Purdue University Department of Physics

Physics 21900

General Physics II

Electricity, Magnetism and Optics

Lecture 15 – Chapter 18.9-10

AC Voltage and Transformers

Fall 2015 Semester

Prof. Matthew Jones

Announcement

Exam #2 will be on November 5th
in Phys 112 at 8:00 pm

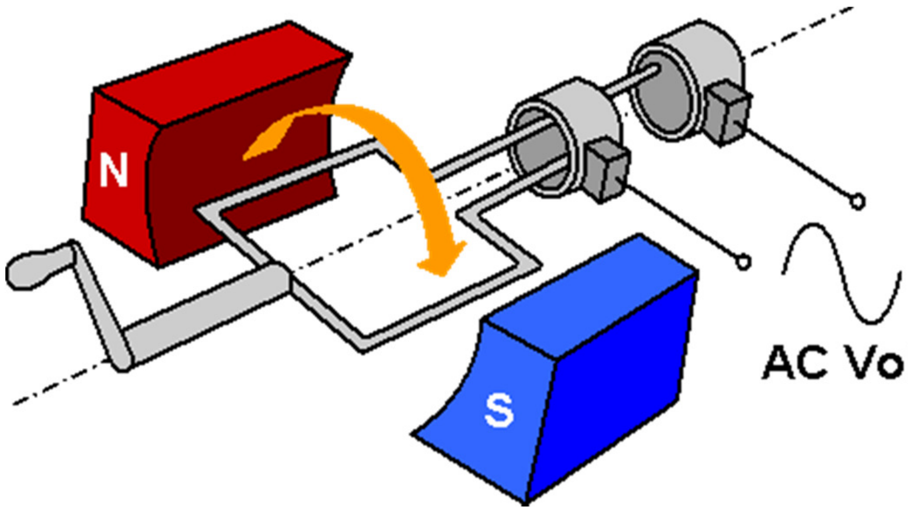
Electric current, DC circuits, Kirchhoff's Rules

Magnetic Fields, Lorentz Force, Forces on Currents

Ampere's Law, Magnetic Induction, Lenz's Law

Induced EMF, AC Voltage, Transformers

AC Voltage Generators

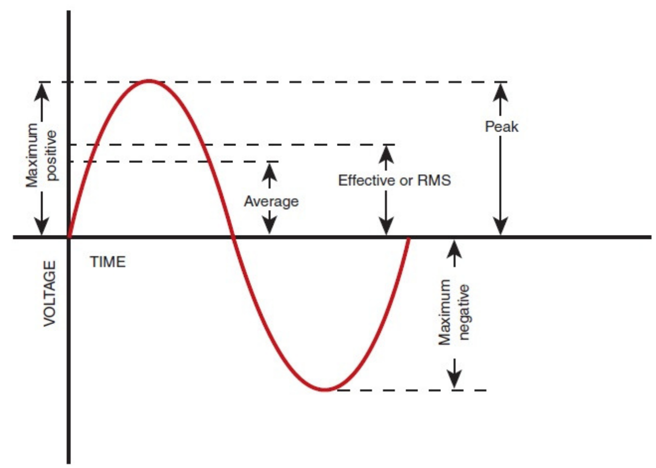


Faraday's law: $\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$

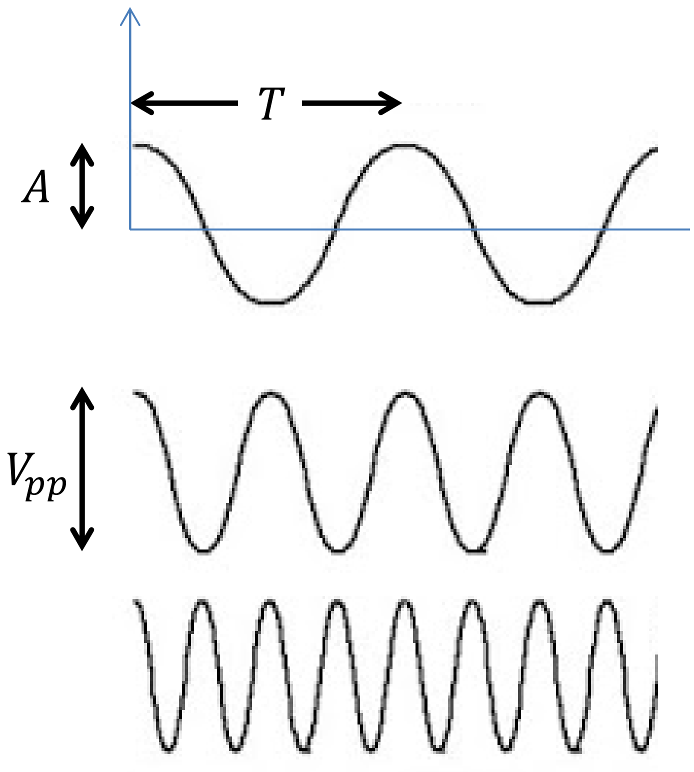
$$\Phi_B = NAB\cos\theta$$
$$= NAB\ \cos(\omega t)$$

AC Voltage, $\mathcal{E} = V_{max}\ sin(\omega t)$

$$V_{max} = NAB\omega$$
$$\omega = 2\pi f$$



Alternating Current/Sine Waves



Period: T (seconds)

Frequency: $f = 1/T$ (Hertz, s^{-1})

Angular frequency: $\omega = 2\pi f$ (s^{-1})

Amplitude: $A = V_{max}$ or I_{max}

$$V(t) = V_{max} \sin \omega t$$

or $V(t) = V_{max} \cos \omega t$

Sometimes we refer to the peak-to-peak amplitude, V_{pp}.

The functions $\sin \omega t$ and $\cos \omega t$ look the same – just shifted in time by 90 degrees.

Alternating Current/Sine Waves

- Example:
  - In North America, the AC line voltage has a frequency of 60 Hz.
  - The angular frequency is

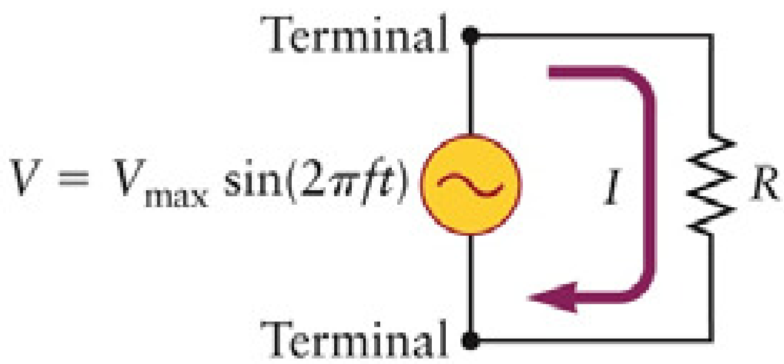
$$\omega = 2\pi f$$

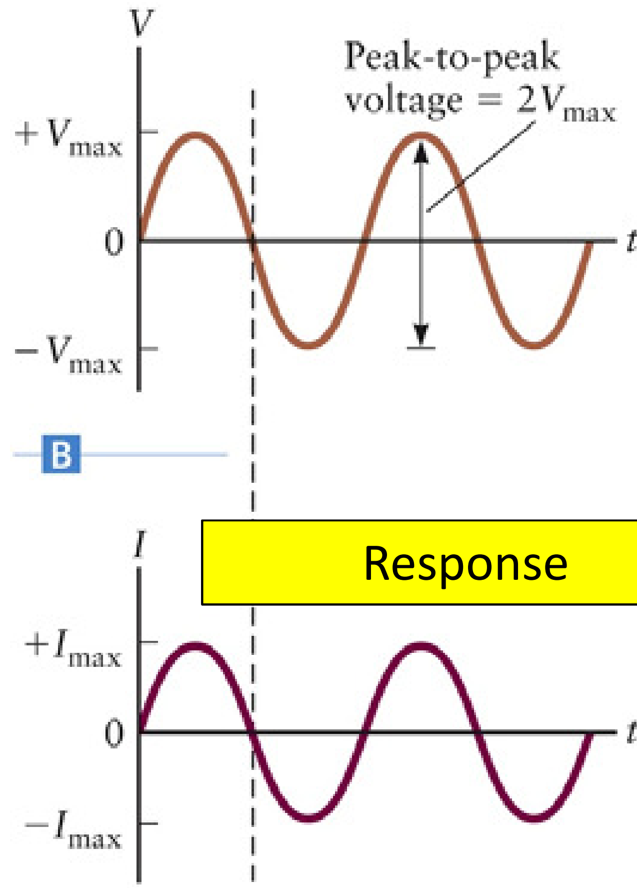
$$= 2 \times 3.141592654 \times (60\ s^{-1})$$

$$= 377 \text{ radians/second}$$

AC Voltage/AC Current

Applied Voltage





Response

Ohm's Law: $I = V/R$

$$V(t) = V_{max} \sin(2\pi f t)$$

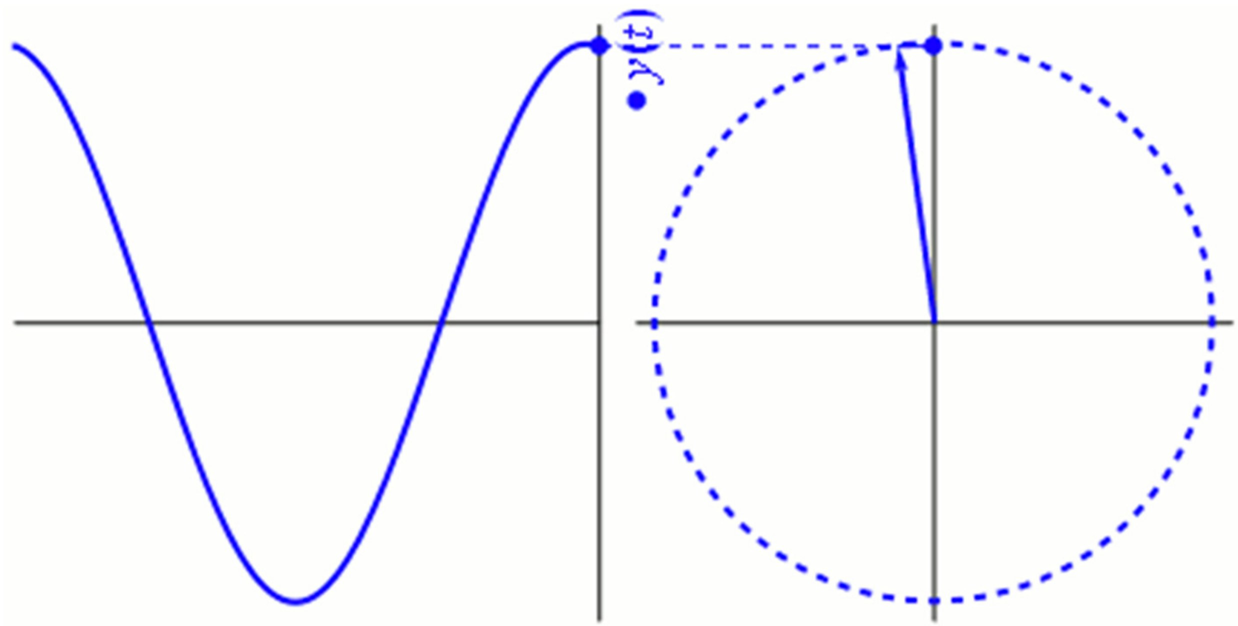
$$I(t) = I_{max} \sin(2\pi f t)$$

$$I_{max} = V_{max}/R$$

$I(t)$ is in phase with $V(t)$

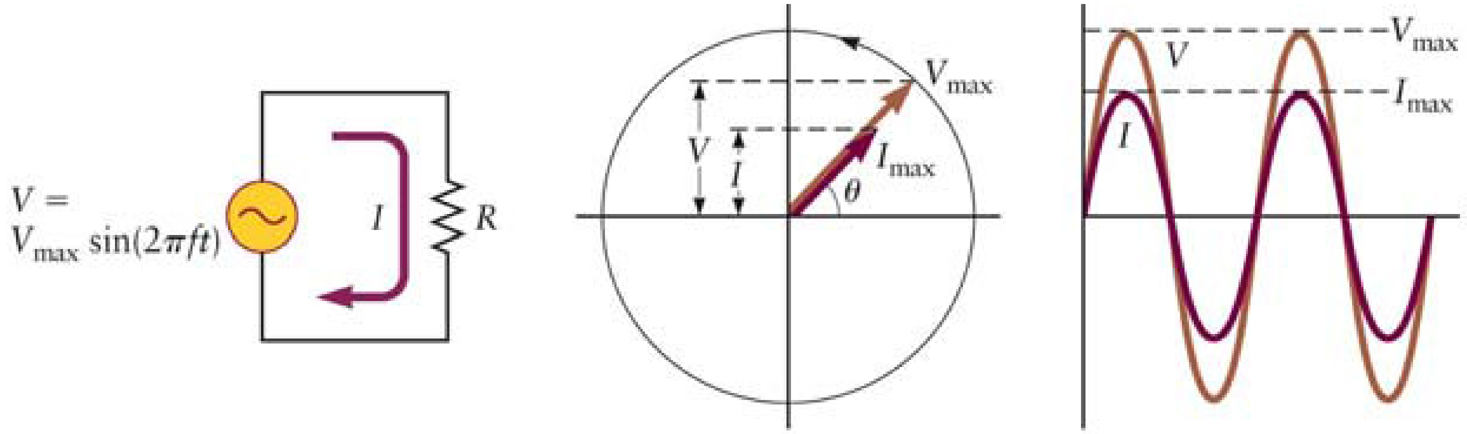
Phasor Diagrams

- A rotating vector is one way to represent a sine wave.



Phasor Diagrams

- These diagrams show the relation between the phase of the current and the voltage:

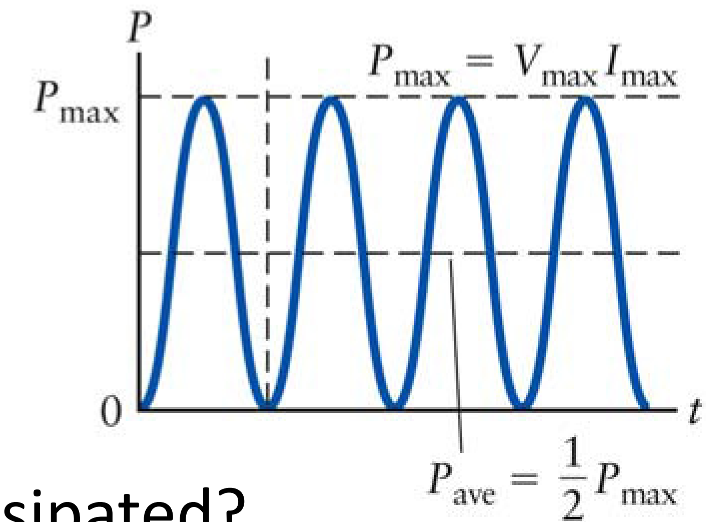


- Both the voltage and current vectors rotate with constant angular velocity, $\omega = 2\pi f$.
- The vectors maintain their relative orientation.
- For a circuit with only resistors, the current and voltage have the same phase.

AC Power

- How much power is dissipated in the resistor?

$$P = IV = I_{max}V_{max}\sin^2(2\pi f t)$$



- What is the average power dissipated?

$$P_{avg} = \frac{1}{2}P_{max}$$

$$= \left(\frac{I_{max}}{\sqrt{2}}\right)\left(\frac{V_{max}}{\sqrt{2}}\right) = I_{rms}V_{rms}$$

AC Power

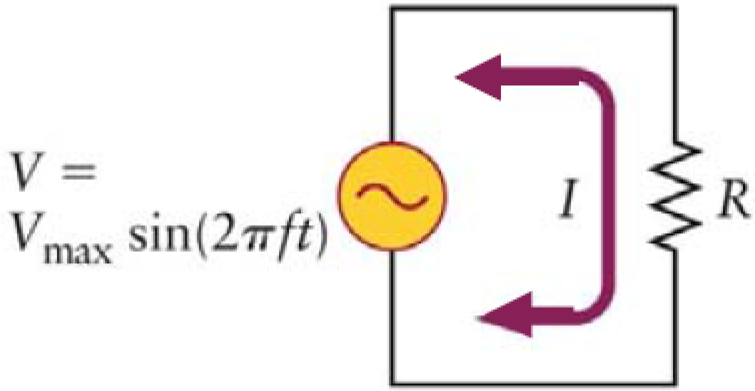
- For sine or cosine waves, the RMS voltage or current is just

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} \qquad V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

- Expressed using RMS voltage and current, Ohm's law can be used to express the average power:

$$P_{avg} = \frac{V_{rms}^2}{R} = I_{rms}^2 \, R$$

Example: ac circuit analysis

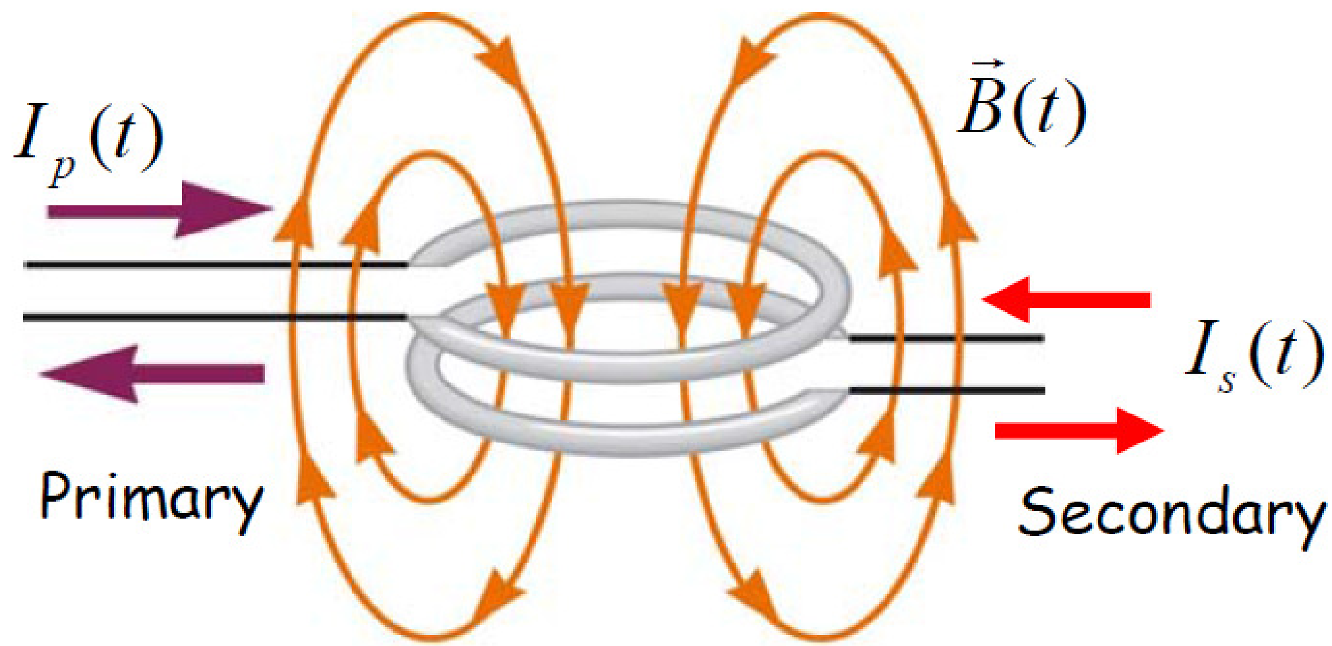


V_{max} = 170 V
f=60 Hz
R=20 Ω

What are V_{rms}, I_{rms}, and average power dissipated in load?
What are V_{max}, I_{max}, and maximum power dissipated in load?

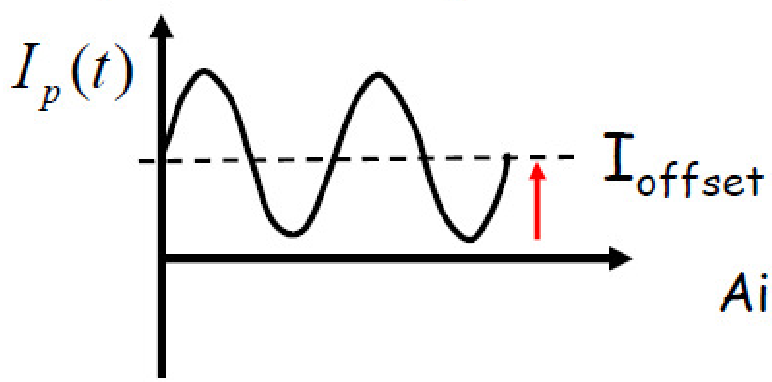
Average values	Maximum values
$V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{170\,V}{1.414} = 120\,V$	$V_{max} = 170\,V$
$I_{rms} = \frac{V_{rms}}{R} = \frac{120\,V}{20\,\Omega} = 6\,A$	$I_{max} = \frac{V_{max}}{R} = \frac{170\,V}{20\,\Omega} = 8.5\,A$
$P_{ave} = I_{rms}V_{rms} = (6\,A)(120\,V) = 720\,W$	$P_{max} = I_{max}V_{max} = (8.5\,A)(170\,V) = 1445\,W$

An ac isolator



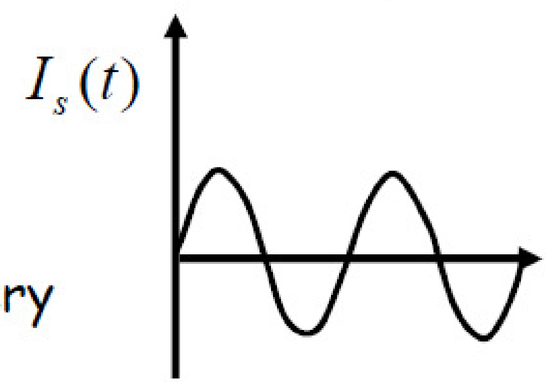
The Secondary is isolated from the Primary

Input: ac +dc signal



Air coupling not very efficient

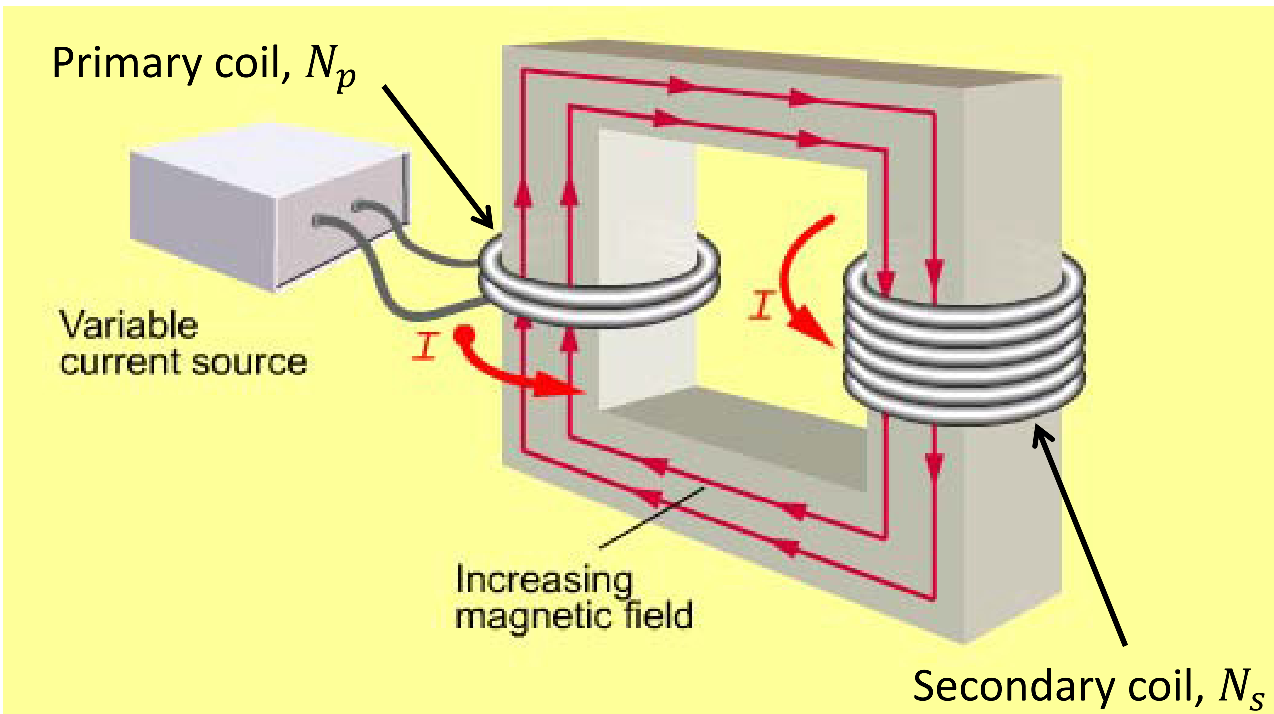
Output: ac signal ONLY



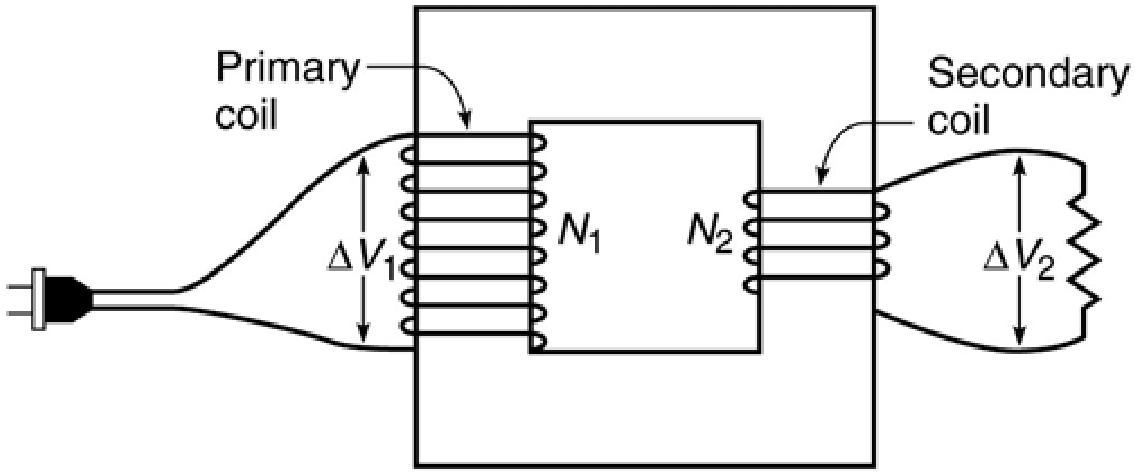
A Practical Transformer

- Magnetic flux is trapped by an iron core.
- The magnetic flux through each coil is the same.

$$\Phi_B(t) \propto N_p I_p(t) = N_s I_s(t)$$



Transformers

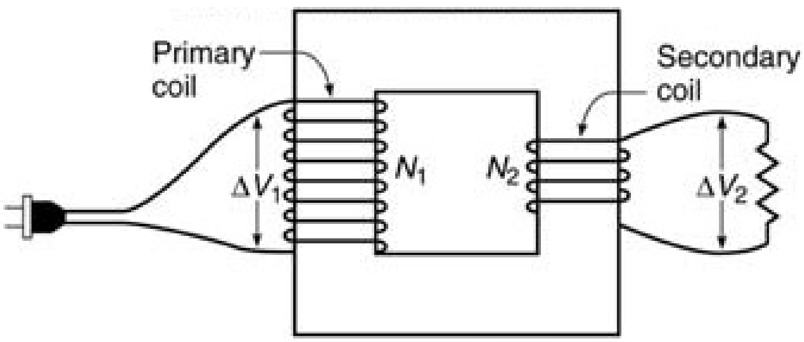


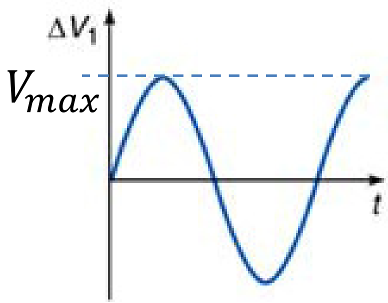
- The input voltage must vary with time so that $\Delta\Phi_B/\Delta t \neq 0$
- Energy must be conserved: $P_p = P_s$

$$I_p \Delta V_1 = I_s \Delta V_2$$

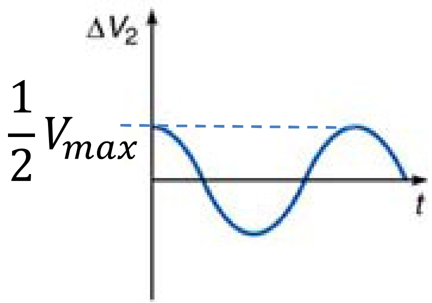
$$\frac{\Delta V_2}{\Delta V_1} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

Examples

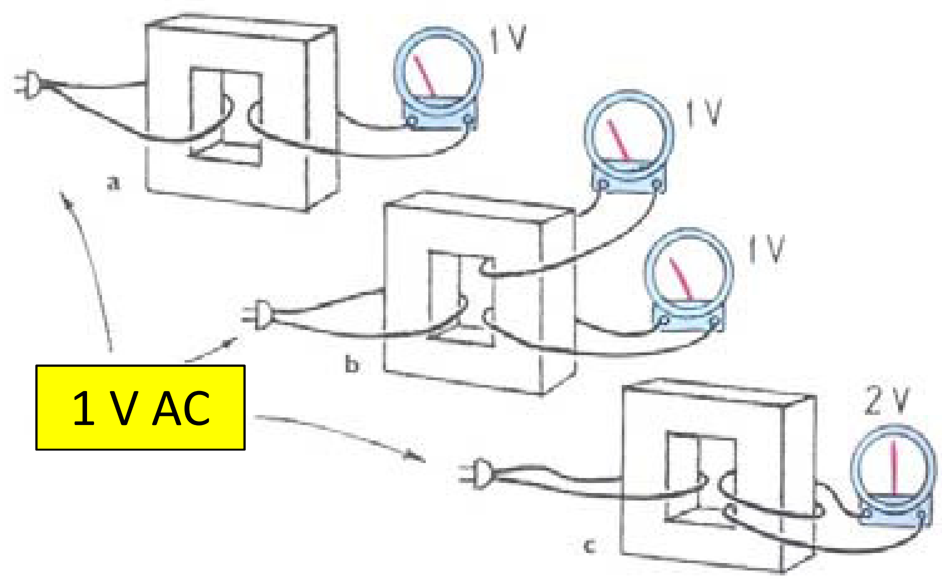




Primary
8 turns



Secondary
4 turns



EXAMPLE

A transformer has 10 turns on the primary and 100 turns on the secondary. If 110V rms is applied to the primary, what is the **peak-to-peak** voltage across the secondary?

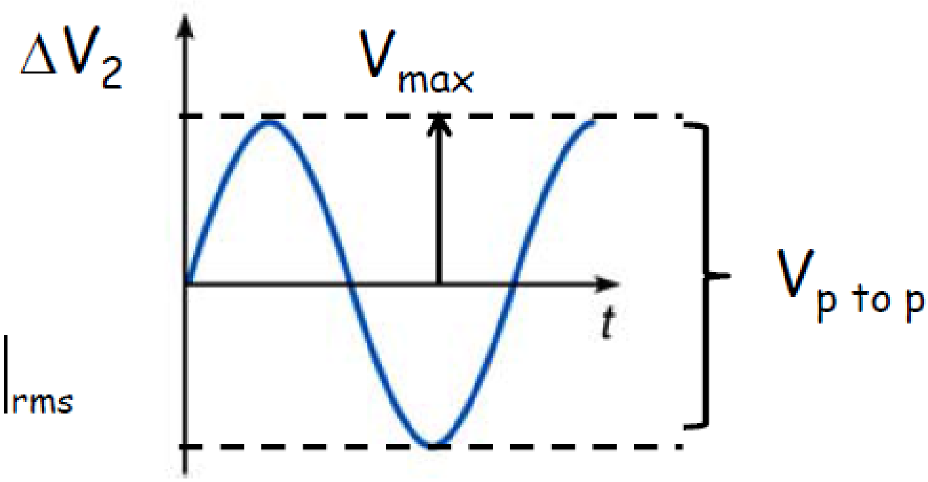
$$\frac{\Delta V_1}{\Delta V_2} = \frac{N_1}{N_2}$$

$$\frac{110\,V|_{rms}}{\Delta V_2|_{rms}} = \frac{10}{100}$$

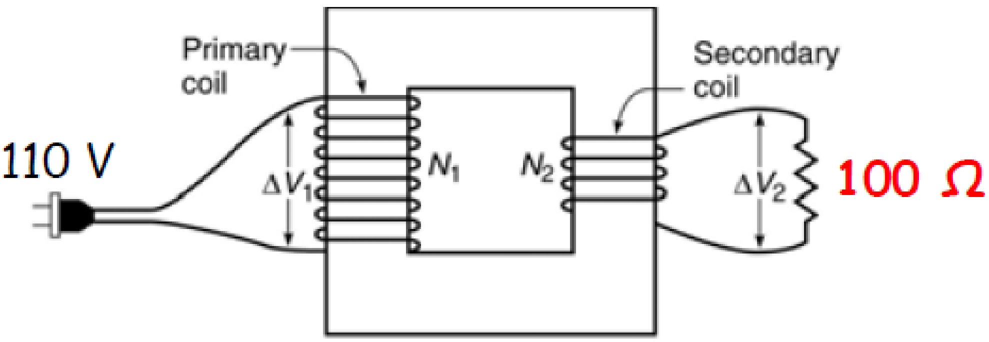
$$\Delta V_2|_{rms} = 10\left(110\,V|_{rms}\right) = 1100\,V|_{rms}$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} \quad \Rightarrow \quad V_{max} = 1555V$$

$$V_{p\ to\ p} = 2V_{max} = 3111\,V$$



If a 100 Ω load is connected to the secondary, what is the rms current in the primary?



Primary
10 turns

Secondary
100 turns

$$\frac{\Delta V_1}{\Delta V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$\frac{110\,V|_{rms}}{\Delta V_2|_{rms}} = \frac{10}{100}$$

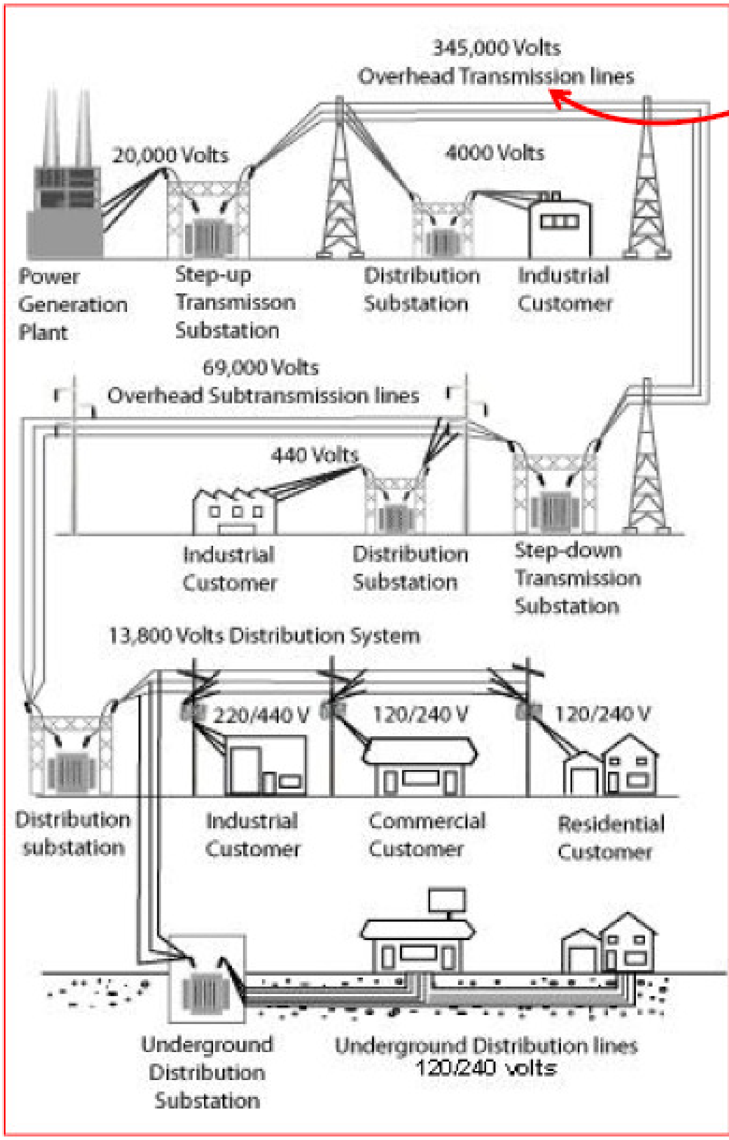
$$\Delta V_2|_{rms} = 10\left(110\,V|_{rms}\right) = 1100\,V|_{rms}$$

$$I_2|_{rms} = \frac{\Delta V_2|_{rms}}{R_{load}} = \frac{1100\,V|_{rms}}{100\,\Omega} = 11\,A|_{rms}$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1} \Rightarrow \frac{10}{100} = \frac{11\,A}{I_1}$$

$$I_1|_{rms} = \left(\frac{100}{10}\right) 11\,A = 110\,A$$

APPENDIX: Transformers in the US Power Grid



Why is power transmitted at such high voltages?

$V_{rms} = 345{,}000\,V$, $I_{rms} = 1000\,A$ } Typical values

$$P_{Transmitted} = V_{rms} I_{rms} = (345{,}000\,V)(1000\,A) = 345\,MW$$

SUPPOSE 10% of power is lost

$$P_{Lost} = 10\%\,P_{Transmitted} = 34.5\,MW$$

$$P_{Lost} = I^2_{rms} R_{line}$$

$$R_{line} = 34.5\,\Omega$$

Now suppose V_{rms} is decreased by 9/10 and I_{rms} is increased by 10/9, so the average power transmitted remains the same:

$$V_{rms} = \frac{9}{10} \times (345{,}000\,V) = 310{,}500\ V$$

$$I_{rms} = \frac{10}{9} \times (1000\,A) = 1111\,A$$

$$P_{Transmitted} = V_{rms} I_{rms} = (310{,}500\ V)(1111\,A)$$

$$= 345\,MW \quad \text{(same as before)}$$

How much power is now lost?

$$R_{line} = 34.5\,\Omega \quad \text{(same as before)}$$

$$P_{Lost} = I^2_{rms} R_{line} = (1111\,A)^2 (34.5\,\Omega) = 42.6\,MW$$

$$\frac{P_{Lost}}{P_{Transmitted}} = \frac{42.6\,MW}{345\,MW} = \text{power lost increases by } 12.3\%$$

http://adogreen.com/blog/power-generation/overview-from-power-plant-to-consumer/