$$

- Compare this with the acceleration of gravity:
  $$g = 9.8\, m/s^2$$
- The ratio is 0.35%...

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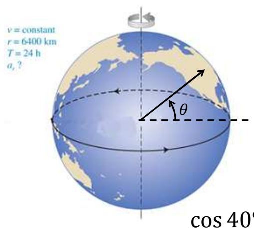
## Example

- What is your radial acceleration as you sleep in your dorm room in West Lafayette?




- Latitude of West Lafayette is
  $\theta = 40°\ North$
- Radius of circular path is
  $r_{WL} = r\ \cos\theta$
- Using our previous result for $\theta = 0°$,

$a_r = \frac{4\pi^2 r}{T^2}$ ➔ $a_{WL} = \frac{4\pi^2 r \cos\theta}{T^2}$

$\cos 40° = 0.77$ so $a_{WL} = 0.026\ m/s^2$

## Is the Earth a Non-Inertial Reference Frame?

- Newton's laws are valid only for observers in inertial reference frames (nonaccelerating observers).
  - Observers on Earth's surface are accelerating due to Earth's rotation.
- Does this mean that Newton's laws do not apply?
  - The acceleration due to Earth's rotation is much smaller than the accelerations we experience from other types of motion.
- In most situations, we can assume that Earth is not rotating and, therefore, does count as an inertial reference frame.

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## Newton's Second Law for Radial Components of Circular Motion

**Circular motion component form of Newton's second law** For the radial direction (the axis pointing toward the center of the circular path), the component form of Newton's second law is

$$a_r = \frac{\Sigma F_r}{m} \quad \text{or} \quad ma_r = \Sigma F_r \tag{4.6}$$

where $\Sigma F_r$ is the sum of the radial components of all forces exerted on the object moving in the circle (positive toward the center of the circle and negative away from the center) and $a_r = v^2/r$ is the magnitude of the radial acceleration of the object.

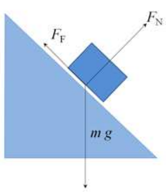
For some situations (for example, a car moving around a highway curve or a person standing on the platform of a merry-go-round), we also include in the analysis the force components along a perpendicular vertical $y$-axis:

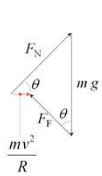
$$ma_y = \Sigma F_y = 0 \tag{4.7}$$

When an object moves with uniform circular motion, both the $y$-component of its acceleration and the $y$-component of the net force exerted on it are zero.

## Another Example

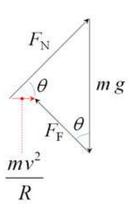
- Highway curves are banked to prevent cards from skidding off the road.
- The angle of the bank depends on the expected speed and the radius of curvature.



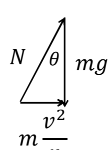




## Banked Highway Curves

- The radial acceleration is $a_r = v^2/r$
- The radial force is $F_r = m\,a_r = mv^2/r$




- $F_F$ is zero when the curve is banked correctly.




$$N\ \cos\theta = mg$$
$$N\ \sin\theta = m\,v^2/r$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{v^2}{gr}$$

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## Banked Highway Curves

- Suppose $r = 250\ m$ and $v = 100\ km/h$

$$\tan\theta = \frac{v^2}{gr} = \frac{\left[(10^5\ m/h)\left(\frac{1}{3600}\frac{h}{s}\right)\right]^2}{(9.8\ m/s^2)(250\ m)} = 0.31$$

$$\theta = \tan^{-1}(0.31) = 17°$$

- The component of the normal force in the radial direction provides the force needed to maintain the circular path.
- If the road were flat, we would rely on static friction to provide this radial force.

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## Tip for Circular Motion

- There is no special force that causes the radial acceleration of an object moving at constant speed along a circular path.
- This acceleration is caused by all of the forces exerted on the system object by other objects.
- Add the radial components of these regular forces.
- This sum is what causes the radial acceleration of the system object.

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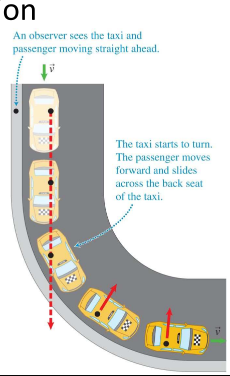
## Conceptual Difficulties with Circular Motion

- When sitting in a car that makes a sharp turn, you feel thrown outward, inconsistent with the idea that the net force points toward the center of the circle (inward)

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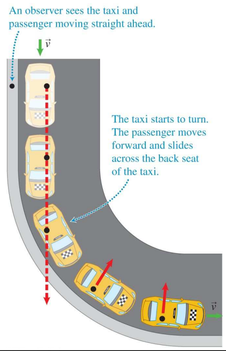
## Conceptual Difficulties with Circular Motion

- Because the car is accelerating as it rounds the curve, passengers in the car are not in an inertial reference frame.
  - A roadside observer would see the car turn left and you continue to travel straight because the net force exerted on you is zero

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