Physics 22000

General Physics

Lecture 24 – Oscillating Systems

Fall 2016 Semester

Prof. Matthew Jones

SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in **PHYS 220**

Tuesday and Thursday	7:30-8:30PM Shreve C113

Office Hour

Tuesday 1:30-2:30 4th floor of Krach

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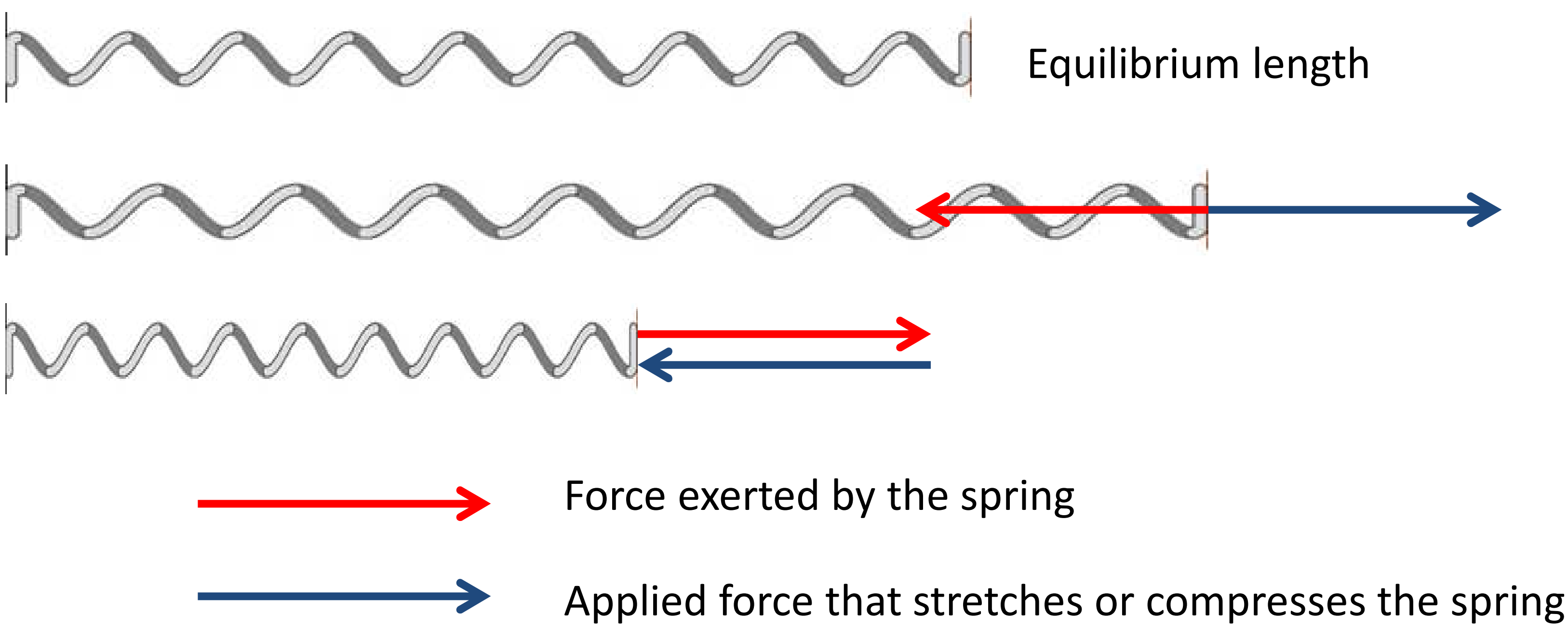
Oscillating Motion

- We have studied linear motion—objects moving in straight lines at either constant velocity or constant acceleration.
- We have also studied objects moving at constant speed in a circle.
- In this chapter we encounter a new type of motion, in which both direction and speed change.

Hooke's Law

- A spring exerts a force in the direction opposite to the extension or compression:



Equilibrium Position

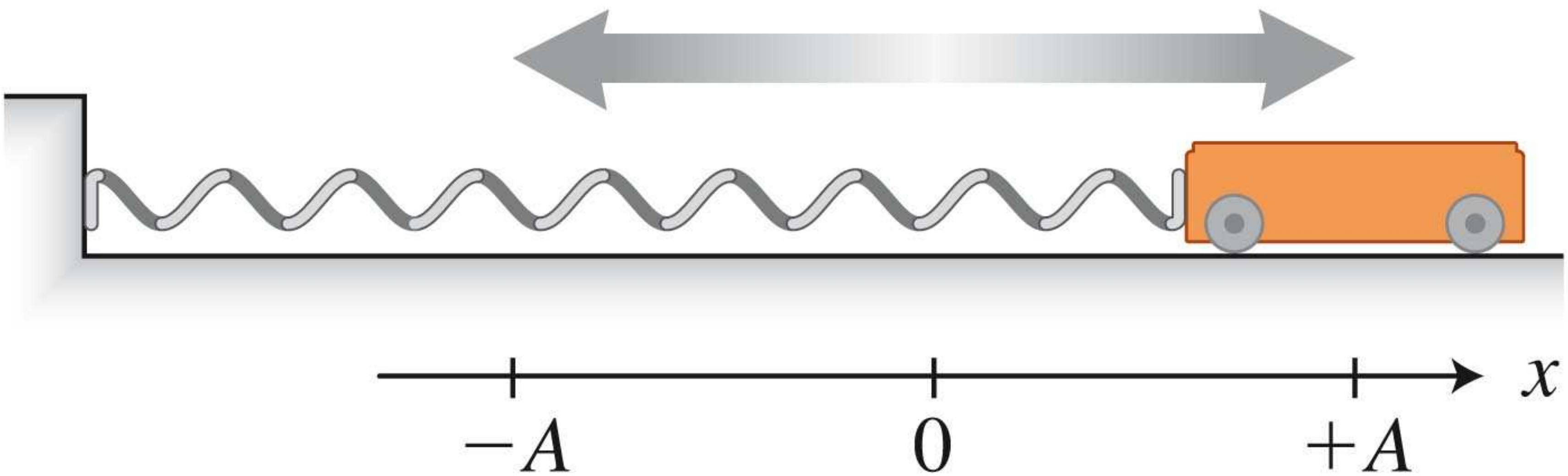
Equilibrium position (or just **equilibrium**) The position at which a vibrating object resides when not disturbed. When resting at this position, the sum of the forces that other objects exert on it is zero. During vibrational motion the object passes back and forth through this position from two opposite directions.

Restoring Force

Restoring force When an object is displaced from equilibrium, some other object exerts a force with a component that always points opposite the direction of the vibrating object's displacement from equilibrium. This force tends to restore the vibrating object back toward equilibrium.

Amplitude

Amplitude The amplitude A of a vibration is the magnitude of the maximum displacement of the vibrating object from its equilibrium position.



Patterns Observed in Vibrational Motion

- An object passes through the same positions, moving first in one direction and then in the opposite direction.
- The object passes the equilibrium position at high speed. When it overshoots, a restoring force exerted on it by some other object points back toward equilibrium.
- A system composed of the vibrating object and the object exerting the restoring force has maximum potential energy when at extreme positions and maximum kinetic energy at equilibrium.

Frequency

Frequency The frequency f of vibrational motion is the number of complete vibrations of the system during one second. Frequency is related to period:

$$f = \frac{1}{T} \tag{19.1}$$

The unit for frequency is the hertz (Hz), where $1\text{ Hz} = 1\text{ vib/s} = 1\text{ s}^{-1}$.

Kinematics of Vibrational Motion

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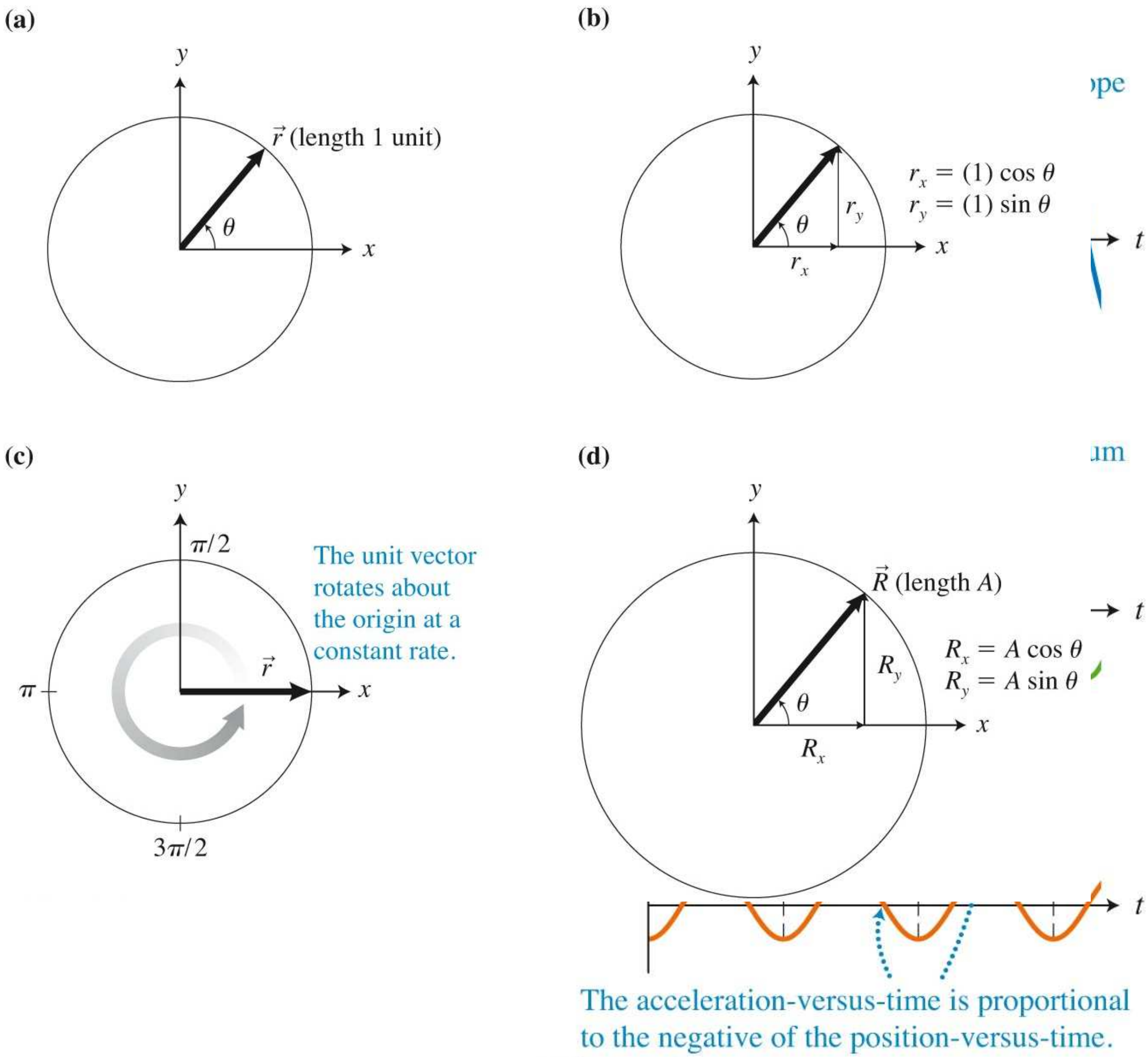
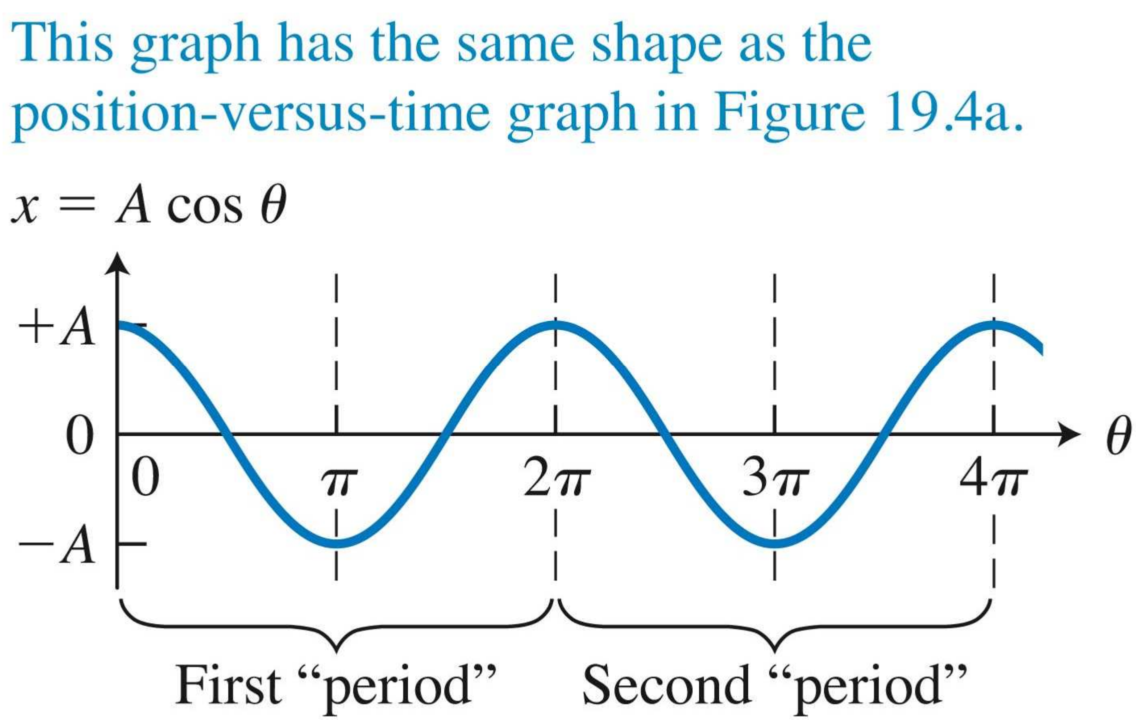


Mathematical Description of Position as a Function of Time

- A graph of $x = A\cos(\theta)$ looks very similar to the position-versus-time graph produced by the motion detector for a cart on a spring.



Mathematical Description of Position as a Function of Time

- We can write the period function $x(t)$ to represent the position-versus-time graph:

$$x = A\ \cos\left(\frac{2\pi}{T}\ t\right)$$

- Notice that $x = +A$ at $t = 0$…
- If an object is at $x = 0$ at $t = 0$, you can either adjust the cos function by adding $-(\pi/2)$ or use the sine function.

Simple Harmonic Motion

- Simple harmonic motion (SHM) is motion that can be described by the following equation:

$$x = A\ \cos\left(\frac{2\pi}{T}\ t\right)$$

- It is a mathematical model of motion.

Position of a Vibrating Object as a Function of Time

Table 19.3 Position of a vibrating object as a function of time.

Clock reading t of the vibrating object shown in Figure 19.4a	Position x of the vibrating object shown in Figure 19.4a	Angle of the radius vector θ (radians) for the function $x = A\cos\theta$	Value of the function $x = A\cos\theta$
0 (0 s)	A	0	A
$T/4$ (1 s)	0	$\pi/2$	0
$T/2$ (2 s)	$-A$	π	$-A$
$3T/4$ (3 s)	0	$3\pi/2$	0
T (4 s)	A	2π	A
$2T$ (8 s)	A	4π	A
$3T$ (12 s)	A	6π	A

Position of a Vibrating Object as a Function of Time

- If the position function is given by:

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

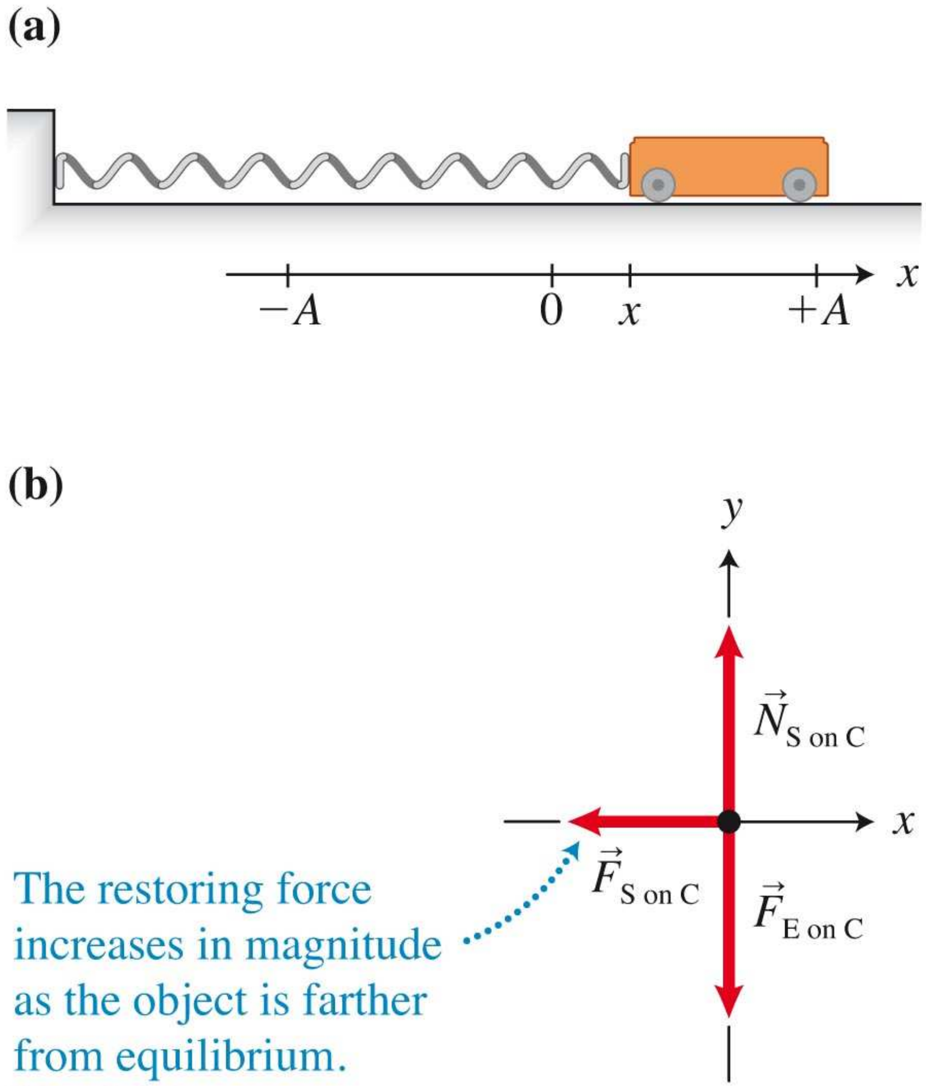
- Then the velocity and acceleration functions are:

$$v_x = -\left(\frac{2\pi}{T}\right)A\sin\left(\frac{2\pi}{T}t\right)$$

$$a_x = -\left(\frac{2\pi}{T}\right)^2 A\cos\left(\frac{2\pi}{T}t\right)$$

- *A* is the amplitude of the vibration; *T* is the period of the vibration.

Dynamics of Simple Harmonic Motion



Spring Forces and Acceleration for a Cart on a Spring

- According to Hooke's law, the force that the stretched spring exerts on a cart in the x-direction is:

$$F_{\mathrm{S\,on\,C}\,x} = -kx$$

- Using Newton's second law, we get:

$$a_x = \frac{-kx}{m} = -\frac{k}{m}x$$

- The cart's acceleration a_x is proportional to the negative of its displacement x from the equilibrium position.

Period of Vibrations of a Cart Attached to a Spring

- Starting with:

$$a_x = \frac{-kx}{m} = -\frac{k}{m}x$$

- And using:

$$x = A\cos\left(\frac{2\pi}{T}t\right) \quad a_x = -\left(\frac{2\pi}{T}\right)^2 A\cos\left(\frac{2\pi}{T}t\right)$$

- We get:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- In this expression for period, there is no dependency on the amplitude.

The Frequency of Vibration of an Object Attached to a Spring

- Using the equation $T = 2\pi\sqrt{m/k}$ and the relation f=1/T, we find

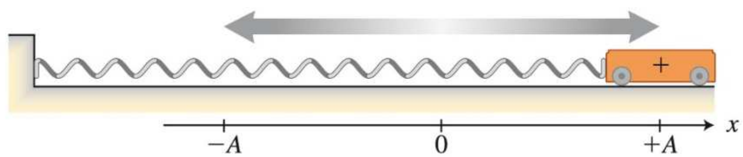
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- We assume that the spring obeys Hooke's law, that the spring has zero mass, and that the cart is a point-like object
- We also ignore friction.

Energy of Vibrating Systems

- As the cart-spring system vibrates, the energy of the system continuously changes from all elastic to all kinetic.

Table 19.4 Variation of energy during one vibration.



Clock reading t	Displacement	Elastic potential energy U_s	Kinetic energy K	Total energy U_{tot}
$\frac{1}{2}T$	$-A$	$\frac{1}{2}kA^2$	0	$U_{tot} = \frac{1}{2}kA^2$
$\frac{1}{4}T$	0	0	$\frac{1}{2}m{v_{max}}^2$	$U_{tot} = \frac{1}{2}m{v_{max}}^2$
$\frac{3}{4}T$	0	0	$\frac{1}{2}m{v_{max}}^2$	
0	A	$\frac{1}{2}kA^2$	0	$U_{tot} = \frac{1}{2}kA^2$
T	A	$\frac{1}{2}kA^2$	0	

Relationship Between Amplitude and Maximum Speed

- Using the equation

$$U = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

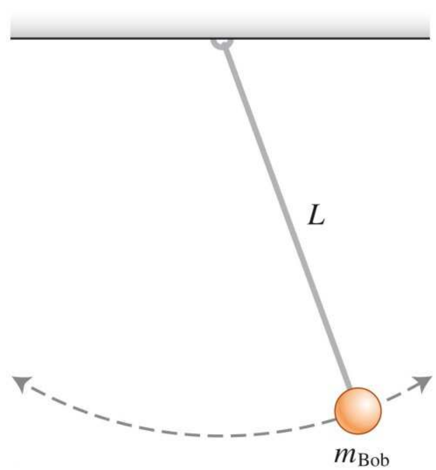
we can solve for v_{max}^2:

$$v_{max} = A\sqrt{\frac{k}{m}}$$

- This makes sense conceptually:
  - When the mass of the cart is large, it should move slowly.
  - If the spring is stiff, the cart will move more rapidly.

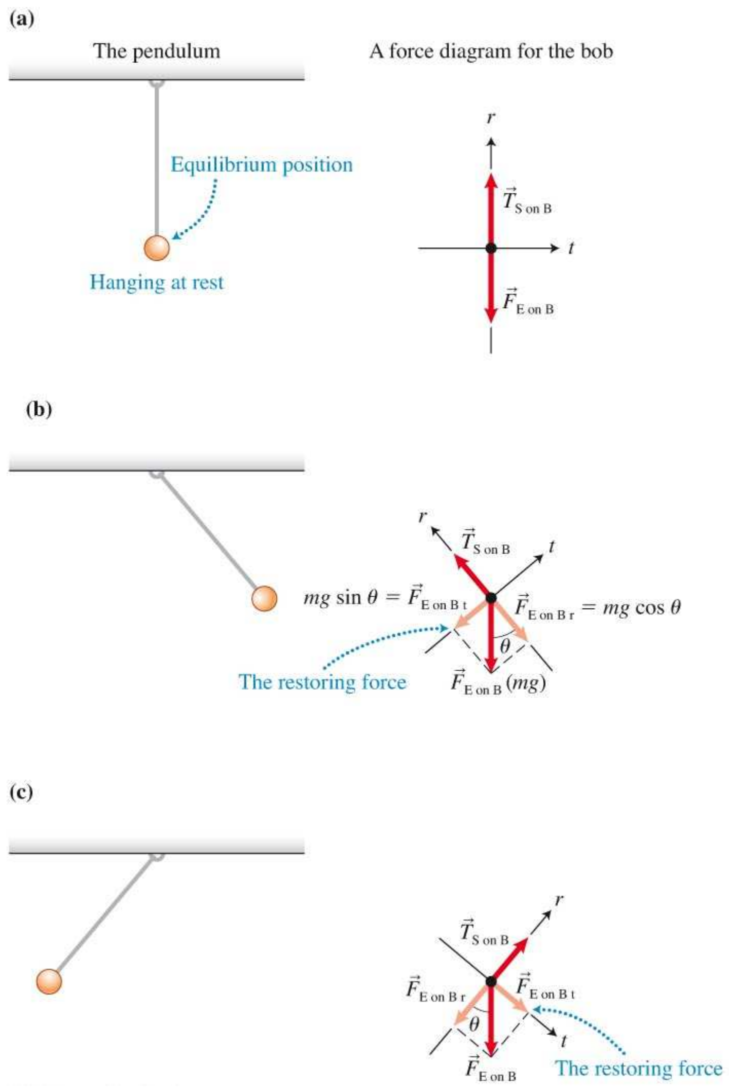
The Simple Pendulum

- A pendulum is a vibrating system in which the motion is very apparent.
- Consider a simplified model of a pendulum system that has a compact object (a bob) at the end of a comparatively long and massless string and that undergoes small-amplitude vibrations.
  - This idealized system is called a simple pendulum



The Simple Pendulum

- Two objects interact with the bob of the pendulum
  - The string S exerts a force that is always perpendicular to the path of the bob
  - Earth always exerts a downward gravitational force.

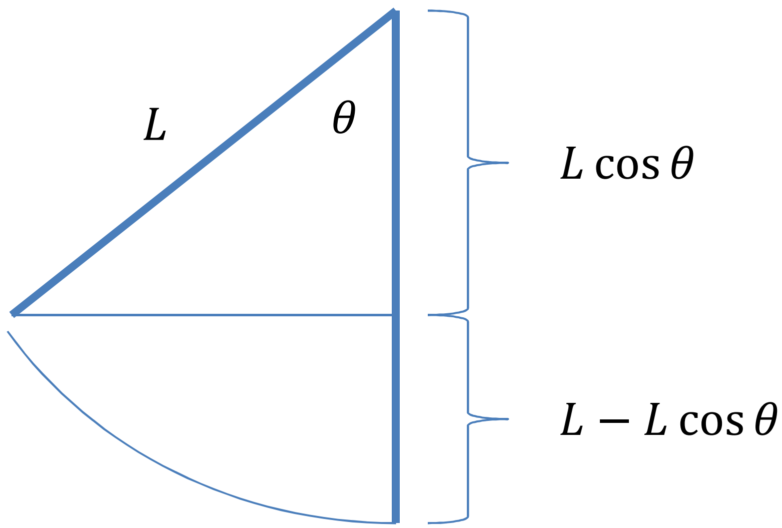


The Simple Pendulum in Relation to the Cart on a Spring

- The motion of the pendulum has the same patterns as the motion of the cart on a spring:
  - It passes the equilibrium position from two different directions.
  - There is a restoring force exerted on the bob.
  - The system's energy oscillates between maximum potential and maximum kinetic.

Energy of a Simple Pendulum

- Gravitational potential energy:



$$U = mg(L - L\cos\theta)$$
$$\approx \frac{1}{2} mgL\,\theta^2$$

$$\cos\theta = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24} - \cdots$$

Energy of a Simple Pendulum

- Simple pendulum:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}mgL\,\theta^2 = \frac{1}{2}mL^2\boldsymbol{\omega}^2 + \frac{1}{2}mgL\boldsymbol{\theta}^2$$

- Compare this with the mass and the spring:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

- Frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Frequency and Period of a Simple Pendulum

- Frequency:

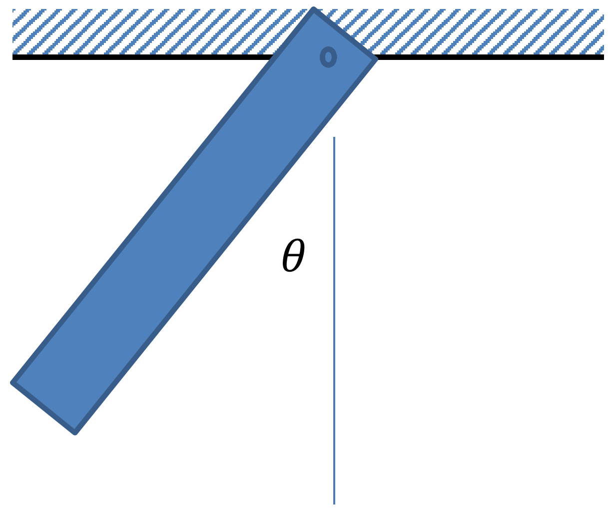
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

- Period:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- The period does not depend on the mass or the amplitude.

The Physical Pendulum

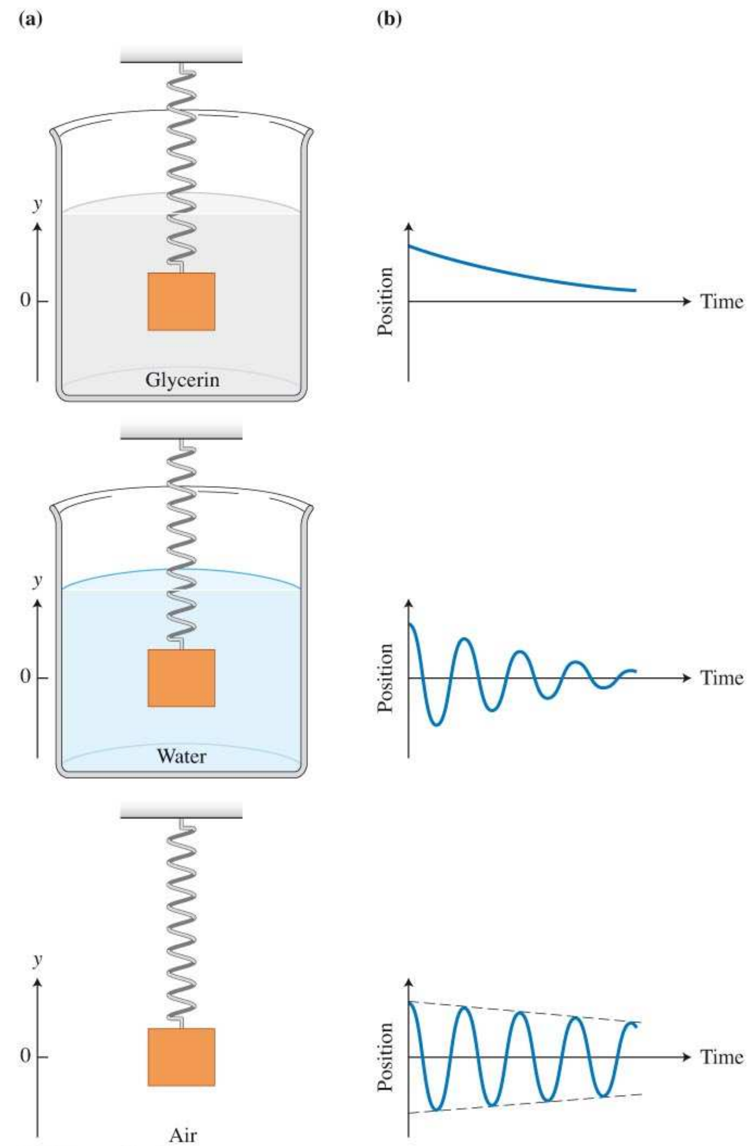


- The kinetic energy is $K = \frac{1}{2} I\omega^2$
- The moment of inertia is $I = \frac{1}{3} mL^2$
- The gravitational force acts through the center of mass

- $U = \frac{1}{2} m \left(\frac{L^2}{3}\right) \omega^2 + \frac{1}{2} m\, g \frac{L}{2} \theta^2 \quad \rightarrow \quad \boldsymbol{T = 2\pi \sqrt{\frac{2L}{3g}}}$

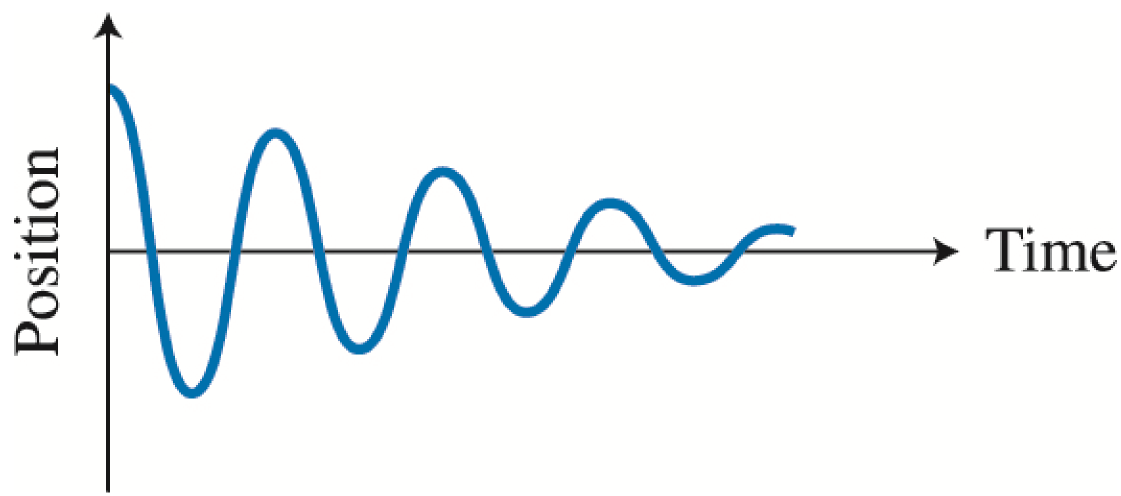
Damped and Undamped Oscillations

- You can observe the effects of friction on a simple system.
- The viscous damping force is proportional to velocity.



Damping

- The phenomenon of decreasing vibration amplitude and increasing period is called damping.

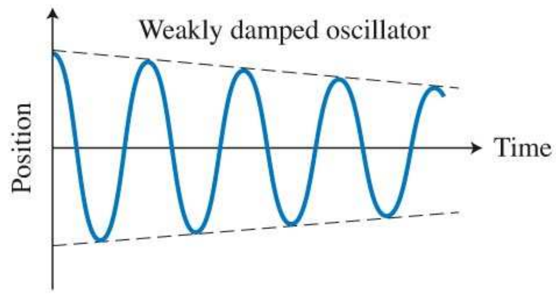


- Damping is a useful aspect of the design of vehicles and bridges.

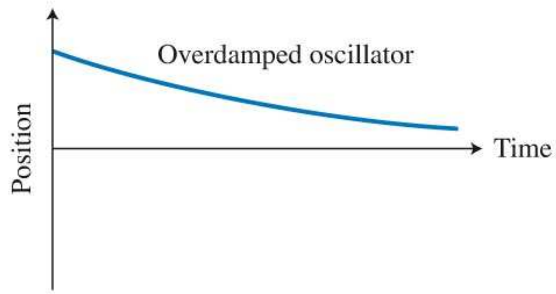
Including Friction in Vibrational Motion

- A weakly damped system continues to vibrate for many periods.
- In an overdamped system, the vibrating system takes a long time to return to the equilibrium position, if it ever does.
- In a critically damped system, the vibrating object returns to equilibrium in the shortest time possible.

(a)



(b)



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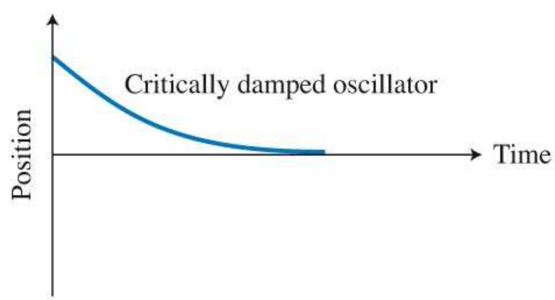
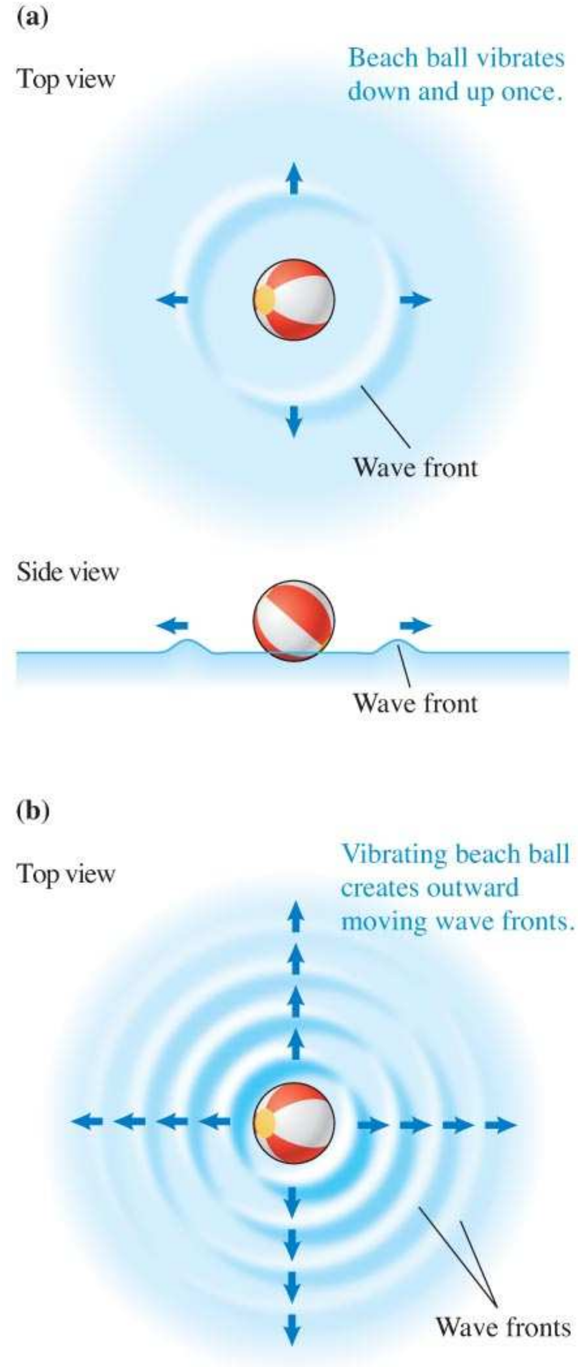


Mechanical Waves

- Waves and wave fronts:

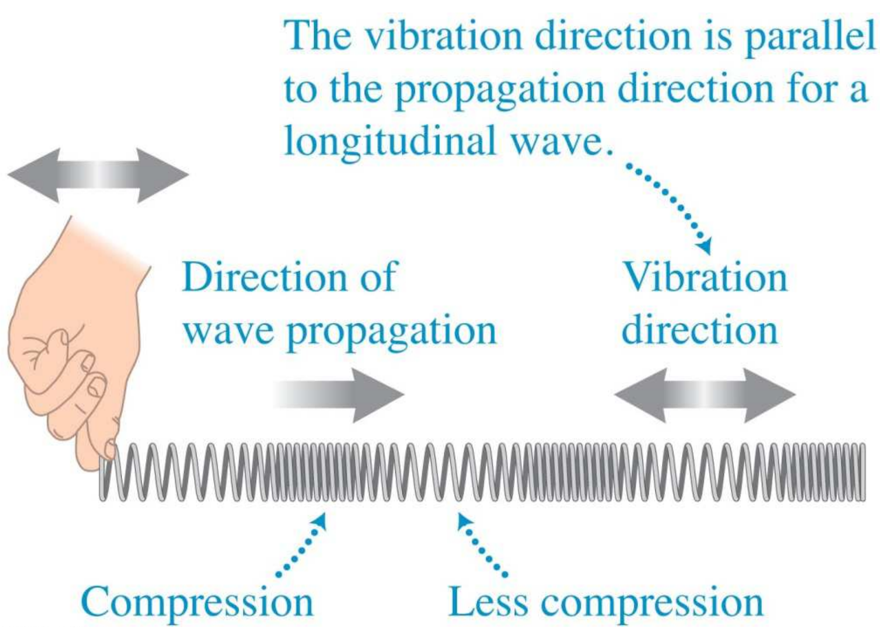


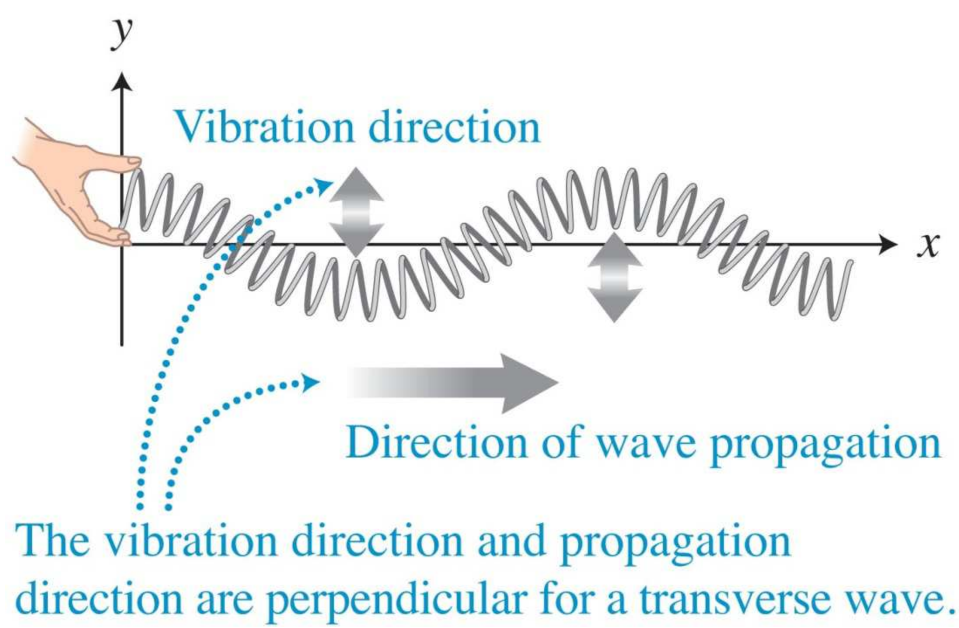
Wave Motion

Wave motion involves a disturbance produced by a vibrating object (a source). The disturbance moves, or propagates, through a medium and causes points in the medium to vibrate. When the disturbed medium is physical matter (solid, liquid, or gas), the wave is called a **mechanical wave.**

Two Kinds of Waves

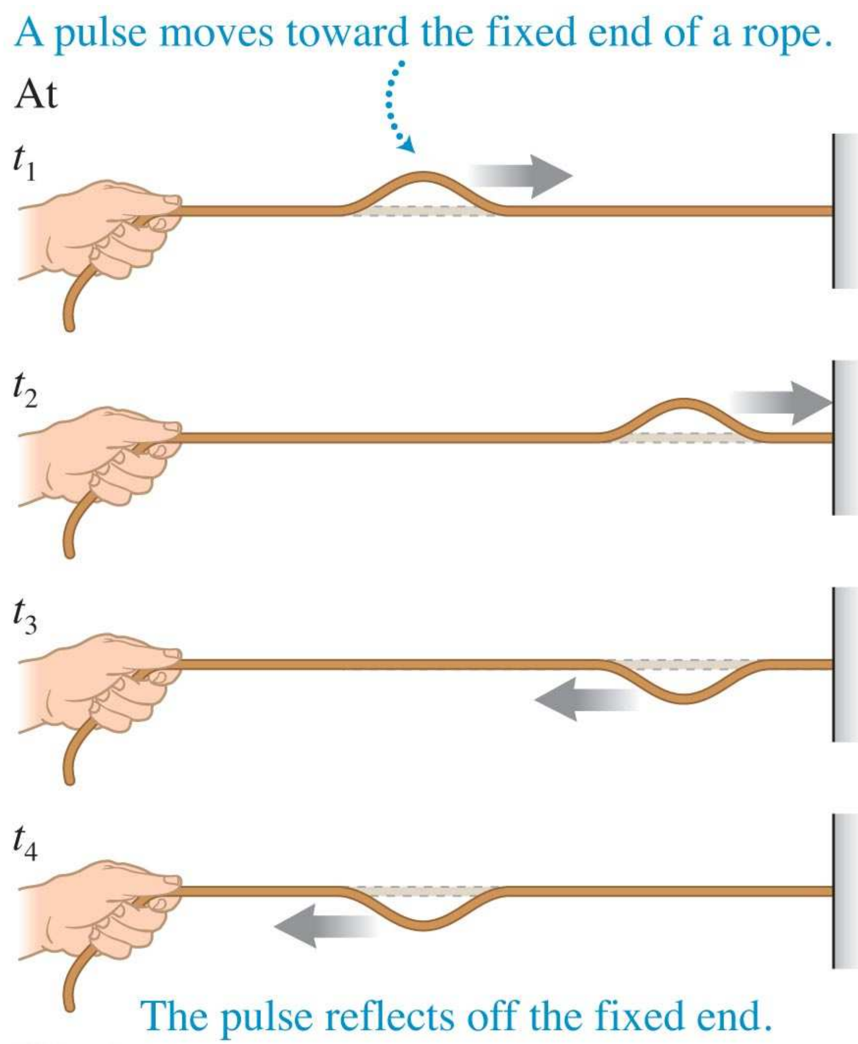
Longitudinal and transverse waves In a *longitudinal wave* the vibrational motion of the particles or layers of the medium is parallel to the direction of propagation of the disturbance. In a *transverse wave* the vibrational motion of the particles or layers of the medium is perpendicular to the direction of propagation of the disturbance.





Reflection of Waves

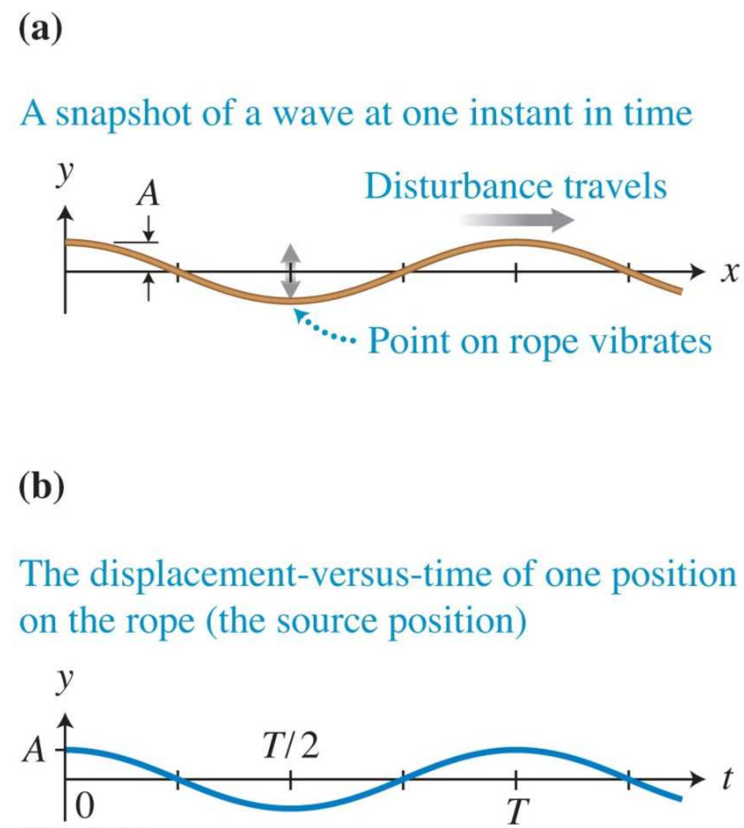
- When a wave reaches the wall of the container or the end of the Slinky or rope, it reflects off the end and moves in the opposite direction.
  - When a wave encounters any boundary between different media, some of the wave is reflected back.



Mathematical Description of Waves

- A wave can be created in a rope by a motor that vibrates the end of a rope up and down, producing a transverse wave.
- The displacement is described by a sinusoidal function of time:

$$y = A\ \cos\left(\frac{2\pi}{T}\ t\right)$$



Mathematical Description of Waves

Period T in seconds is the time interval for one complete vibration of a point in the medium anywhere along the wave's path.

Frequency f in Hz (s^{-1}) is the number of vibrations per second of a point in the medium as the wave passes.

Amplitude A is the maximum displacement of a point of the medium from its equilibrium position as the wave passes.

Speed v in m/s is the distance a disturbance travels during a time interval, divided by that time interval.

Mathematical Description of a Traveling Sinusoidal Wave

- We know the source oscillates up and down with a vertical displacement given by:

$$y = A\cos\left(\frac{2\pi}{T}t\right)$$

- We can mathematically describe the disturbance *y*(*x*, *t*) of a point of the rope at some positive position *x* to the right of the source (at *x* = 0) by:

$$y(x,t) = A\cos\left[\frac{2\pi}{T}\left(t - \frac{x}{v}\right)\right]$$

Wavelength

Wavelength λ equals the distance between two nearest points on a wave that at any clock reading have exactly the same displacement and shape (slope). It is also the distance between two consecutive wave fronts:

$$\lambda = Tv = \frac{v}{f} \tag{20.3}$$

Mathematical Description of a Traveling Sinusoidal Wave

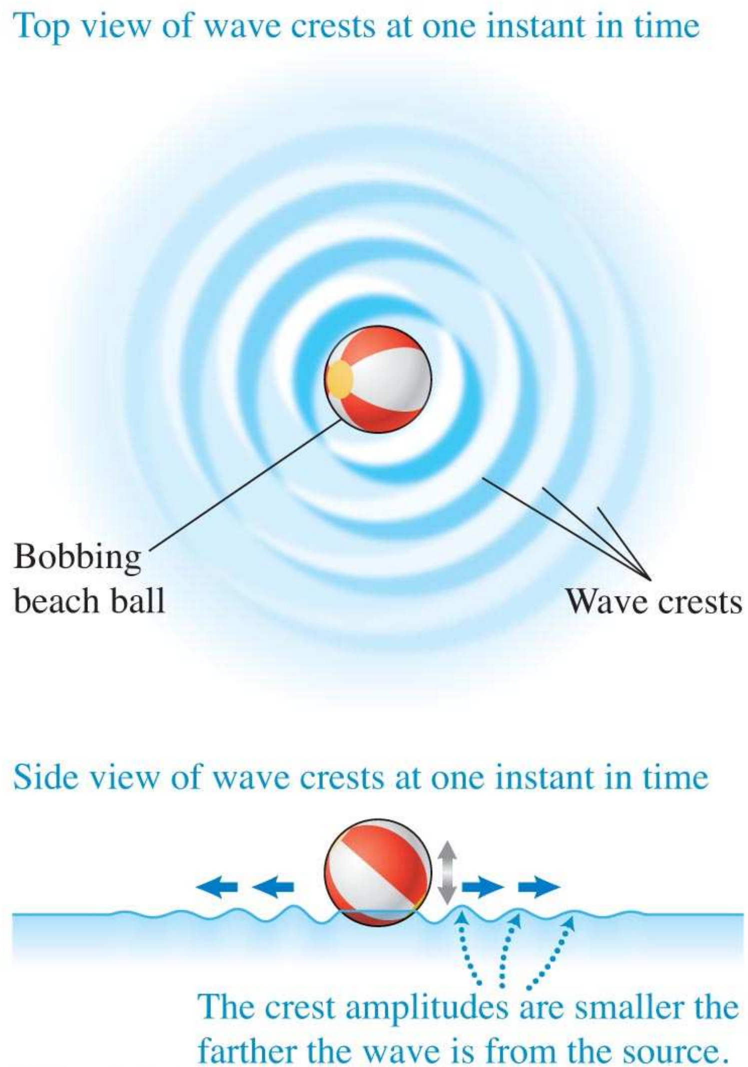
Mathematical description of a traveling sinusoidal wave The displacement from equilibrium y of a point at location x at time t when a wave of period T travels at speed v in the positive x-direction through a medium is described by the function

$$y = A\cos\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] \tag{20.4}$$

The wavelength λ of this wave equals $\lambda = Tv$.

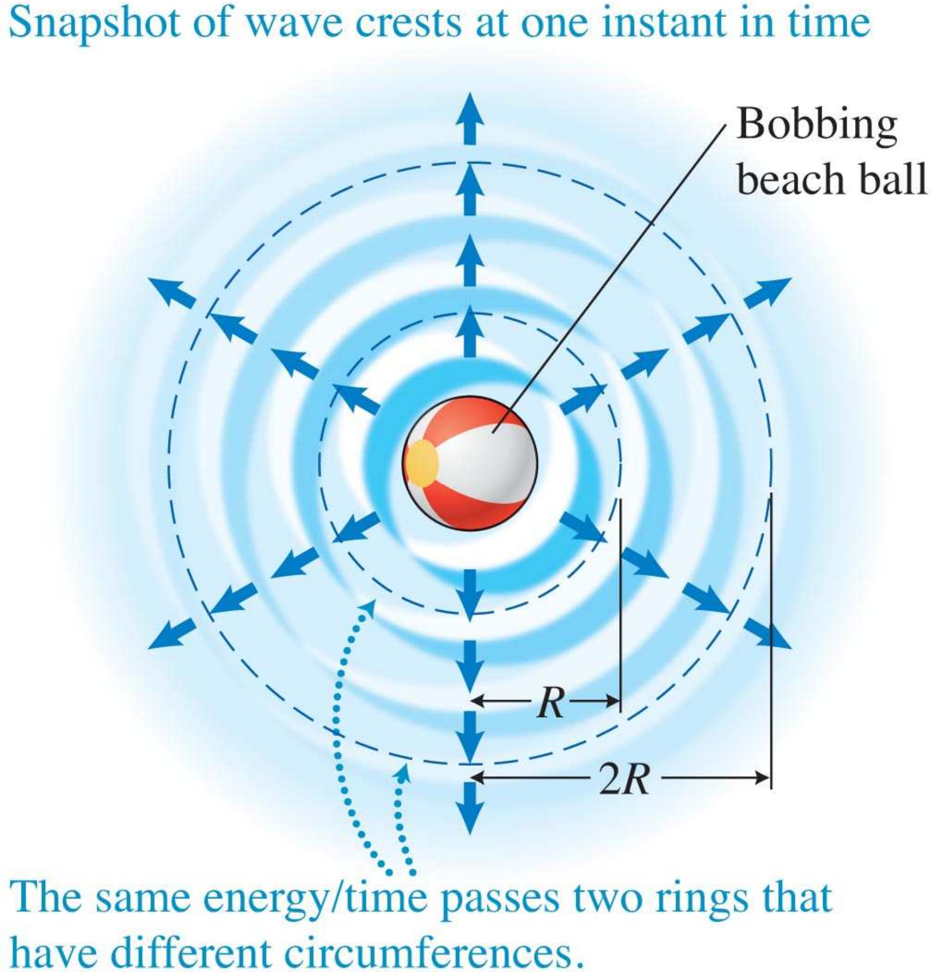
Amplitude and Energy in a Two Dimensional Medium

- A beach ball bobs up and down in water in simple harmonic motion, producing circular waves that travel outward across the water surface in all directions.
  - The amplitudes of the crests decrease as the waves move farther from the source.



Amplitude and Energy in a Two Dimensional Medium

- The circumference of the second ring is two times greater than the first, but the same energy per unit time moves through it.
  - The energy per unit circumference length passing through the second ring is one-half that passing through the first ring.

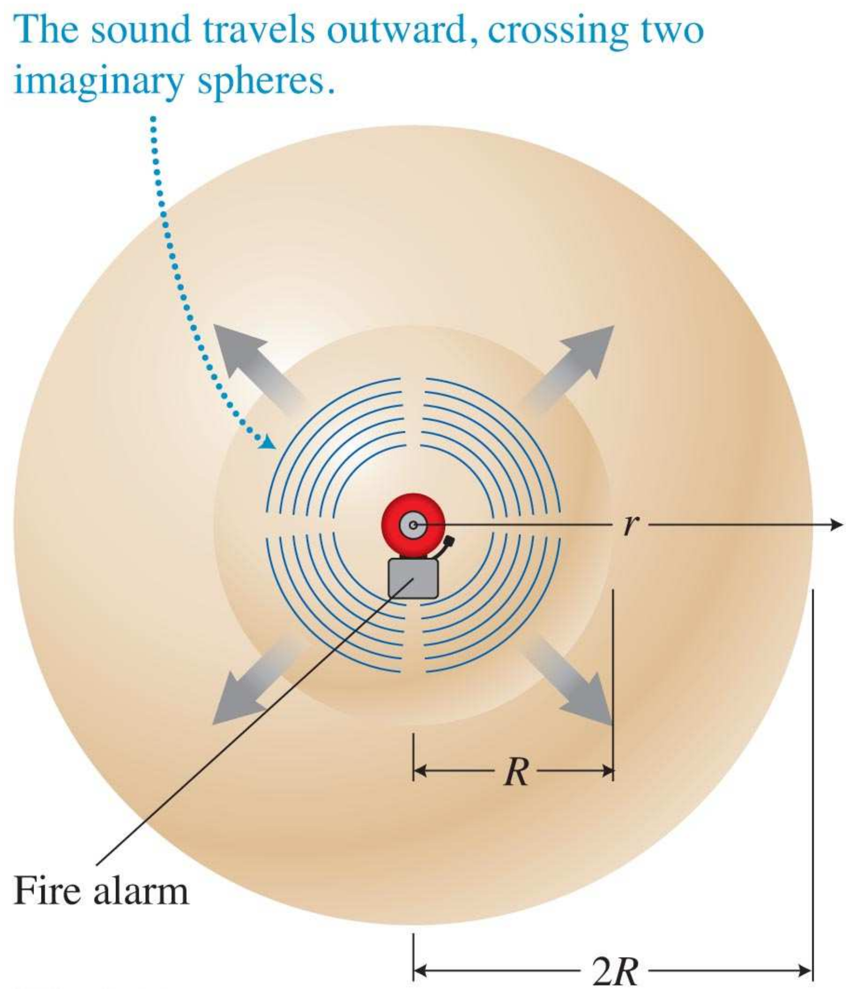


Two-dimensional waves produced by a point source

Two-dimensional waves produced by a point source The energy per unit circumference length and per unit time crossing a line perpendicular to the direction that the wave travels decreases as $1/r$, where r is the distance from the point source of the wave.

Three-Dimensional waves produced by a point source

- The area of the second sphere is four times the area of the first sphere, but the same energy per unit time moves through it.
  - The energy per unit area through the second sphere is one-fourth that through the first sphere.



Three-dimensional waves produced by a point source

Three-dimensional waves produced by a point source The energy per unit area per unit time passing across a surface perpendicular to the direction that the wave travels decreases as $1/r^2$, where r is the distance from the point source of the wave.

Wave Power and Wave Intensity

- The intensity of a wave is defined as the energy per unit area per unit time interval that crosses perpendicular to an area in the medium through which it travels:

$$\text{Intensity} = \frac{\text{Energy}}{\text{Time } \cdot \text{Area}} = I = \frac{\Delta U}{\Delta t \cdot A} = \frac{P}{A}$$

- The unit of intensity *I* is equivalent to joules per second per square meter or watts per square meter.