Purdue University Department of Physics

Physics 21900

General Physics II

Electricity, Magnetism and Optics

Lecture 3 – Chapter 14.5-6

Electric Potential Energy

Fall 2015 Semester

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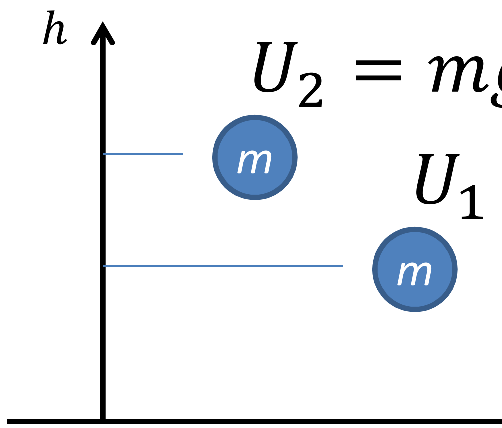
Mechanics Review

- Mechanical potential energy was a useful concept in mechanics
- Energy is always conserved
- A decrease in potential energy must result in an increase in other forms of energy
  - kinetic energy
  - Heat
- Potential energy can be changed by moving an object when subjected to certain types of force

Mechanics Review

- A good example is gravitational potential energy:



$$U_2 = mgh_2$$

$$U_1 = mgh_1$$

Constant force near the surface of the earth when changes in height are small

$$F_g = mg$$

$$g = 9.81\ N/kg$$

- How much did the potential energy increase when the mass was moved from height h_1 to height h_2?

$$\Delta U = mg(h_2 - h_1) = mg\Delta h\ > 0$$

Mechanics Review

- How did we move the mass?
  - We pushed it up to a greater height
  - Physicists like to say “We did work **ON** the mass.”
- Energy is conserved:

$$\Delta U + W = 0$$

- Using this convention, the work is a negative number.
- It doesn’t matter where we define $h = 0$.
- The change in potential energy only depends on the change in height.

Mechanics Review

- Negative work (you do something to increase the potential energy of the system):
  - You carry water up a hill
  - You compress a spring
  - You climb stairs
- Positive work (the system uses its potential energy to do work for you):
  - The water flows downhill and turns a turbine
  - The spring unwinds (and does something for you)
  - You slide down the handrail (increase kinetic energy)

Mechanics Review

- What good is this?
- If the mass was initially at rest and fell a height h, how fast would it be moving?
- We can define $h_i = h$ and $h_f = 0$

$$\Delta h = h_f - h_i = -h$$

- Change in potential energy:

$$\Delta U = mg\Delta h < 0$$

- Conservation of energy:

$$\Delta U + \Delta K = 0$$

$$\Delta K = \frac{1}{2}mv^2 = \frac{1}{2}m(v_f^2 - \cancel{v_i^2}) \quad (v_i^2 \to 0)$$

$$v = \sqrt{2gh}$$

Mechanics Review

- The gravitational force is called ***conservative***:
  - The potential energy only depends on the position of an object.
  - The potential energy does not depend on the path the object takes.
- Other *conservative* forces:
  - Spring force, $F = -kx$ (Hooke's law)
  - *Electrostatic force*
- Forces that are not conservative:
  - Friction
  - Viscous damping

These dissipate energy rather than store energy.

Mechanics Review

Gravitational potential energy:

- Constant force, $F_g = m\ g$

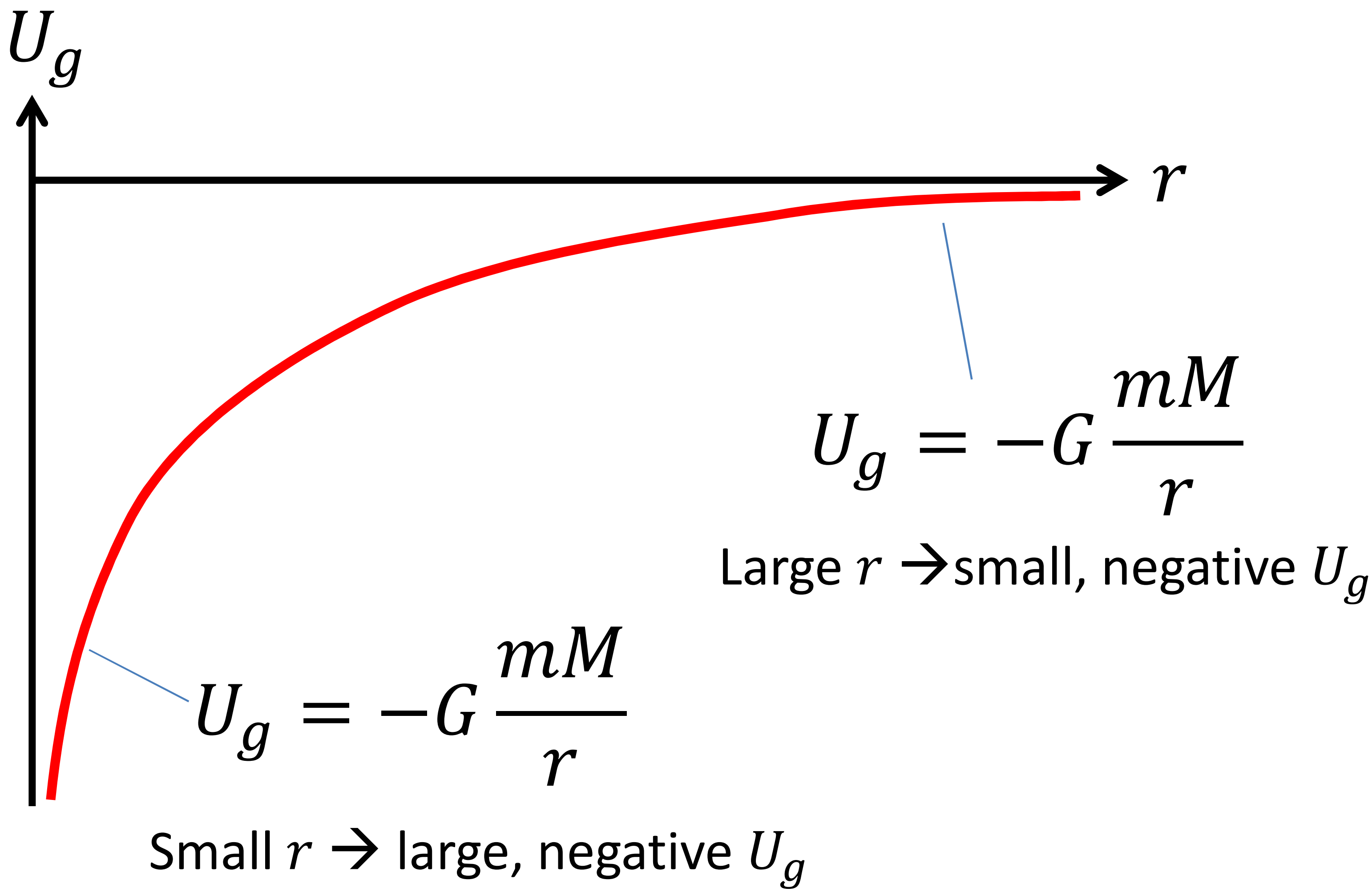
$$U = mgh$$

  - Defined so that $U = 0$ when $h = 0$
  - h can have both positive and negative values.
- Near a large mass, M, over large distances, r:

$$F_g = G\frac{mM}{r^2} \qquad\qquad U = -G\frac{mM}{r}$$

  - This is defined so that $U \to 0$ as $r \to \infty$.
  - F_g is always attractive
  - r is always positive.
  - If r decreases, then U decreases and the gravitational potential energy most be converted into some other form (eg. kinetic energy)
    - For example, the speed of a comet increases as it falls towards the sun.

Gravitational Potential Energy



Electrostatic Potential Energy

- Suppose a charge q_1 is fixed in one place.
- If a charge q_2 is located at a distance r then the force is

$$F_q = k\frac{q_1 q_2}{r^2}$$

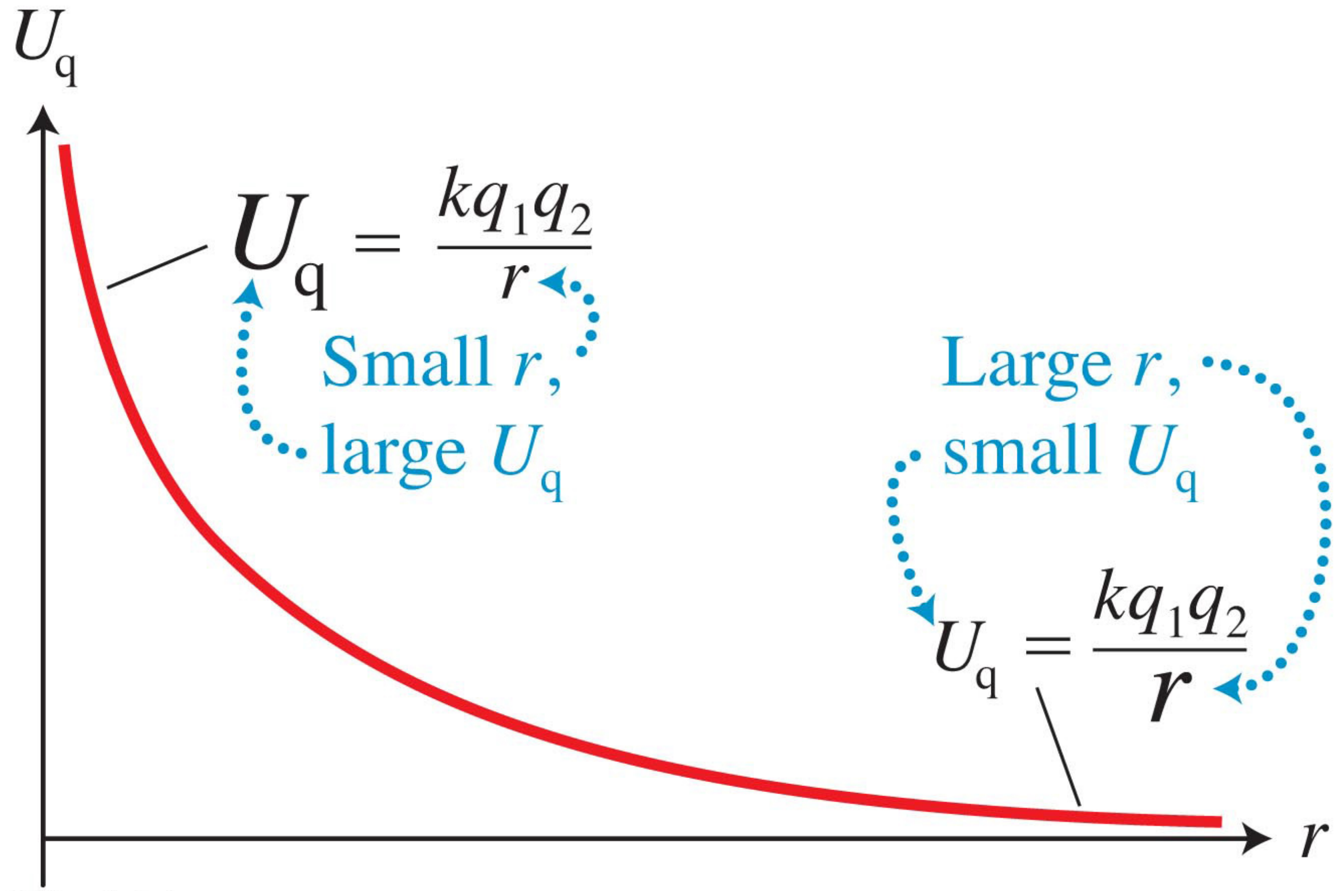
  - F_q can be attractive or repulsive
  - When q_1 and q_2 have the same sign, F_q is repulsive.
- The electrostatic potential energy is

$$U_q = k\frac{q_1 q_2}{r}$$

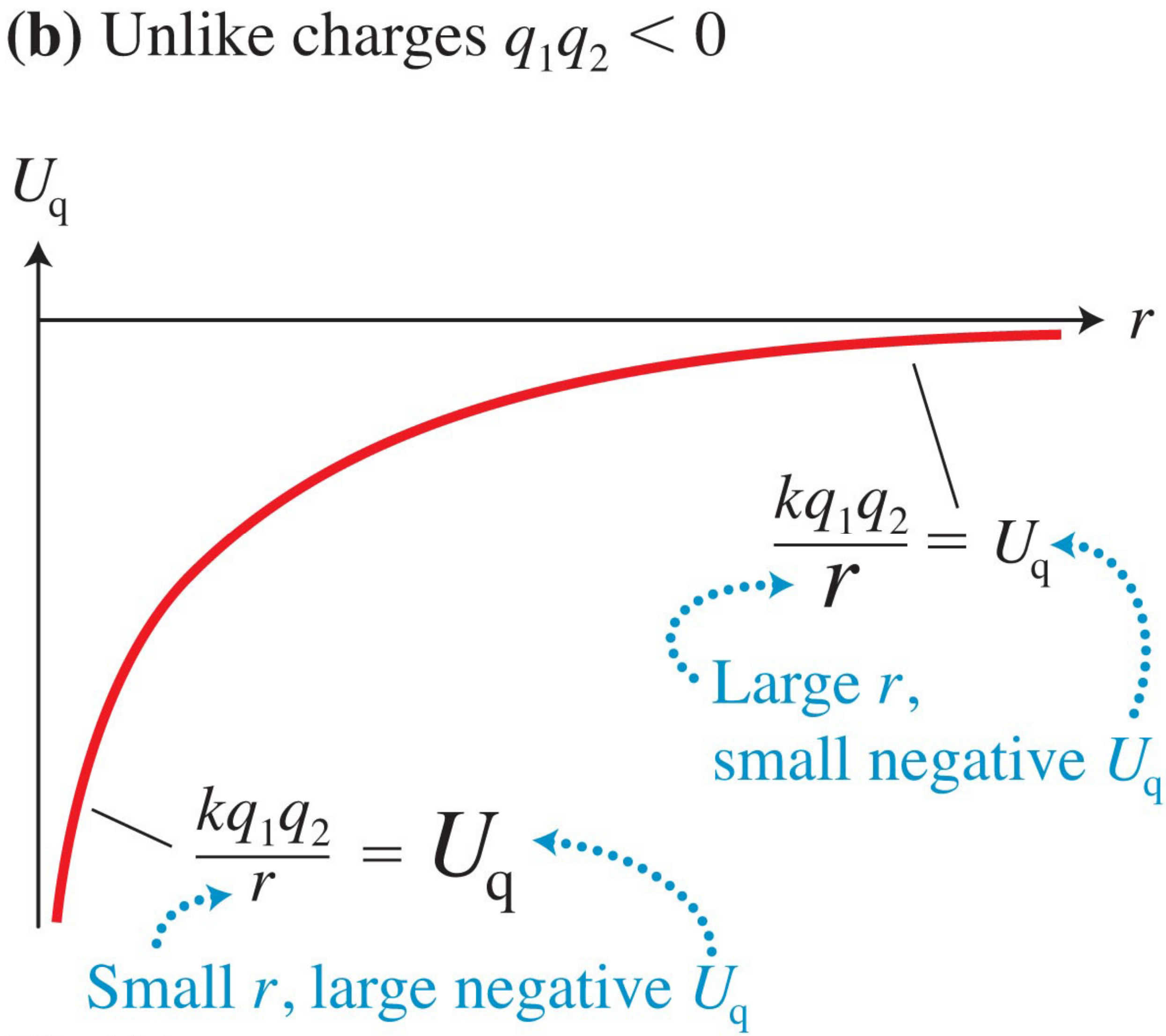
  - Defined so that $U_q \to 0$ as $r \to \infty$.

Electrostatic Potential Energy

(a) Like charges $q_1 q_2 > 0$



Electrostatic Potential Energy



Key Points

It is important to use the signs associated with each charge.

When the signs of the two charges are opposite, the potential energy of the pair is negative.

When the signs of the two charges are the same, the potential energy of the pair is positive.

Examples

How much work do we have to do if we want to place two $1\ \mu C$ charges at a distance of $1\ cm$ from each other?

– The force is repulsive, so we have to push them towards each other (just like lifting a mass)
– If they are initially separated by a large distance, $r \to \infty$ then they initially have $U_q = 0$.
– In their final configuration,

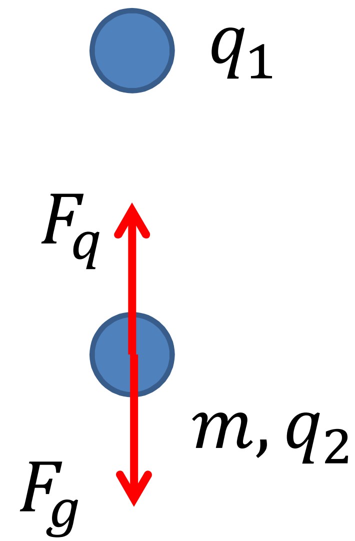
$$U_q = k\frac{q_1 q_2}{r} = (9 \times 10^9\ N \cdot m^2 \cdot C^{-2})\frac{(1 \times 10^{-6}\ C)^2}{0.01\ m}$$
$$= 9 \times 10^{-1}\ N \cdot m = 0.9\ N \cdot m = 0.9\ J$$

– How much work did we do?

$$\Delta U + W = 0 \text{ so } W = -0.9\ J.$$

Example

- One charge $q_1 = 10\ \mu C$ is fixed in place at the origin. A second charge $q_2 = -10\ \mu C$ is initially located $10\ cm$ from q_1.
- If q_2 has a mass of $m = 1\ kg$, and the electrostatic force is used to lift it, how fast will it be moving when the charges are separated by $5\ cm$?
  - Use energy concepts to analyze this problem



Example

- One charge $q_1 = 10\ \mu C$ is fixed in place at the origin. A second charge $q_2 = -10\ \mu C$ is initially located $10\ cm$ from q_1.
  - What is the initial potential energy of the system?

$$U_i = k\frac{q_1 q_2}{r_i}$$

$$= (9 \times 10^9\ N \cdot m^2 \cdot C^{-2})\frac{(10^{-5} C)(-10^{-5} C)}{(0.1\ m)}$$

$$= -9\ N \cdot m = \boldsymbol{-9\ J}$$

Example

- The final separation is $r_f = 5\ cm$
  - What is the final electrical potential energy of the system?

$$U_f = k\frac{q_1 q_2}{r_f}$$

$$= (9 \times 10^9\ N \cdot m^2 \cdot C^{-2})\frac{(10^{-5}C)(-10^{-5}C)}{(0.05\ m)}$$

$$= -18\ N \cdot m = \mathbf{-18\ J}$$

Example

- What is the change in electrical potential energy?

$$\Delta U_q = U_f - U_i = (-18\text{ J}) - (-9\text{ J}) = -9\text{ J}$$

- What is the change in the gravitational potential energy?

$$\Delta U_g = mg\left(h_f - h_i\right) = (1\,kg)(9.81\,N/kg)(0.05\,m)$$
$$= 0.49\,J$$

- Conservation of energy:

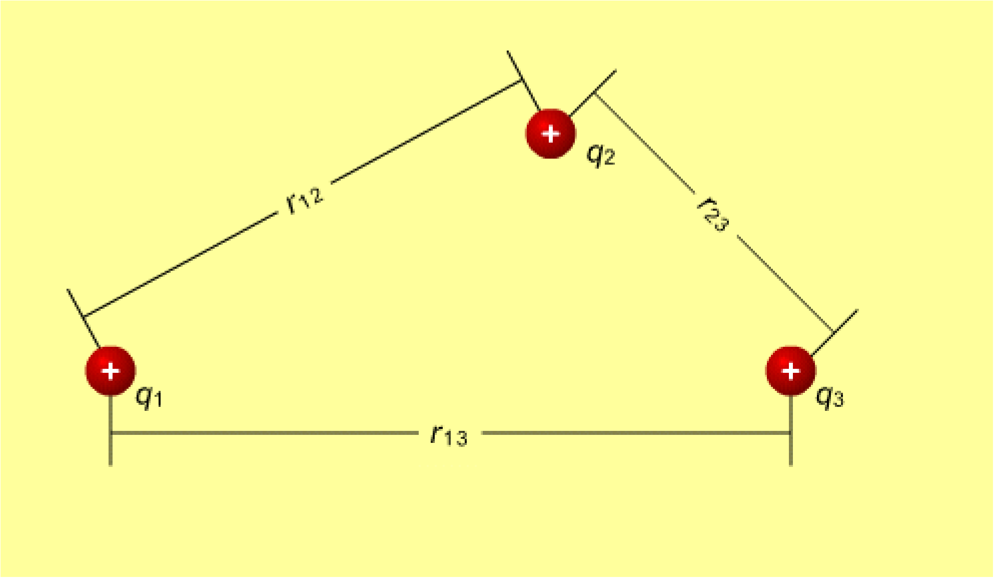
$$\Delta U_q + \Delta U_g + \Delta K = 0$$

$$\Delta K = \frac{1}{2} m\, v^2 = -(-9\,J + 0.49\,J) = 8.51\,J$$

$$v = \sqrt{2\frac{8.51\,J}{1\,kg}} = \sqrt{2\frac{8.51\,kg\,m^2/s^2}{1\,kg}} = 4.13\,m/s$$

More than two point charges

- How much work does it take to assemble a more complex charge configuration consisting of charges q_1, q_2, q_3 separated by distances r_{12}, r_{13}, r_{23}?



More than two point charges

- How much work does it take to assemble a more complex charge configuration consisting of charges q_1, q_2, q_3 separated by distances r_{12}, r_{13}, r_{23}?
  - In the absence of any other charges (so no forces), it takes no work to place the first charge anywhere
  - Placing the second charge increases the potential energy by the amount

$$\Delta U_q = k\frac{q_1 q_2}{r_{12}}$$

More than two point charges

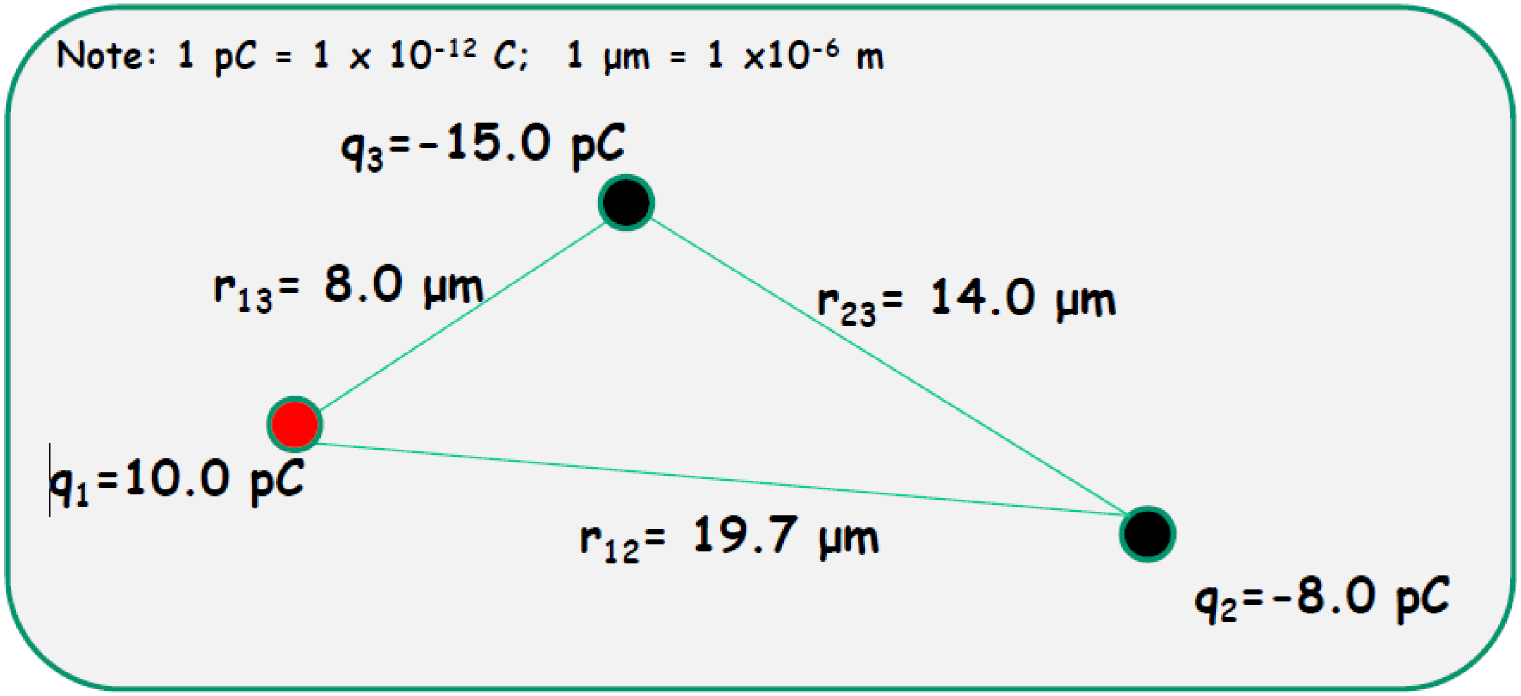
- How much work does it take to assemble a more complex charge configuration consisting of charges q_1, q_2, q_3 separated by distances r_{12}, r_{13}, r_{23}?
  - When placing the third charge, it feels forces from both q_1 and q_2.

$$\Delta U_q = k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$

  - Total work needed to assemble the three charges:

$$U_q = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$

Example: What is the PE_E of this system of charges?



$$PE_E = \left[\frac{k\,q_1q_2}{r_{12}} + \frac{k\,q_2q_3}{r_{23}} + \frac{k\,q_1q_3}{r_{13}}\right]$$

$$= \left(9\times10^{9}\,\mathrm{Nm}^2/\mathrm{C}^2\right)\left(\frac{(10\times10^{-12}\mathrm{C})(-8\times10^{-12}\mathrm{C})}{19.7\times10^{-6}\mathrm{m}} + \frac{(-15\times10^{-12}\mathrm{C})(-8\times10^{-12}\mathrm{C})}{14.0\times10^{-6}\mathrm{m}} + \frac{(10\times10^{-12}\mathrm{C})(-15\times10^{-12}\mathrm{C})}{8.0\times10^{-6}\mathrm{m}}\right)$$

$$= \left(9\times10^{9}\right)\left(-4.06\times10^{-18} + 8.57\times10^{-18} - 1.88\times10^{-17}\right)\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\times\frac{\mathrm{C}^2}{\mathrm{m}}$$

$$= \left(9\times10^{9}\right)\left(-4.06 + 8.57 - 18.8\right)\times10^{-18}\,\mathrm{Nm} = \left(9\times10^{9}\right)(-14.3)\times10^{-18}\,\mathrm{J}$$

$$= -1.29\times10^{-7}\,\mathrm{J} = -12.9\,\mu\mathrm{J}$$

Summary

- Energy can be stored in the configuration of two or more charges.
- The energy of the system can be increased or decreased by moving the charges around.
- Changing the electrical potential energy must be balanced by changes in other types of energy
  - Kinetic
  - Gravitational
  - Chemical
  - Thermal
  - Etc…