PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 21900
General Physics II

Electricity, Magnetism and Optics
Lecture 5 – Chapter 15.3-5
Electric Potential

Fall 2015 Semester

Prof. Matthew Jones

Reminder

- The first mid-term exam will be on Thursday, September 24th.
- Material to be covered is chapters 14 and 15
  - Coulomb's law
  - Electric potential energy
  - Electric field
  - Electric potential
  - Capacitors

Electric Field and Electrostatic Force

- Coulomb's law:

$$\vec{F} = k\frac{Qq}{r^2}\,\hat{r}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

(points from Q to q)

  – Describes the interaction between two charges.

- Electric field:

$$\vec{E} = k\frac{Q}{r^2}\,\hat{r}$$

  – points away from Q when Q is positive
  – The electric field is a property of space around a single charge, Q

Electric Potential Energy

- Electrostatic potential energy stored in a configuration of two point charges:

$$U = k\frac{Qq}{r}$$

- We now define a new quantity V, called the ***Electric Potential***, as follows:

$$V(r) = k\frac{Q}{r}$$

  – Depends on only one charge, Q.

  – Depends on the distance r from the charge.

Electric Potential

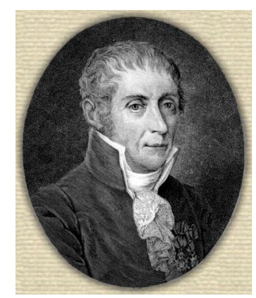
$$V(r) = k\frac{Q}{r}$$

- The electric potential energy that a charge q would have if it were placed a distance r from Q would be

$$U = q\,V(r)$$

- Units for electric potential:

$$\frac{\text{Joules}}{\text{Coulomb}} \equiv \text{Volt}$$



named after Count Alessandro Giuseppe Antonio Anastasio Volta
1745-1827

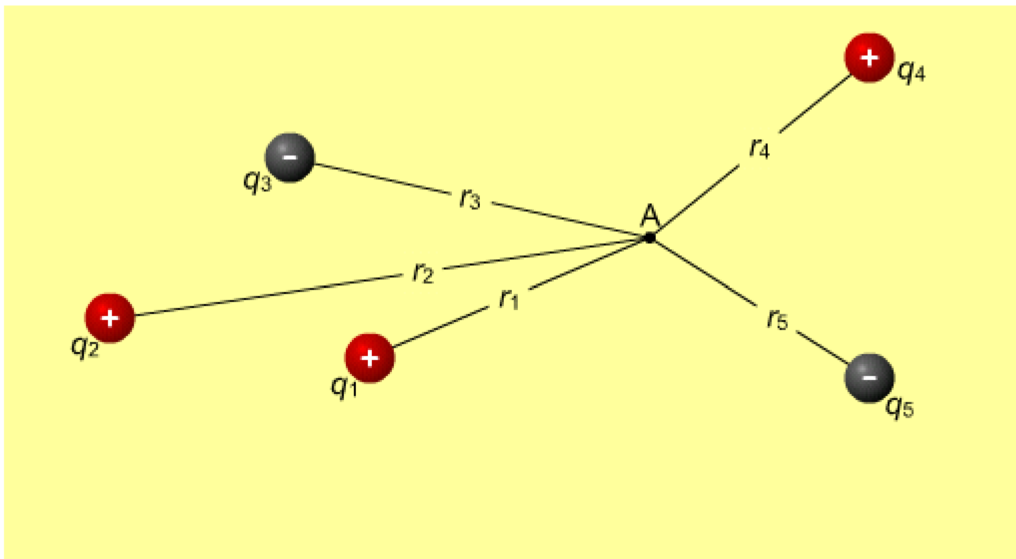
Electric Potential

TIP

- Both the V field and the $\vec{E}$ field at a specific location are independent of the test charge and characterize the properties of space at that location.
- Unlike the $\vec{E}$ field, which is a vector quantity, the V field (electric potential) is a scalar quantity. It can have a positive or a negative value depending on the sign of the electric charge Q of the object that creates the V field at a particular location.

Electric Potential

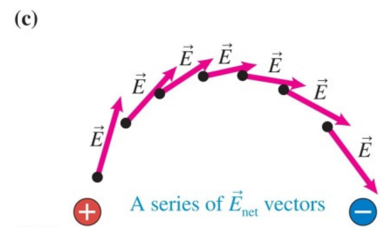
- The electric potential at a point in space is the sum of the electric potentials due to multiple sources (principle of superposition)

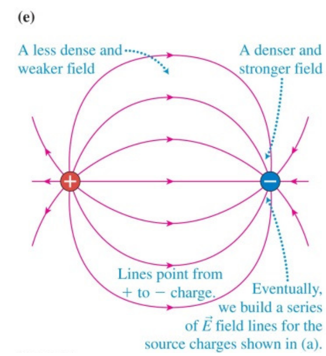


$$V_A = k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + k\frac{q_3}{r_3} + k\frac{q_4}{r_4} + k\frac{q_5}{r_5}$$

Visualizing Electric Potential

- The electric field is a *vector field*.
  - At each point in space it has a magnitude and a direction.
- We drew pictures of the electric field using
  a) Vectors (lots of little arrows)
  b) Electric field line diagrams
- The electric potential is a *scalar field*.
  - At each point in space it has a value (positive or negative) but no direction





Visualizing Electric Potential

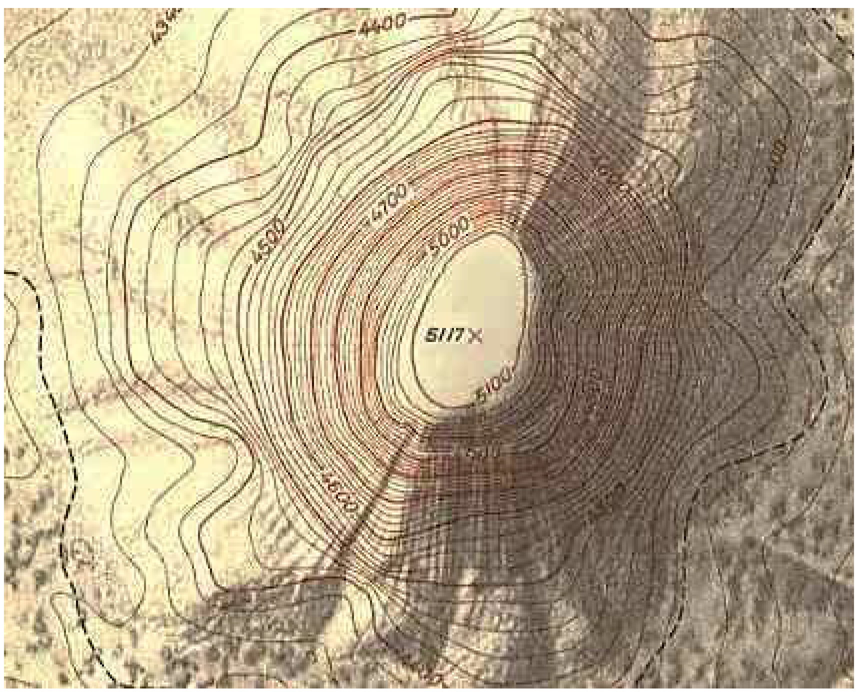
- Every point at a distance r from a single, point charge Q has the same electric potential.

$$V(r) = k\frac{Q}{r}$$

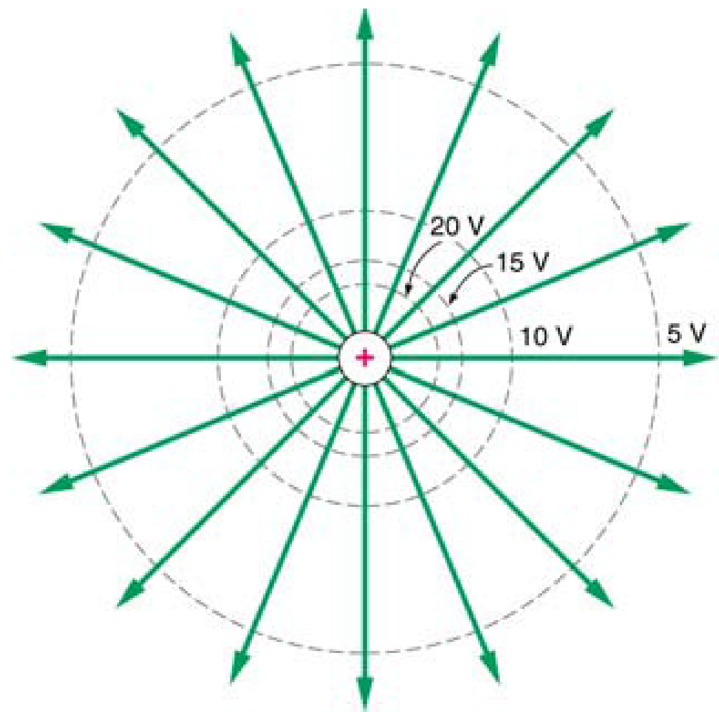
- All these points form an *equipotential surface.*
- We can’t easily draw this because it is a 3-dimensional surface in space.
- If it has sufficient symmetry, we can draw a projection of this surface in 2-dimensions.

Visualizing Electric Potential

Gravitational equipotentials – contour lines on a topographic map:



Electrostatic equipotentails:



Equipotential Surfaces

- Gravity example:
  - Moving an object without changing its height *does not* change its gravitational potential energy.
  - Gravity exerts a force on the object, but motion that does not change the height is always perpendicular to the direction of the force.

$$\Delta U = -\vec{F} \cdot \Delta\vec{x}$$

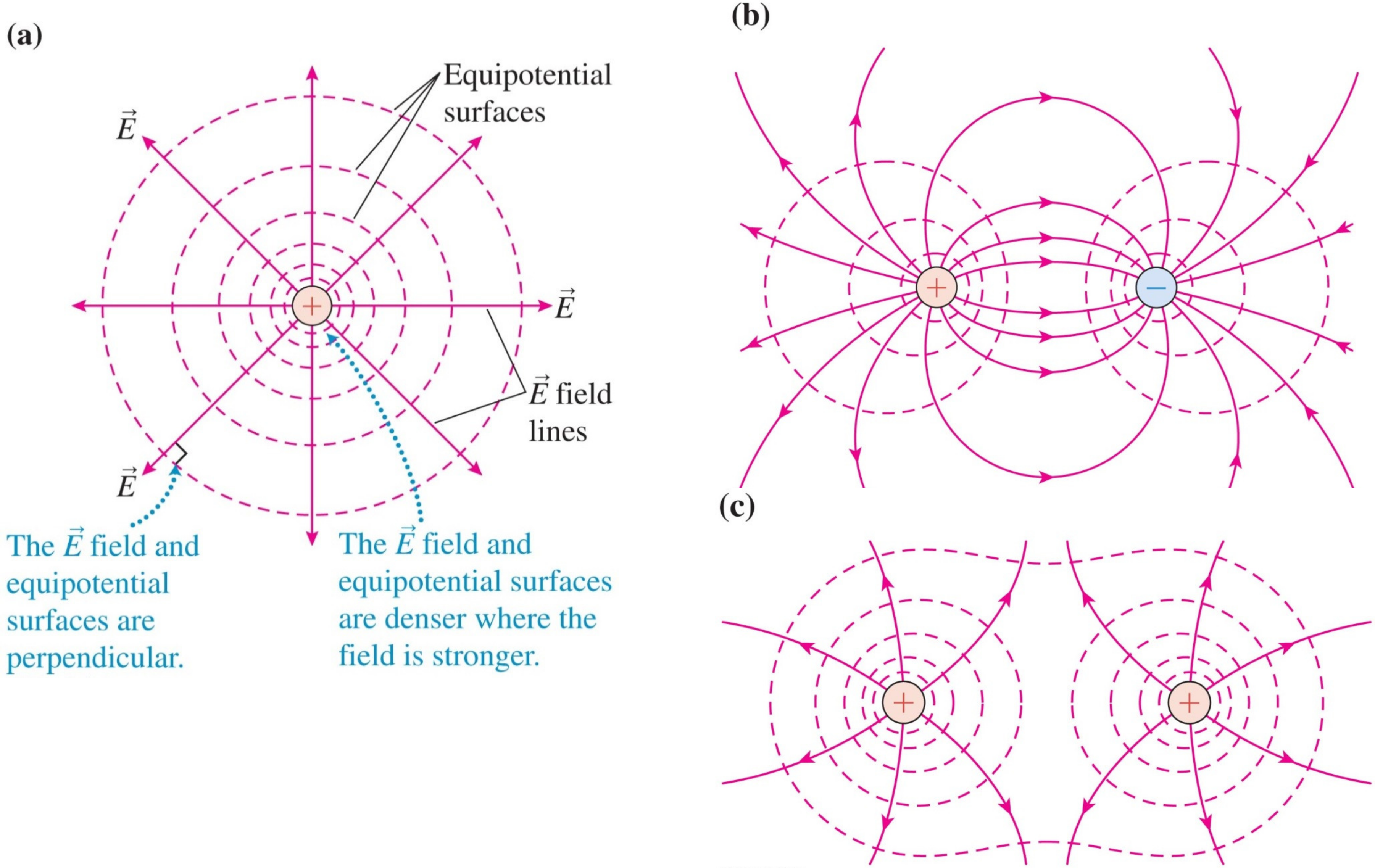
(if $\Delta\vec{x}$ is in the opposite direction of $\vec{F}$ then the potential energy increases because $\vec{F} \cdot \Delta\vec{x} < 0$)

Equipotential Surfaces

- Electrostatic case:
  - Moving a charge around on an equipotential surface does not change its electric potential energy
  - There is an electrostatic force on the charge but it is always perpendicular to the surface
  - Therefore, the electric field is also perpendicular to the surface.
- Relation between the components of the electric field and electric potential:

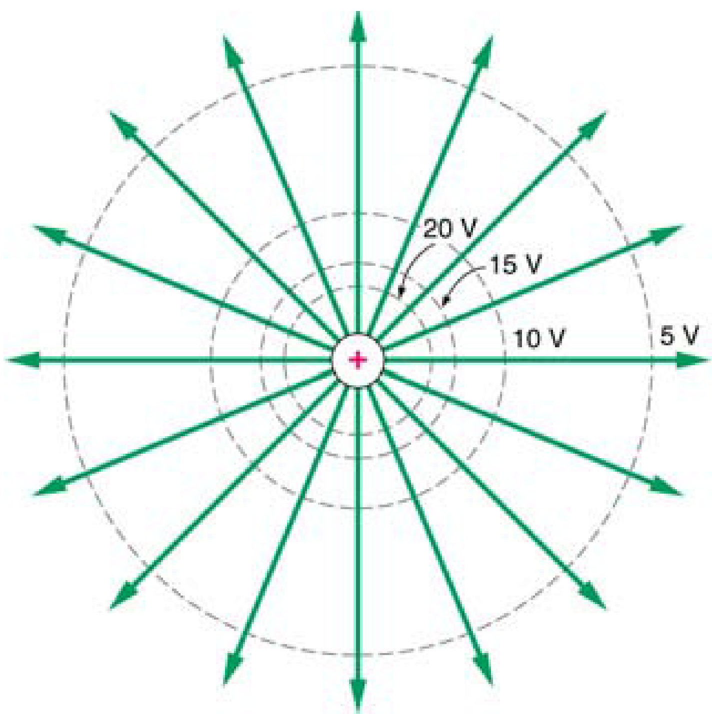
$$E_x = -\frac{\Delta V}{\Delta x} \qquad E_y = -\frac{\Delta V}{\Delta y} \qquad E_z = -\frac{\Delta V}{\Delta z}$$

Electric Field Lines and Electric Potential Surfaces



Equipotential Surfaces

- We will only consider some simple cases.
- Equipotential surfaces around a point charge:

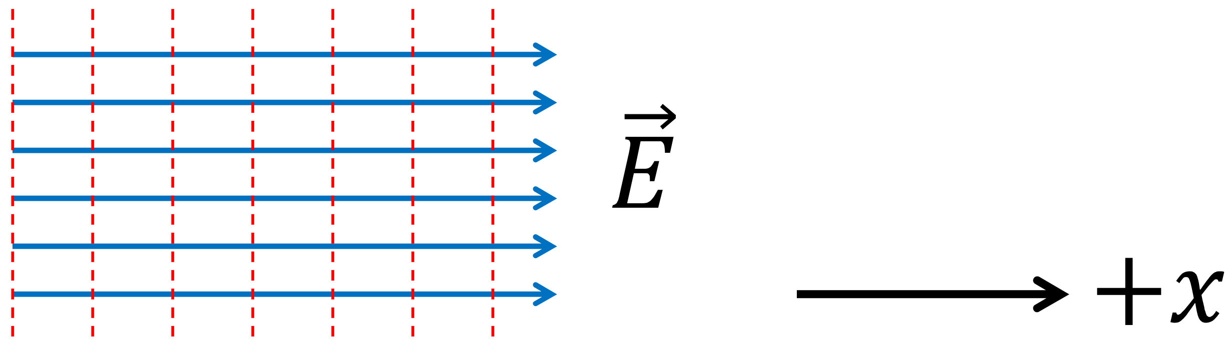


The electric field always points away from the charge (assuming it's positive)

Equipotential surfaces are spherical, with the charge at the center.

Equipotential Surfaces

Constant, uniform electric field:



- Pick a coordinate axis
  - It may as well be in the same direction as $\vec{E}$
- Relation between $\vec{E}$ and V:

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$\Delta V = -E_x \Delta x$$

- Electric potential decreases linearly to the right.

Equipotential Surfaces

- Another great example: *no electric field at all!*
- Does this mean that $V = 0$?
- No! It means that the electric potential is *constant*.

$$E_x = -\frac{\Delta V}{\Delta x}$$

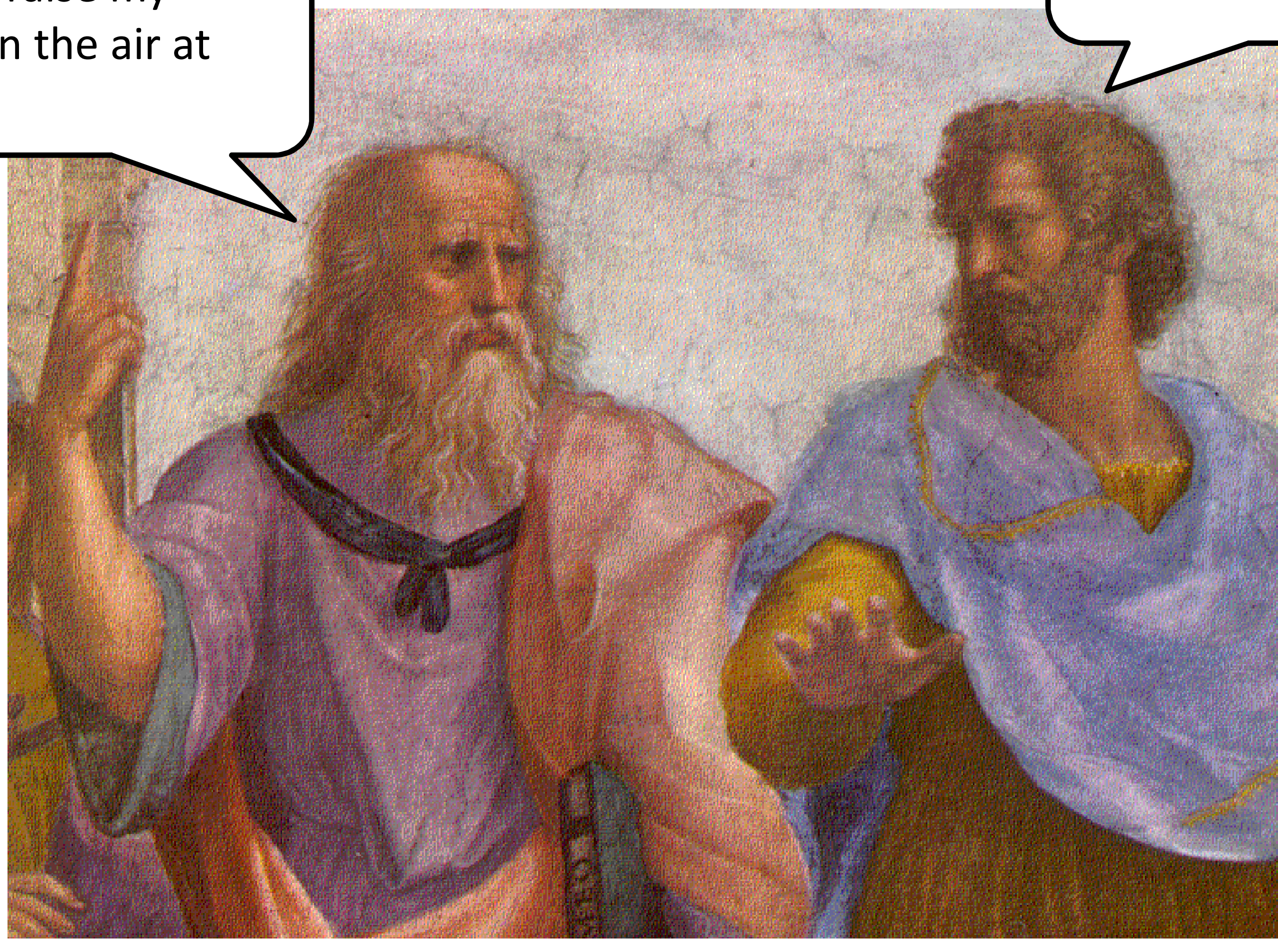
(if V is constant, then $\Delta V = 0$ and so $E_x = 0$)

Electrostatic Equilibrium

- Charges will exert forces on each other
- Unless they are stuck to something, the force will cause them to move around
- They will move until the electrostatic forces balance out
  - So the *net* force acting on them is zero
- When all the charges eventually stop moving, the system is in a state of *electrostatic equilibrium*
  - Usually this happens “quickly”, especially for “small” systems… so fast that we usually won’t worry about it.
  - For example, in much less than 1 μs for something that is about 1 cm in size.

Electric Fields in Conductors

Time for some Greek philosophy…



Electric Fields in Conductors

Socrates: “Dude, what beith the electric field in a conductor?”

Plato: “I don’t know. Is it in a state of electrostatic equilibrium?”

Socrates: “Yes.”

Plato: “Then suppose there was an electric field. What would happen?”

Socrates: “It’s a conductor so it would cause the electrons to move.”

Plato: “But that can’t be, because then it wouldn’t be in a state of electrostatic equilibrium!”

Socrates: “So the only thing that makes sense is that there must not be any electric field in the conductor.”

Plato: “…If it’s in a state of electrostatic equilibrium.”

Socrates: “Yeah, whatever…”

(In philosophy this is called *modus tollens*. In mathematics it’s called *proof by contradiction*. But whatever…)

Electric Fields in Conductors

- The point is, that there is *never* an electric field inside a conductor ($\vec{E} = 0$).
  - If it is in a state of electrostatic equilibrium.
  - If there are free charges, they get pushed to the surfaces until the electric field inside vanishes.
  - There can certainly be an electric field outside the conductor.
- What is the electric potential of a conductor?
  - No electric field, so is it zero?
  - No! It is *constant!*
- *The surface and everywhere inside a conductor has the same electric potential.*
- Circuit diagrams: we use lines to represent "wires" or in general, any place in a circuit that has the same electric potential (usually connected together by a good conductor.)

Example

- Electric field *outside* a spherical conductor with total charge Q and radius R:

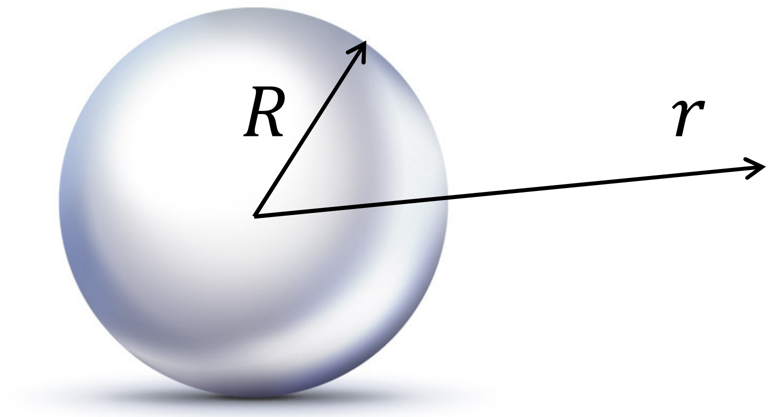
$$\vec{E}(r) = k\frac{Q}{r^2}\hat{r}$$

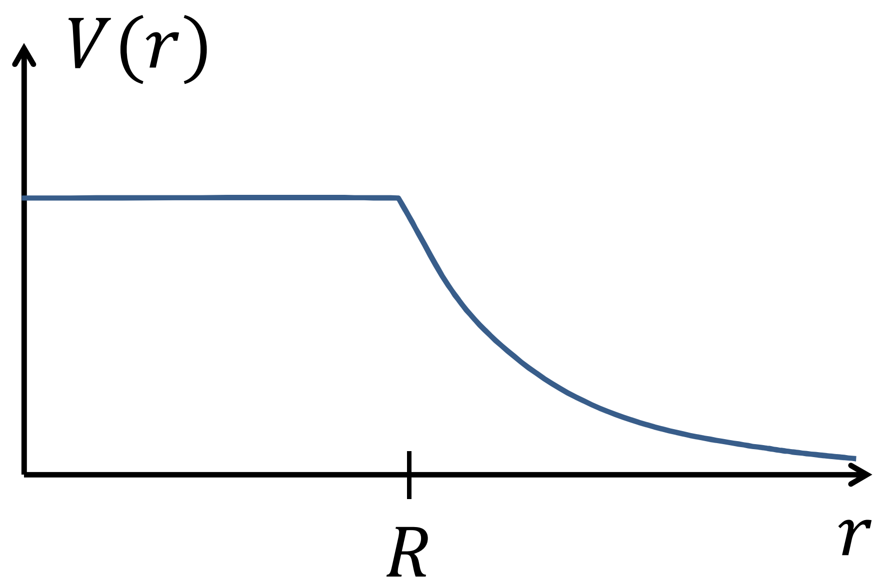
- Electric potential *outside* the spherical conductor:

$$V(r) = k\frac{Q}{r}$$

- Electric potential inside the spherical conductor:

$$V(r) = k\frac{Q}{R}$$



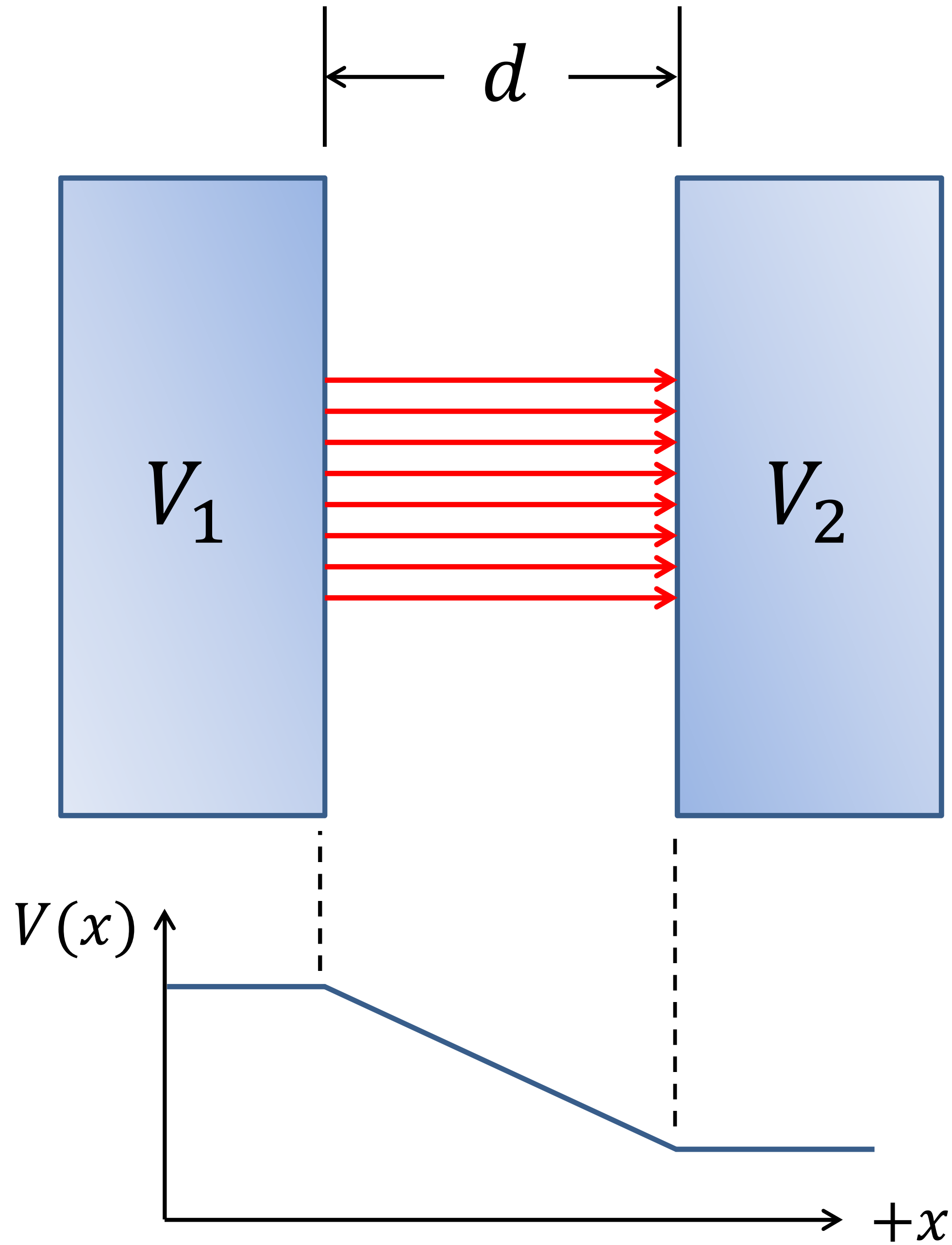


Example

- Two large, parallel conductors at different electric potentials.
- Electric field between them will be uniform and constant.

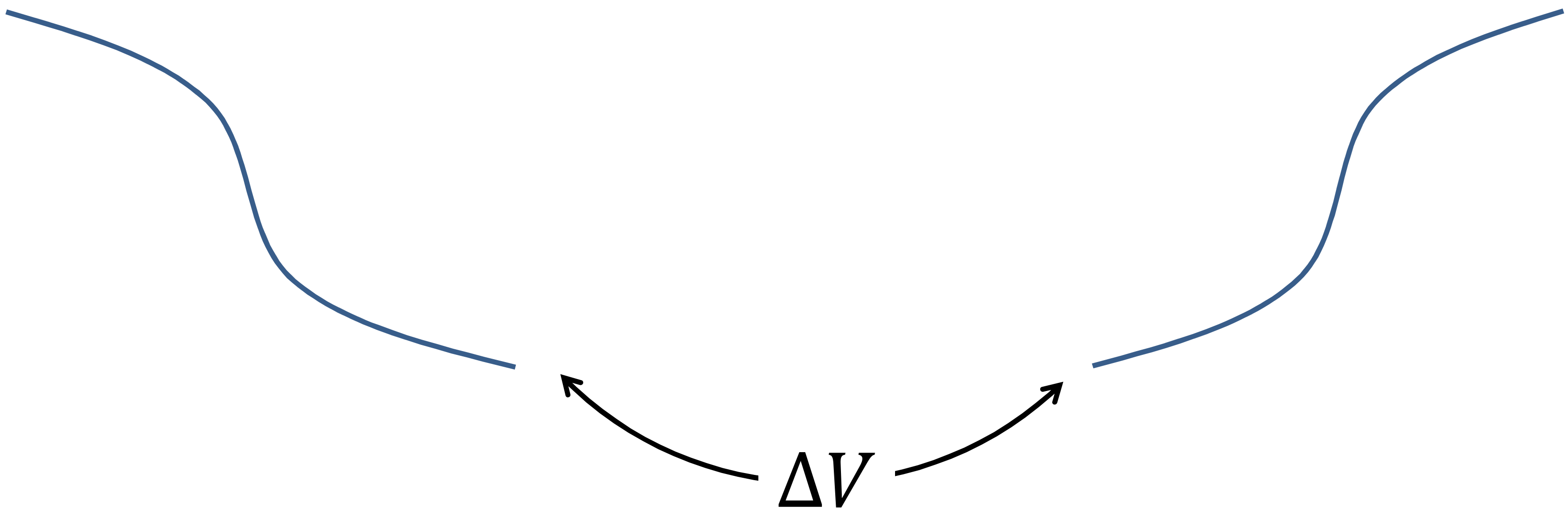
$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{V_2 - V_1}{d}$$

- Important to get the signs right. The drawing assumes that $V_1 > V_2$.



One Final Example

- How can you measure electric potential difference?

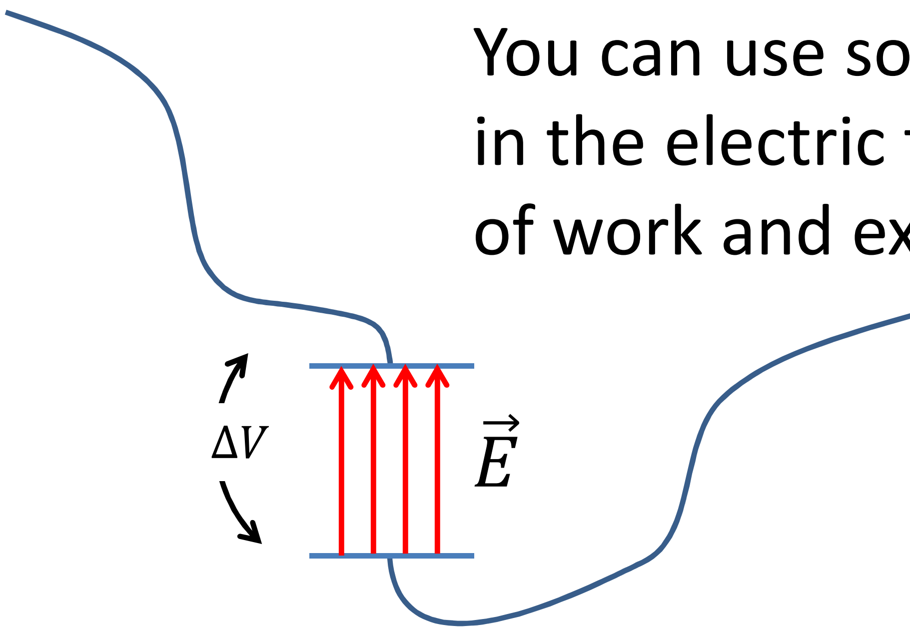


- Not meant to provide a practical recipe, but the basic ideas demonstrate that it can be done.

One Final Example

- The potential difference will result in an electric field between the two conductors

You can use some of the energy stored in the electric field to do a small amount of work and examine the result.



- Lift a small mass against the force of gravity.
- Compress a small spring.
- Polarize the molecules in some medium.
- Move electrons around in semiconductor.

These have observable consequences that depend on ΔV.