Physics 21900

General Physics II

Electricity, Magnetism and Optics

Lecture 26 – Chapter 27

Photons, The Hydrogen Atom, de Broglie Waves

Fall 2015 Semester

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Review

- Photons are *quanta* of electromagnetic radiation
- Energy can be measured in electron-volts:

$$\mathbf{1\ eV = 1.602 \times 10^{-19}\ J}$$

- The energy of a photon depends on its frequency:

$$E = hf$$

$$\mathbf{h = 6.626 \times 10^{-34}\ J \cdot s} \quad \text{(Planck's constant)}$$

$$\mathbf{= 4.14 \times 10^{-15}\ eV \cdot s}$$

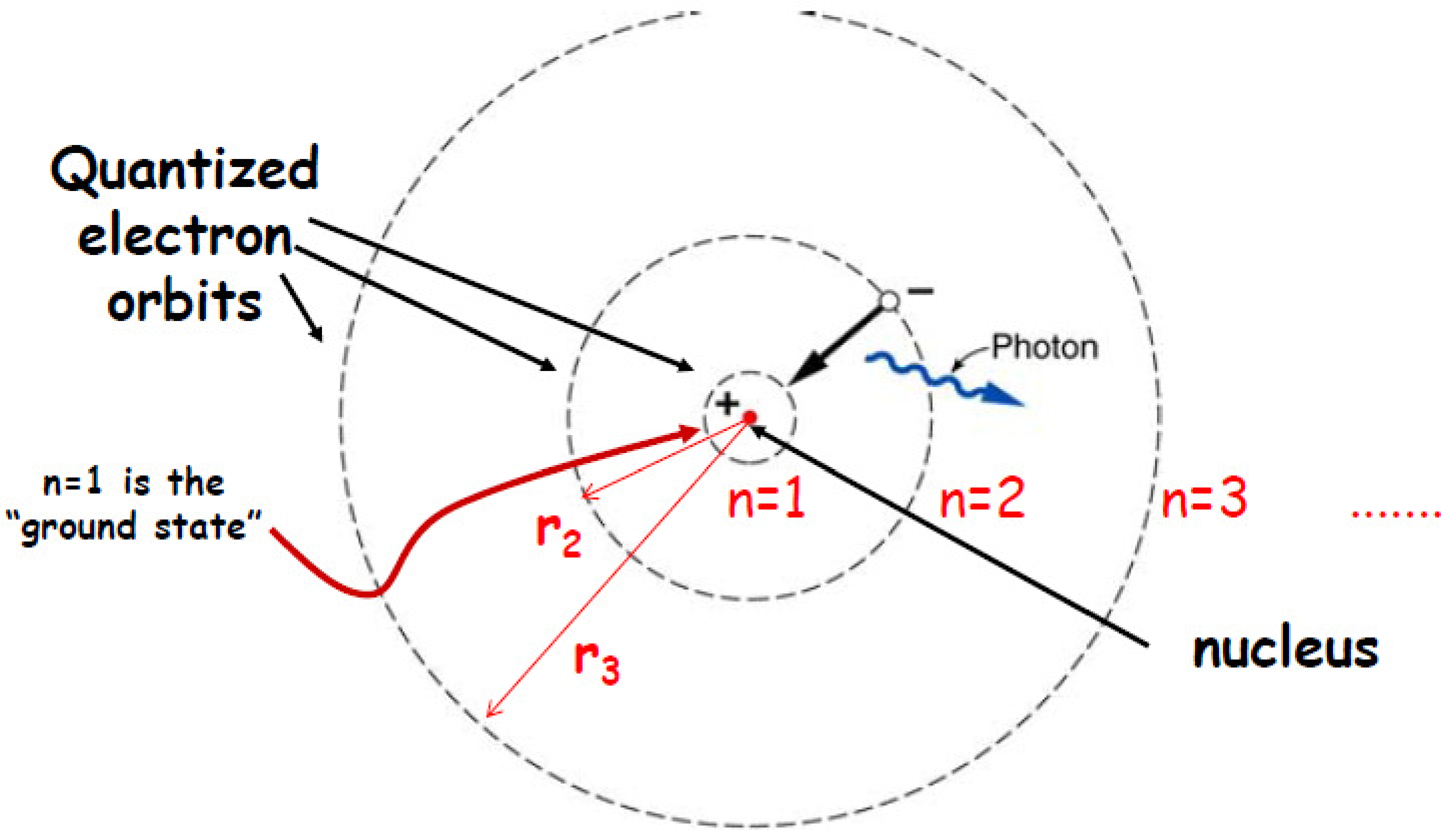
- Wavelength is related to frequency:

$$\lambda = \frac{c}{f}$$

- Energy is related to wavelength:

$$E = \frac{hc}{\lambda}$$

Bohr’s model for light emission from H



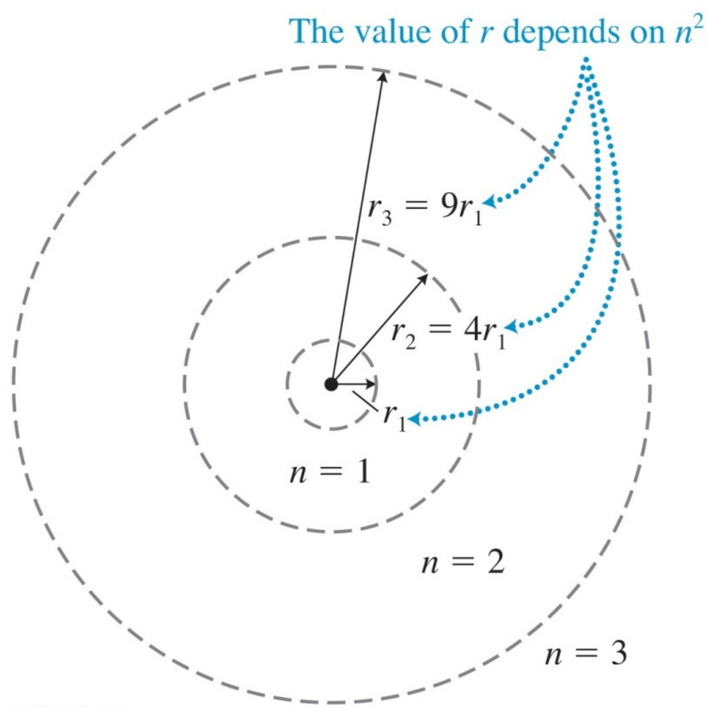
Using this model, you can derive
Balmer's empirical formula

Size of the hydrogen atom

$r_n = (0.53 \times 10^{-10}\ \mathrm{m})n^2$, for $n = 1,2,3, \ldots$

- n is called the principal quantum number and must be a positive integer
- Only certain radii represent stable electron orbits.

$$r_n = n^2 a_0$$
$$a_0 = 0.0529\ \mathrm{nm}$$

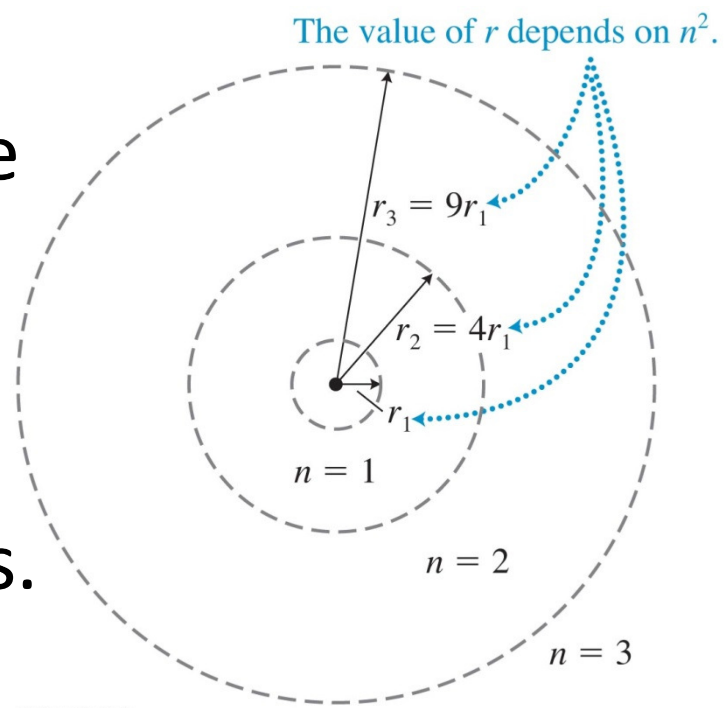


Energy of Electron orbits

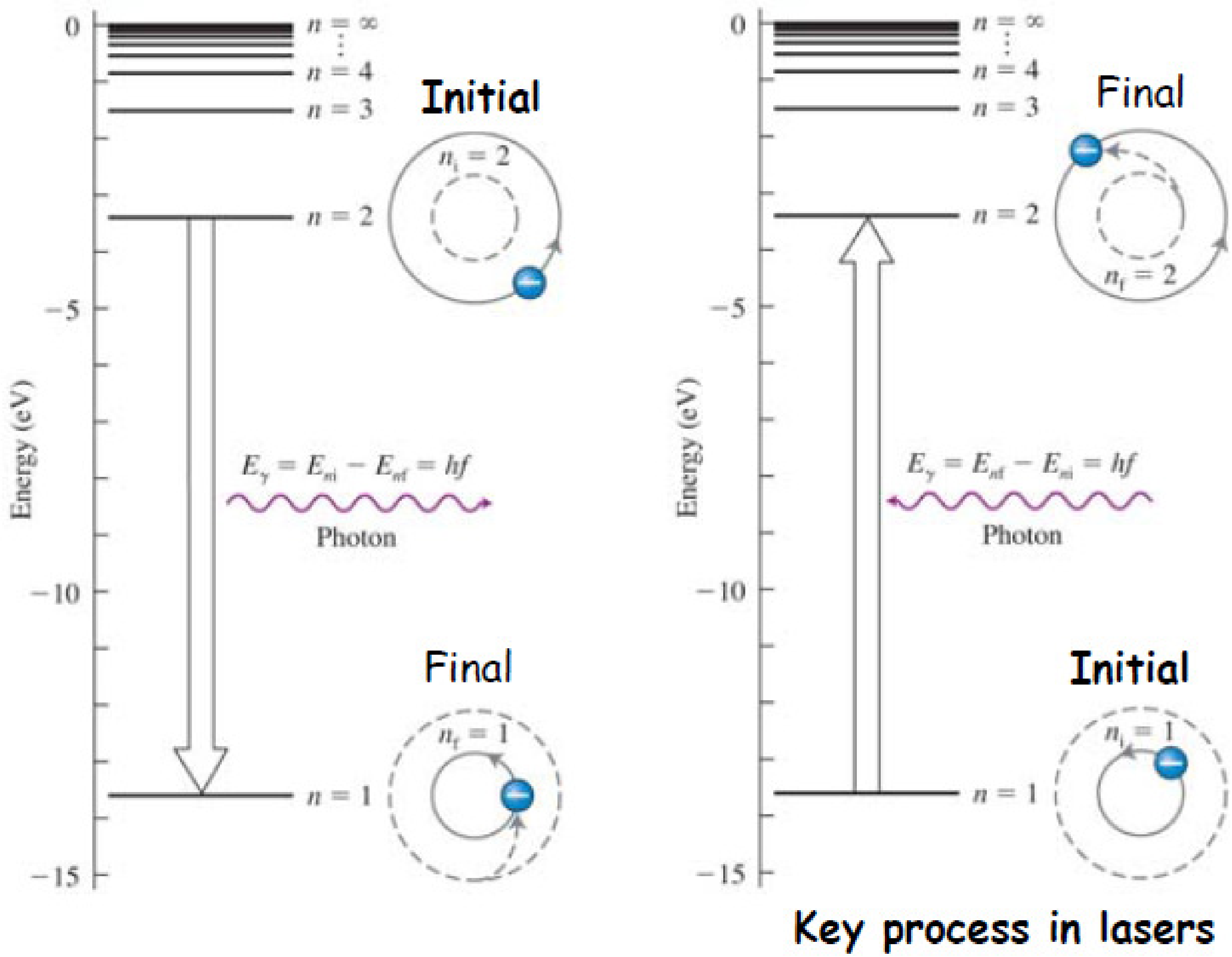
$$E_n = -\frac{13.6\ eV}{n^2}$$

- Negative energies mean the electron is bound to the nucleus.
- $n = 1$ is the lowest possible energy (the ground state).
- A free electron has $E > 0$.
- A photon is absorbed or emitted when an n changes.

$$\Delta E = E_i - E_f$$



Photon Emission and Photon Absorption



Example

- What is the wavelength of a photon emitted when an electron drops from the $n = 3$ orbit to the ground state?

$$E_i = -\frac{13.6\ eV}{3^2} = -1.51\ eV$$

$$E_f = -\frac{13.6\ eV}{1^2} = -13.6\ eV$$

$$\Delta E = E_i - E_f = 12.1\ eV$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.14 \times 10^{-15}\ eV \cdot s)(3 \times 10^{8}\ m/s)}{12.1\ eV}$$

$$= 1.03 \times 10^{-7}\ m = 103\ nm$$

(extreme ultraviolet)

Example

What is the minimum energy needed to ionize a hydrogen atom that has its electron in the $n = 2$ orbit?

- Bound electrons have $E < 0$
- The minimum energy of a free electron is $E = 0$

$$E_i = -\frac{13.6\ eV}{2^2} = -3.4\ eV$$

$$E_f = 0 \text{ (minimum possible)}$$

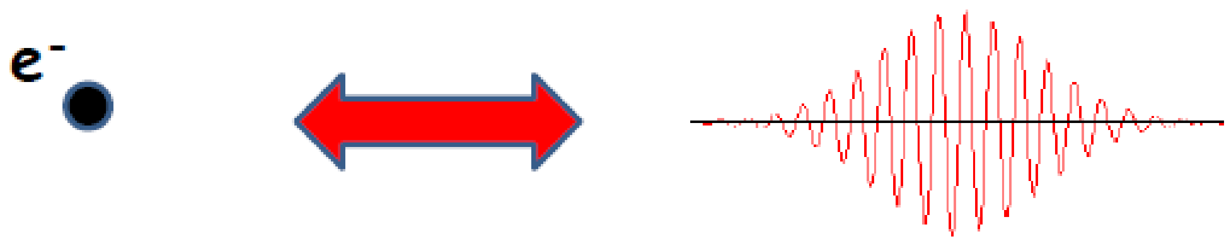
Minimum photon energy is $E = 3.4\ eV$

- Wavelength, $\lambda = \frac{hc}{E} = \frac{(4.14\times10^{-15}\ eV\cdot s)(3\times10^8 m/s)}{3.4\ eV}$

$$= 365\ nm \text{ (ultraviolet)}$$

de Broglie in 1923 postulated that particles (like an electron) might behave as a wave

"...I had a sudden inspiration. Einstein's wave-particle dualism was an absolutely **general** phenomenon extending to **all** physical nature..."

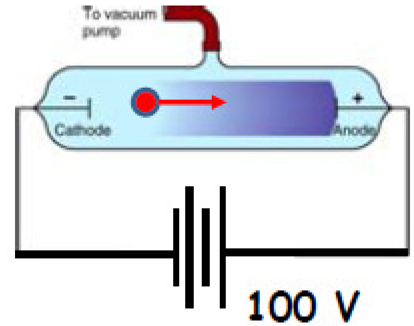


de Broglie asserts that matter, like light, has a wave-particle duality.

Particles with a momentum p have a wavelength λ given by

$$\lambda = \frac{h}{p}$$

Example



What is the de Broglie wavelength of an electron that is accelerated from rest through a potential difference of 100 V?

1. Kinetic energy is

$$KE = \frac{1}{2} m_e v^2 = 100\ eV = 1.602 \times 10^{-17}\ J$$

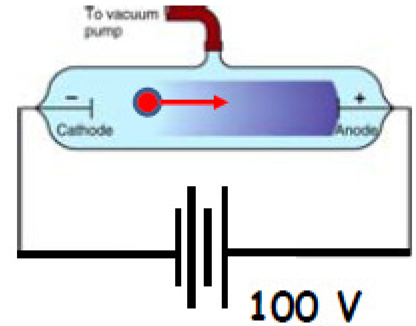
2. Momentum is

$$p = m_e v = \sqrt{2 m_e\ KE}$$

$$= \sqrt{2(9.109 \times 10^{-31}\ kg)(1.602 \times 10^{-17}\ J)}$$

$$= 5.40 \times 10^{-24}\ kg \cdot m/s$$

Example



What is the de Broglie wavelength of an electron that is accelerated from rest through a potential difference of 100 V?

3. de Broglie wavelength is

$$\lambda = \frac{h}{p}$$

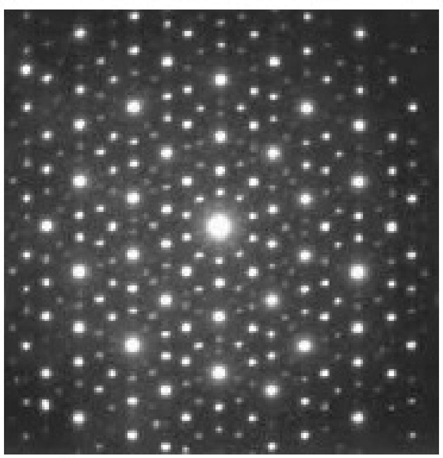
$$= \frac{6.626 \times 10^{-34}\, J \cdot s}{5.40 \times 10^{-24}\, kg \cdot m/s} = 0.123\, nm$$

If electrons behave like waves, can we observe interference phenomena? A double slit experiment will need a slit spacing that is less than 1 nm!

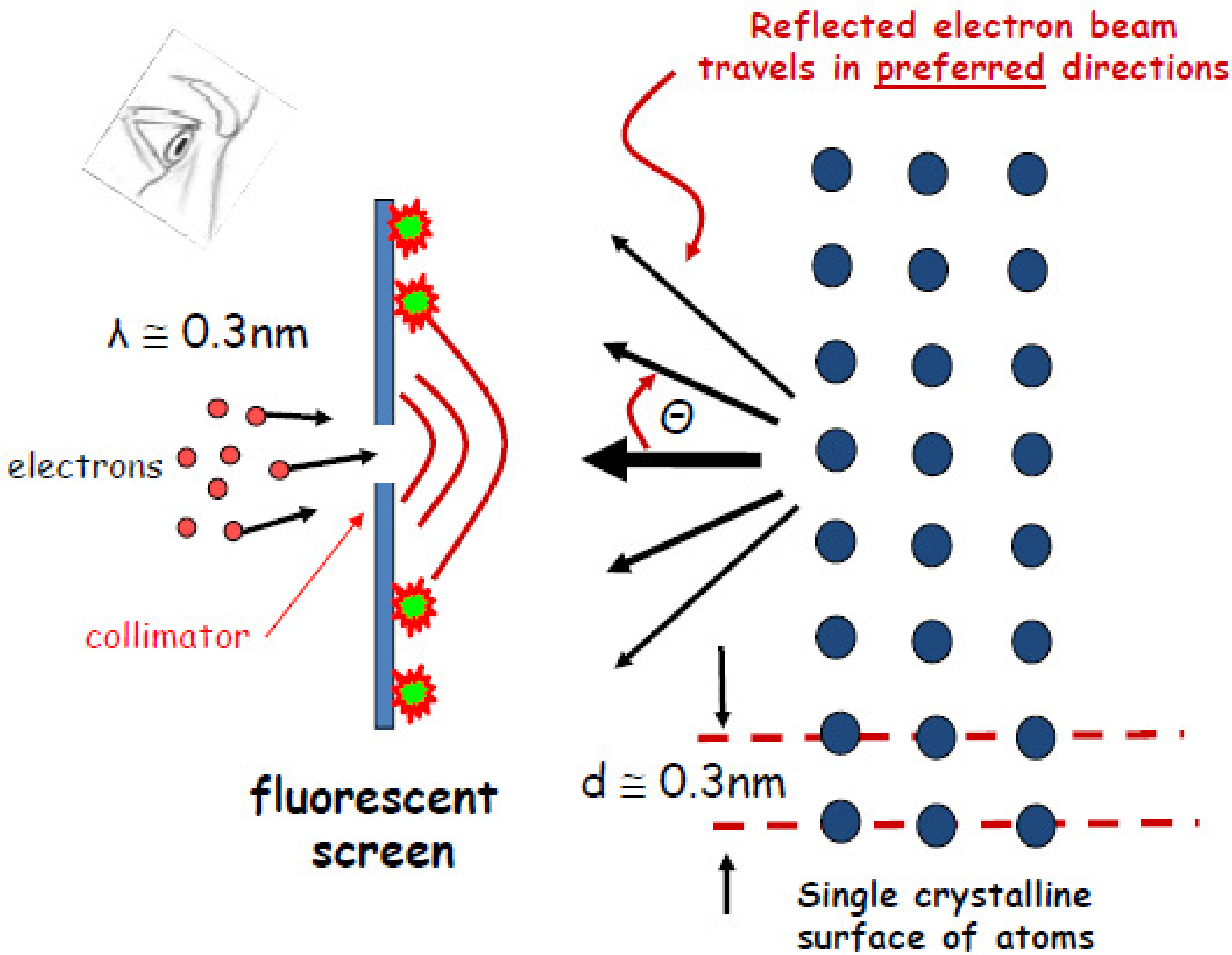
Davisson-Germer Experiment - 1927

Noble Prize 1937

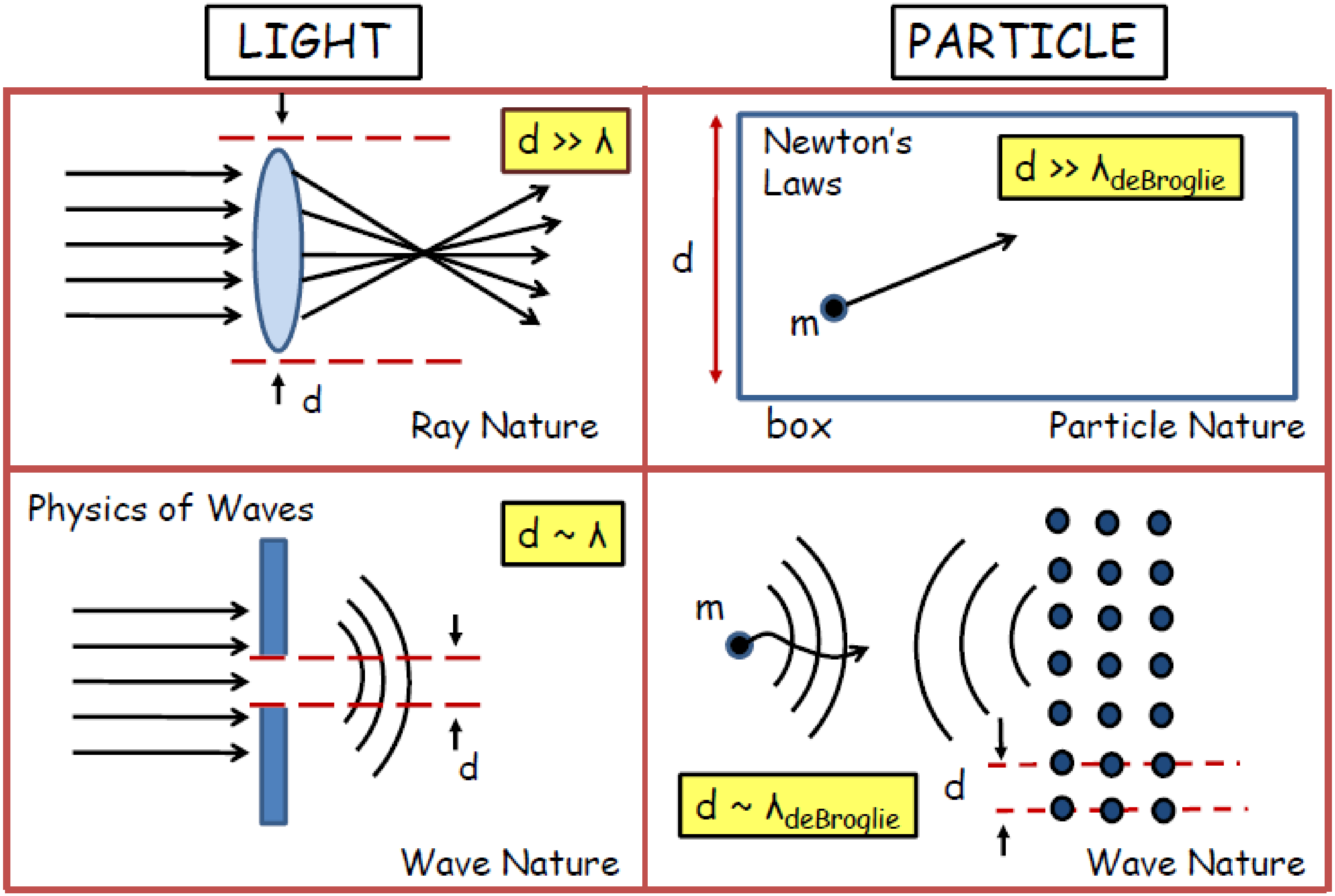
Bell Telephone Laboratories



Representative photograph of screen



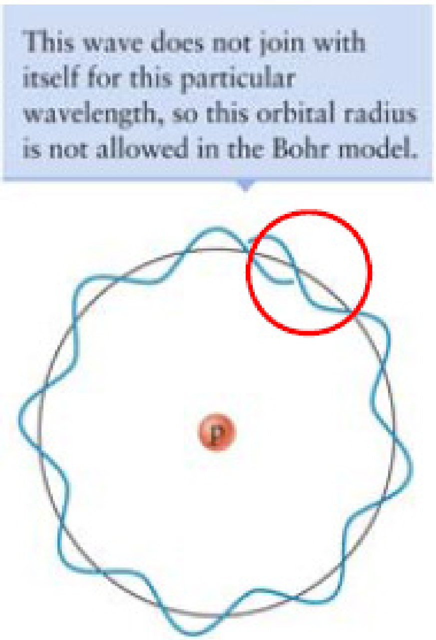
The dual nature of light and particles



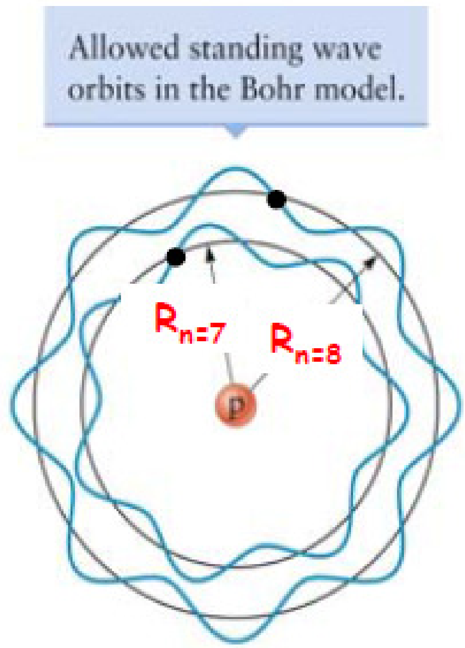
de Broglie waves explain origin of Bohr's electron orbits

Why Quantized Orbits?

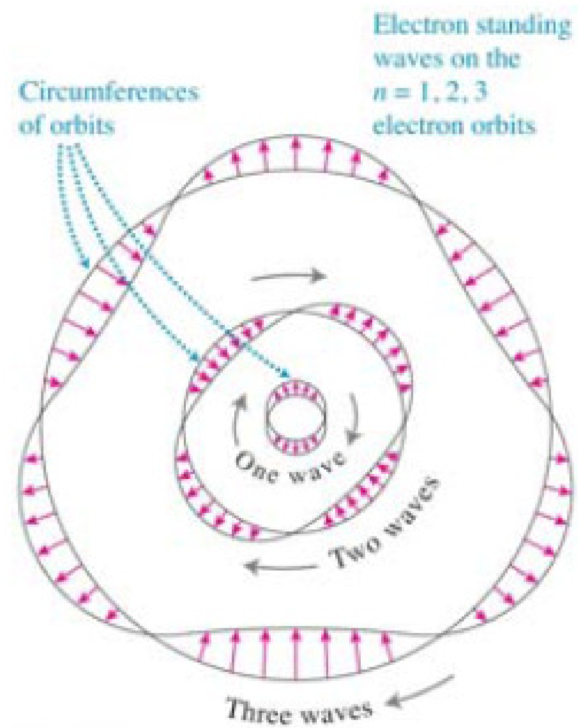
For an electron wave to be stable, an exact integer number of electron wavelengths must fit into a stationary orbit.



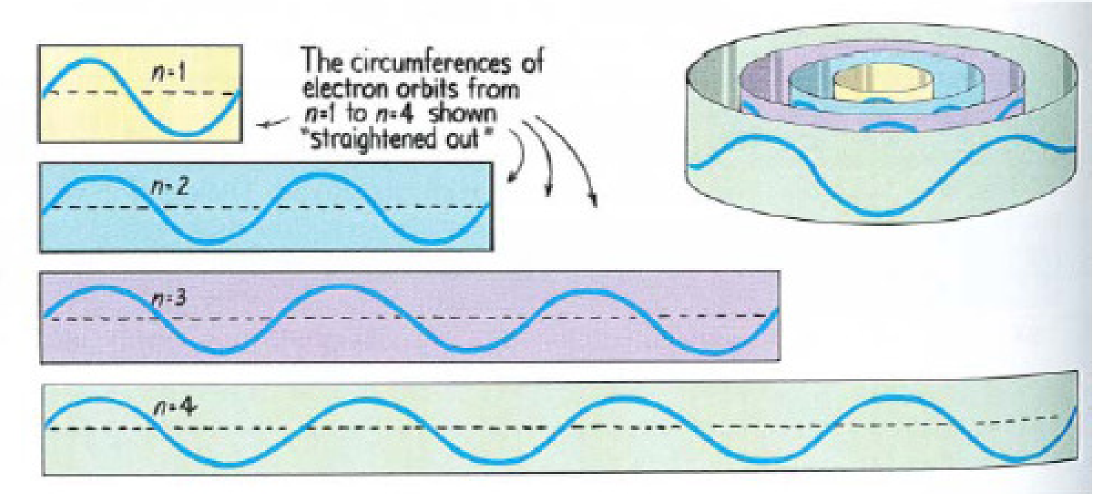
unstable

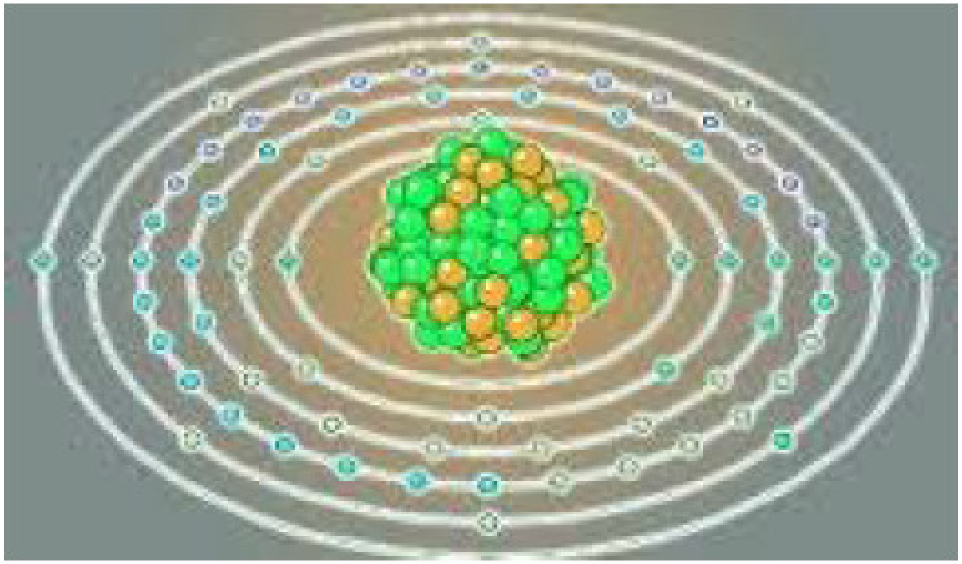


Stable when
Circumference = $2\pi r_n = n\lambda$



Explains Allowed Orbits for Electrons in Atoms





Wave Mechanics Leads to Understanding of Atomic Structure – ~1926 to ~1940

- In 1926, Schrödinger extends Bohr's theory in a significant way.
- The field of quantum wave mechanics describes the behavior of electrons at the nanometer level in terms of particle wave functions predicted by Schrödinger's theory.
- A particle's matter wave (now called the wave function) provides a mathematical description related to the **likelihood** of finding the electron at various locations at a specific time.
- Quantum wave mechanics was rapidly applied to chemistry where it was known for over 50 years that many elementary substances (the chemical elements) showed unique and predictable chemical behavior.
- Focus on pure chemical substances made from identical tiny particles called atoms (Greek for "without division").
- How are chemical elements organized?

Arrange the Elements by Mass

- Dalton's contribution (1804) -

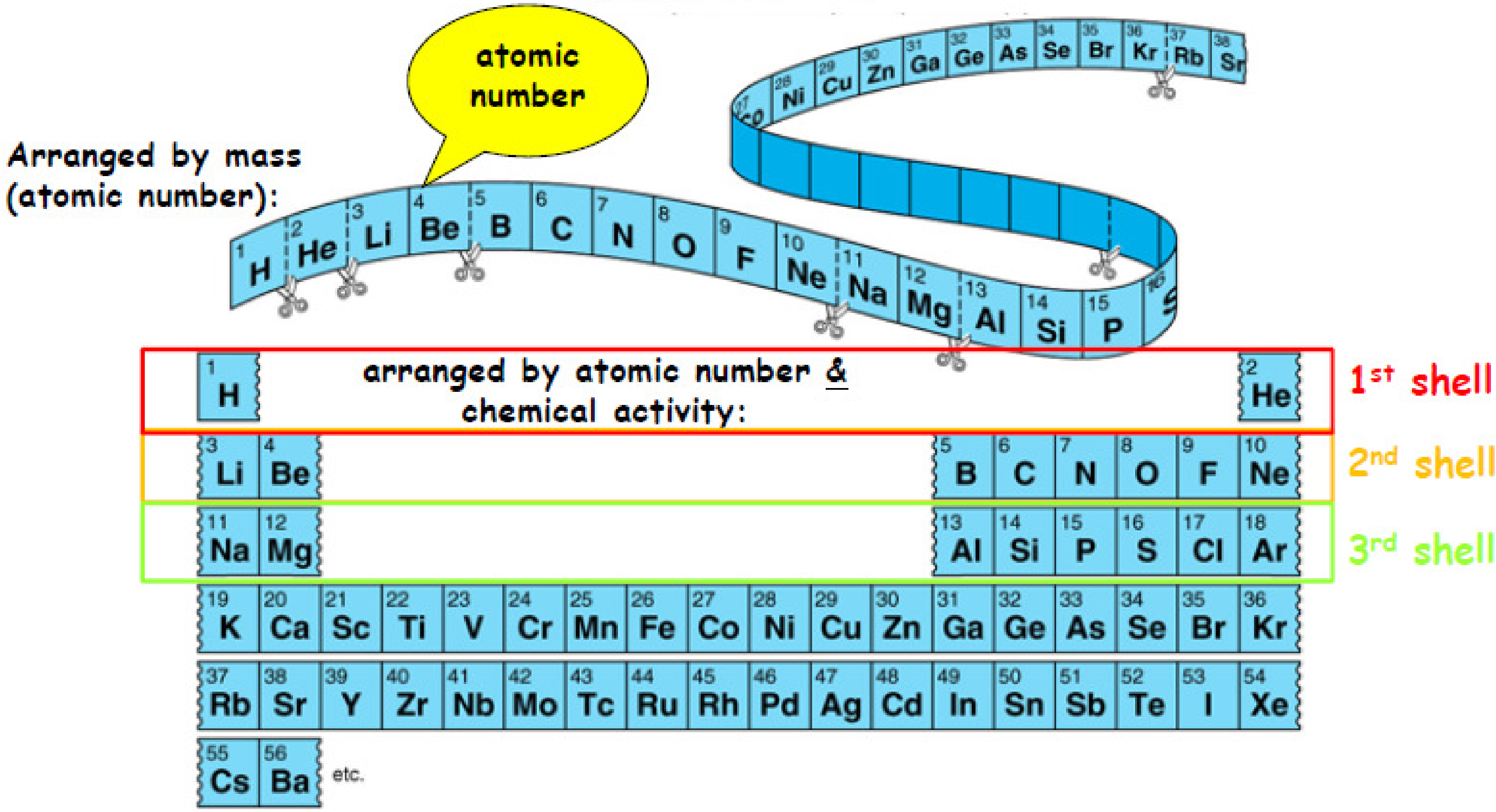
light

heavy

- Hydrogen (H)
- Helium (He)
- Lithium (Li)
- Beryllium (Be)
- Boron (B)
- Calcium (Ca)
- etc.

Periodic Table Sorted by Chemical "Activity"

Mendeleev 1869

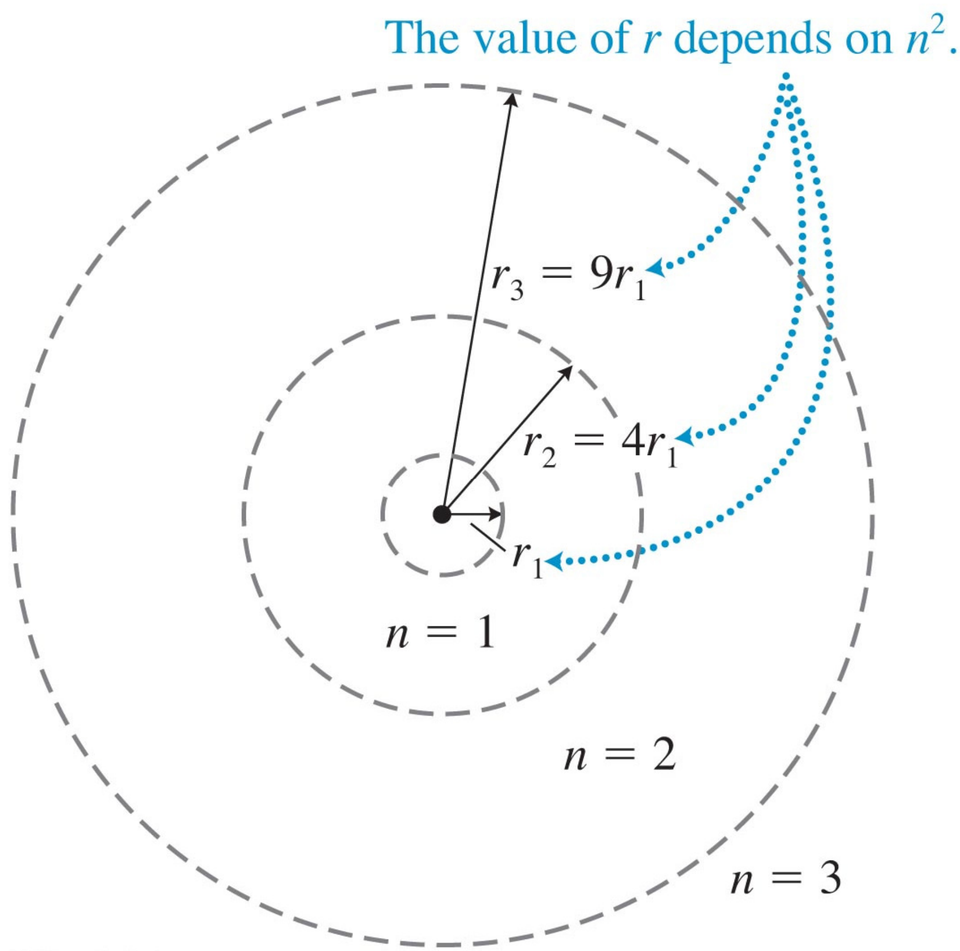


Why does this classification scheme make sense?

Atomic Quantum Numbers

Principal quantum number, n.

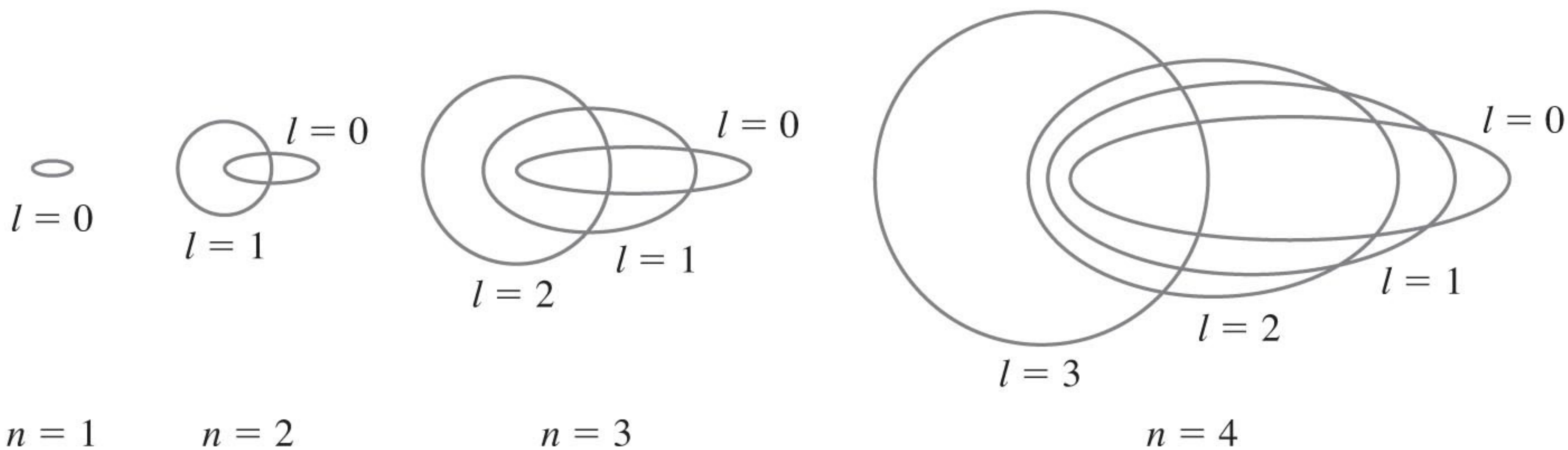
Mainly determines the energies of bound electrons.



Atomic Quantum Numbers

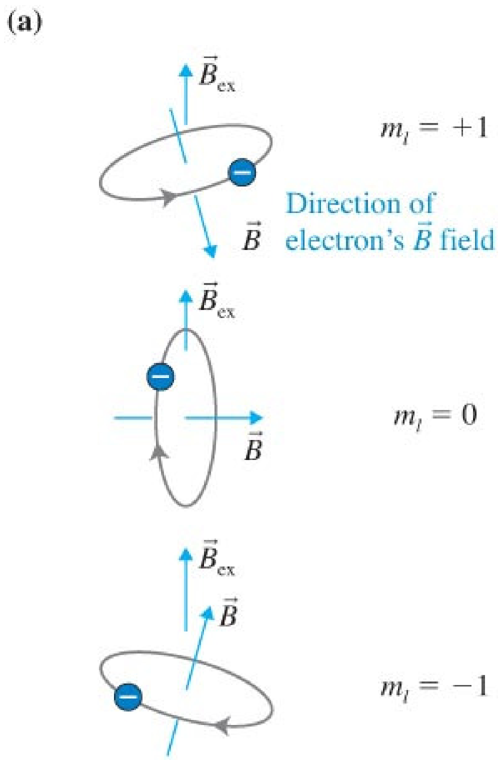
- Sommerfeld extended the Bohr model to account for quantized angular momentum
- A new quantum number, ℓ, known as the orbital quantum number, identifies the orbital angular momentum of a state.

$$0 \leq \ell \leq n - 1$$



Atomic Quantum Numbers

- A third quantum number, the magnetic quantum number, m_ℓ, is related to the orientation of the angular momentum vector



$$L_z = m_\ell \frac{h}{2\pi}$$

$$-\ell \leq m_\ell \leq \ell$$

Atomic Quantum Numbers

- Electrons have an *intrinsic* angular momentum called "spin" which can have two possible values:

$$m_s = \pm\frac{1}{2}$$

- Although electrons are point-like particles, they behave like little bar magnets.
- This property has no analogous concept in classical mechanics.

Summary of Quantum Numbers

- Principal quantum number:

$$n = 1, 2, 3, \ldots$$

- Orbital angular momentum quantum number:

$$l = 0, 1, 2, \ldots, n - 1$$

- Magnetic quantum number:

$$m_l = 0, \pm 1, \pm 2, \ldots, \pm l$$

- Spin magnetic quantum number:

$$m_s = \pm \frac{1}{2}$$

Pauli's Exclusion Principle

- No two electrons can have the same set of quantum numbers.
- Each electron has a unique set of n, ℓ, m_ℓ, m_s
- Example: The number of states and the quantum number designation of each state for the $\ell = 2$ subshell:

Table 27.7 **The number of states and the quantum number designation of each state for the 3*d* subshell.**

n	l	m_l	m_s
3	2	−2	+1/2
3	2	−1	+1/2
3	2	0	+1/2
3	2	1	+1/2
3	2	2	+1/2
3	2	−2	−1/2
3	2	−1	−1/2
3	2	0	−1/2
3	2	1	−1/2
3	2	2	−1/2

Atomic subshells from lowest to highest energy (approximate)

Table 27.8 Atomic subshells from lowest to highest energy (approximate).

Quantum Numbers			Number of Quantum States	
n	l	m_l	In the Subshell	Total
1	0 (s)	0	2	2
2	0 (s)	0	2	
	1 (p)	−1, 0, +1	6	8
3	0 (s)	0	2	
	1 (p)	−1, 0, +1	6	18
	2 (d)	−2, −1, 0, +1, +2	10	
4	0 (s)	0	2	
	1 (p)	−1, 0, +1	6	32
	2 (d)	−2, −1, 0, +1, +2	10	
	3 (f)	−3, −2, −1, 0, +1, +2, +3	14	

Table 27.9 Periodic table of the elements*

I	II											III	IV	V	VI	VII	0
1 H 1.0080																	2 He 4.0026
3 Li 6.941	4 Be 9.0122											5 B 10.81	6 C 12.011	7 N 14.0067	8 O 15.9994	9 F 18.9984	10 Ne 20.179
11 Na 22.9898	12 Mg 24.305	Transition elements										13 Al 26.9815	14 Si 28.086	15 P 30.9738	16 S 32.06	17 Cl 35.453	18 Ar 39.948
19 K 39.102	20 Ca 40.08	21 Sc 44.956	22 Ti 47.90	23 V 50.941	24 Cr 51.996	25 Mn 54.9380	26 Fe 55.847	27 Co 58.9332	28 Ni 58.71	29 Cu 63.54	30 Zn 65.37	31 Ga 69.72	32 Ge 72.59	33 As 74.9216	34 Se 78.96	35 Br 79.909	36 Kr 83.80
37 Rb 85.467	38 Sr 87.62	39 Y 88.906	40 Zr 91.22	41 Nb 92.906	42 Mo 95.94	43 Tc (99)	44 Ru 101.07	45 Rh 102.906	46 Pd 106.4	47 Ag 107.870	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.9045	54 Xe 131.30
55 Cs 132.906	56 Ba 137.34	57 La 138.906	72 Hf 178.49	73 Ta 180.948	74 W 183.85	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.09	79 Au 196.967	80 Hg 200.59	81 Tl 204.37	82 Pb 207.2	83 Bi 208.981	84 Po (210)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra 226.03	89 Ac 227.028	104 Rf (261)	105 Db (262)	106 Sg (266)	107 Bh (264)	108 Hs (269)	109 Mt (268)	110 Ds (271)	111 Rg (272)	112 Cn (285)		114 Fl (289)		116 Lv (293)		
Lanthanide series				58 Ce 140.12	59 Pr 140.908	60 Nd 144.24	61 Pm (147)	62 Sm 150.4	63 Eu 151.96	64 Gd 157.25	65 Tb 158.925	66 Dy 162.50	67 Ho 164.930	68 Er 167.26	69 Tm 168.934	70 Yb 173.04	71 Lu 174.97
Actinide series				90 Th 232.038	91 Pa 231.036	92 U 238.029	93 Np 237.048	94 Pu (242)	95 Am (243)	96 Cm (248)	97 Bk (249)	98 Cf (249)	99 Es (254)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (260)

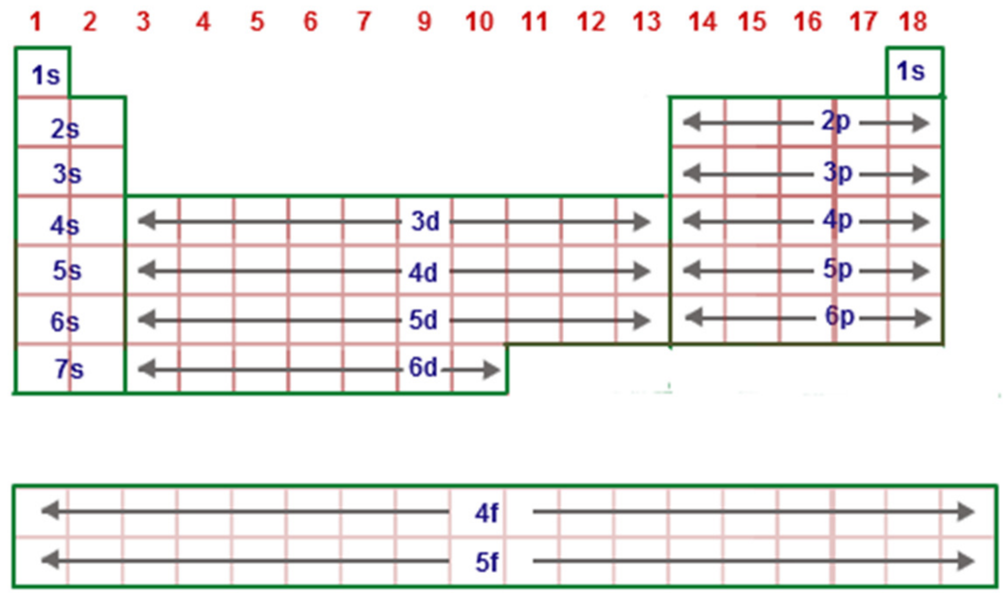
*Atomic weights of stable elements are those adopted in 1969 by the International Union of Pure and Applied Chemistry. For those elements having no stable isotope, the mass number of the "most stable" isotope is given in parentheses.

2 in the top row
8 in the second row
8 in the third row
18 in the fourth row
…

Atomic Electron Configurations

Electron shell configurations:

Element	Z	1st shell	2nd shell	3rd shell	4th shell
hydrogen	1	1			
helium	2	2			
lithium	3	2	1		
beryllium	4	2	2		
boron	5	2	3		
carbon	6	2	4		
nitrogen	7	2	5		
oxygen	8	2	6		
fluorine	9	2	7		
neon	10	2	8		
sodium	11	2	8	1	
magnesium	12	2	8	2	
aluminium	13	2	8	3	
silicon	14	2	8	4	
phosphorus	15	2	8	5	
sulphur	16	2	8	6	
chlorine	17	2	8	7	
argon	18	2	8	8	
potassium	19	2	8	8	1
calcium	20	2	8	8	2



TWO Fundamental NEW lessons of quantum physics

1. Whenever matter is confined to a very small space, the allowed values of certain properties become severely restricted or quantized.

> Example: Energy of an electron in free space
> – no restrictions apply
>
> Energy of an electron when bound to an atom
> – quantized energy restrictions apply

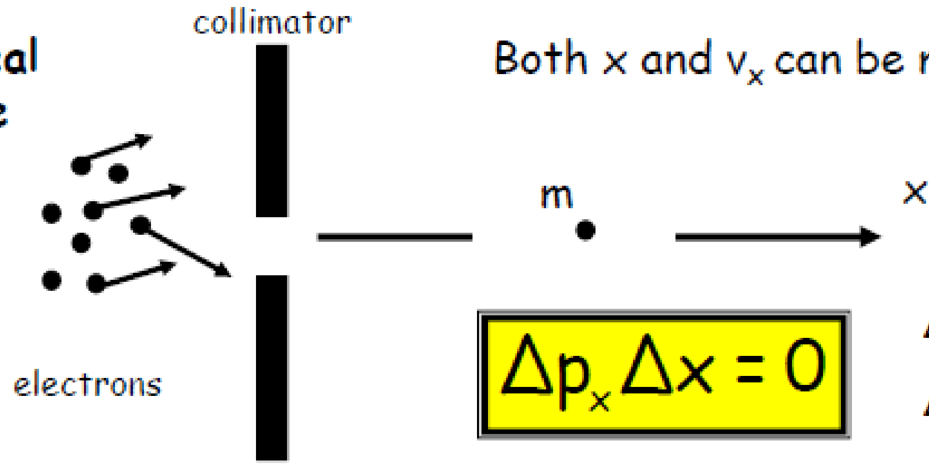
2. *Corollary*: The simultaneous measurement of two closely related quantities with high precision becomes impossible – probabilistic models are required.

> These principles apply to many quantities like angular momentum, vibrational energies of atoms/molecules, etc.

IF a particle behaves like a wave, it will be difficult to specify its position exactly

Heisenberg's Uncertainty Principle

Classical Picture



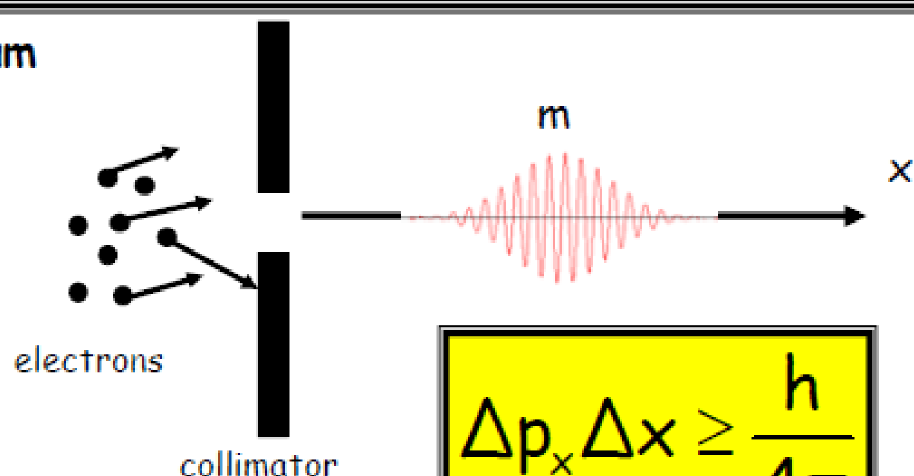
Both x and v_x can be measured exactly

$$\Delta p_x \Delta x = 0$$

Δp_x means uncertainty in p_x

Δx means uncertainty in x

Quantum Picture



$$\Delta p_x \Delta x \geq \frac{h}{4\pi}$$

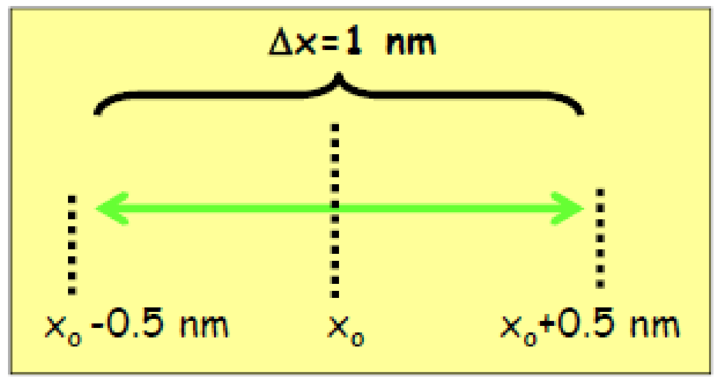
Specifies the **intrinsic** uncertainties in the **simultaneous** determination of position and momentum

EXAMPLE: A hydrogen gas molecule (H_2) at room temperature has an **average** velocity of about 1920 m/s. If the molecule (made from 2 H atoms) is localized to some position $\mathbf{x_o}$ within ±0.5 nm, what is the intrinsic uncertainty in its velocity?

$$m_{H_2} = 2m_p + 2m_e = 2(1.67\times10^{-27}\,\text{kg}) + 2(9.11\times10^{-31}\,\text{kg})$$

$$= 3.34\times10^{-27}\,\text{kg}$$

$$v_x = 1920\ {}^{m}\!/_{s} \quad \text{(given, but not needed)}$$



$$\Delta p_x \Delta x \geq \frac{h}{4\pi}$$

$$\Delta p_x \geq \frac{h}{4\pi}\frac{1}{\Delta x}$$

$$m_{H_2}\Delta v_x \geq \frac{h}{4\pi}\frac{1}{\Delta x} \Rightarrow \Delta v_x \geq \frac{h}{4\pi}\frac{1}{\Delta x}\frac{1}{m_{H_2}}$$

$$\Delta v_x \geq \frac{6.626\times10^{-34}\,\text{Js}}{4\pi}\cdot\frac{1}{1.0\times10^{-9}\,\text{m}}\cdot\frac{1}{3.34\times10^{-27}\,\text{kg}}$$

$$\Delta v_x \geq 16\ \text{m/s}$$

$$\textbf{as } \Delta x \to 0, \quad \Delta v_x \to \infty$$

Heisenberg Uncertainty Principle

The Heisenberg Uncertainty Principle is NOT a statement about the inaccuracy of measurement instruments, nor a reflection on the quality of experimental methods. Rather, it arises from the wave properties inherent in the quantum mechanical description of nature. Even with perfect instruments and technique, the uncertainty is inherent in the nature of things.