PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 21900

General Physics II

Electricity, Magnetism and Optics

Lecture 14 – Chapter 18.4-8

Induced EMF

Fall 2015 Semester

Prof. Matthew Jones

Announcement

Exam #2 will be on November 5th

in Phys 112 at 8:00 pm

Electric current, DC circuits, Kirchhoff's Rules

Magnetic Fields, Lorentz Force, Forces on Currents

Ampere's Law, Magnetic Induction, Lenz's Law

Induced EMF, AC Voltage, Transformers

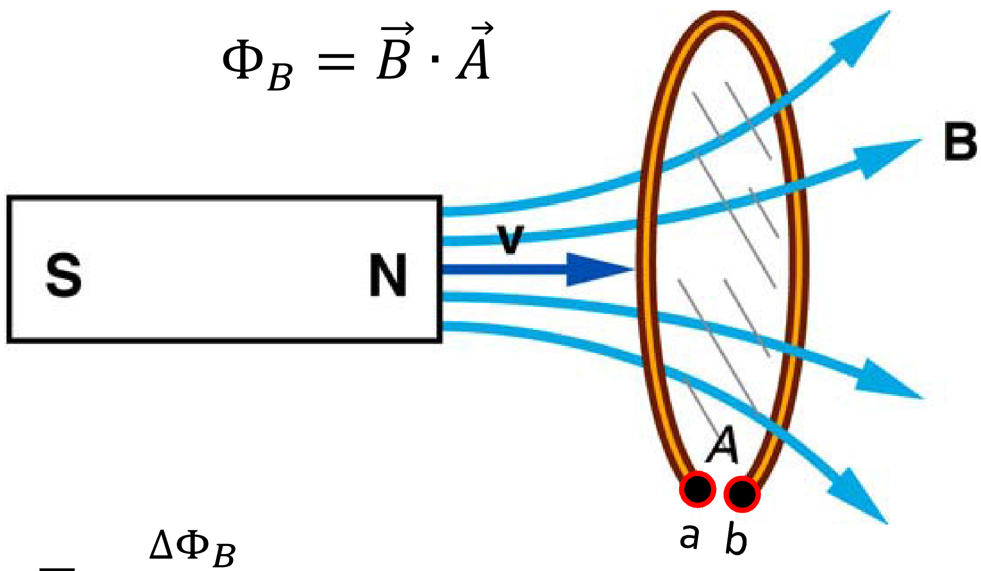
Faraday's Law – Electromagnetic Induction

- Faraday described many magnetic effects on circuits in terms of magnetic flux:

$$\Phi_B = \sum \vec{B} \cdot \Delta\vec{A}$$

- For a constant $\vec{B}$ field:

$$\Phi_B = \vec{B} \cdot \vec{A}$$

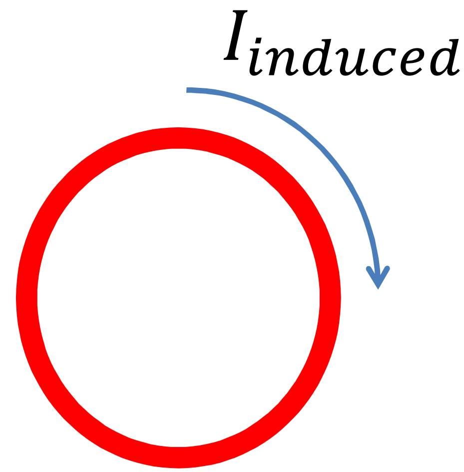


- Faraday's law: $\mathcal{E}_{ab} = -\frac{\Delta\Phi_B}{\Delta t}$
- The minus sign is determined using ***Lenz's Law***…

Faraday's Law

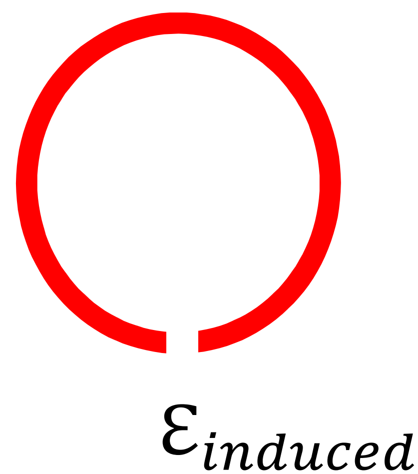
A changing magnetic field through a loop (ie, a changing magnetic flux) will…

i. Cause a current to flow in the direction such that the induced magnetic field opposes the change in flux

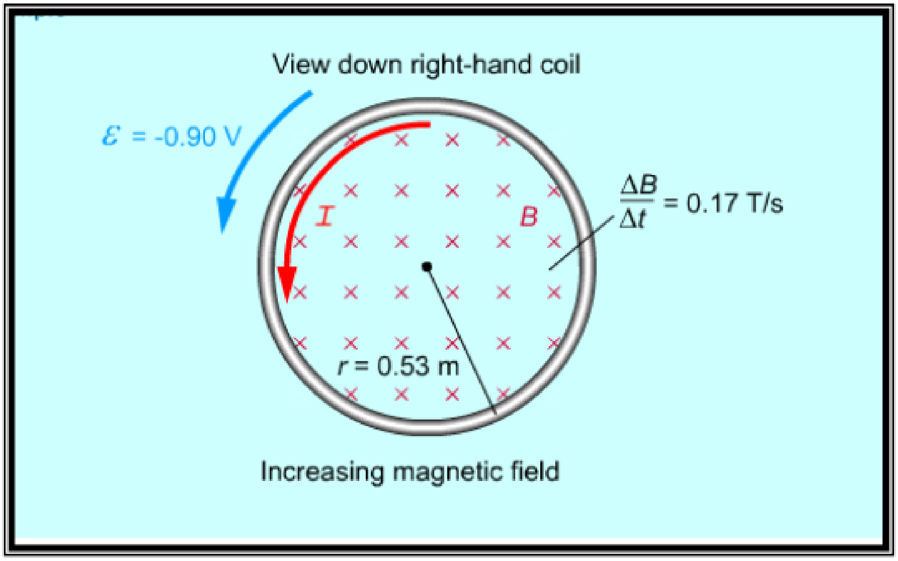


OR

ii. Produce an induced voltage with a polarity that would cause a current to flow and induce a magnetic field that opposes the change in flux.



Example I: What is the induced emf if the magnetic field through a **six** turn coil increases at a rate of 0.17 T/s?



$$\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t}$$

The negative sign indicates that the induced emf acts to "oppose" the change in magnetic flux that causes it

$$|\mathcal{E}| = \frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta(\vec{B}\bullet\vec{A})}{\Delta t} = \frac{\Delta(BA\cos\theta)}{\Delta t}\Bigg|_{\theta=0}$$

$$= A\frac{\Delta(B)}{\Delta t}$$

$$A = \pi R^2 = \pi(0.53\,m)^2 = 0.88\,m^2$$

$$\frac{\Delta B}{\Delta t} = +0.17\ T/s \quad [\text{given}]$$

$$|\mathcal{E}| = (0.88\,m^2)(0.17\ T/s) = 0.15\ V$$

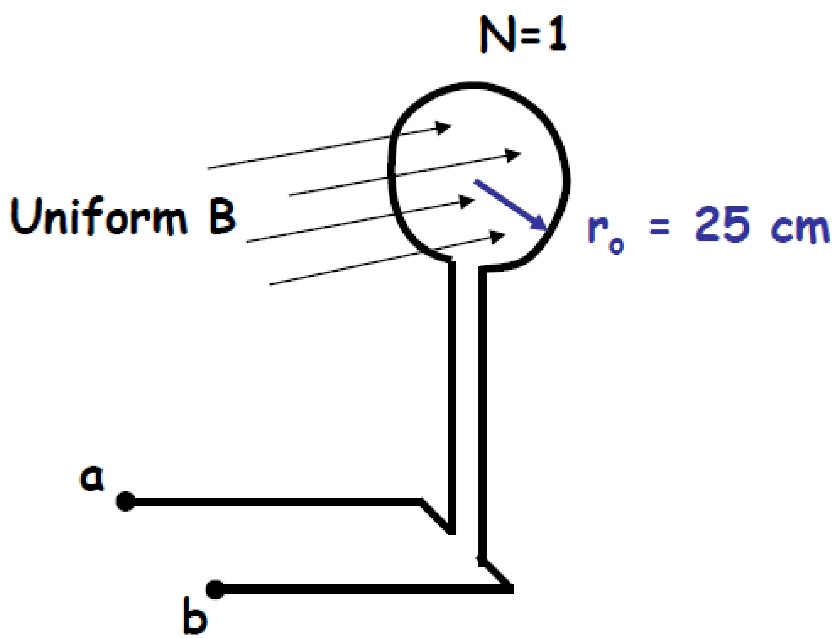
$$\text{Since coil has six turns},\ E = 6\times(0.15\ V) = 0.90\ V$$

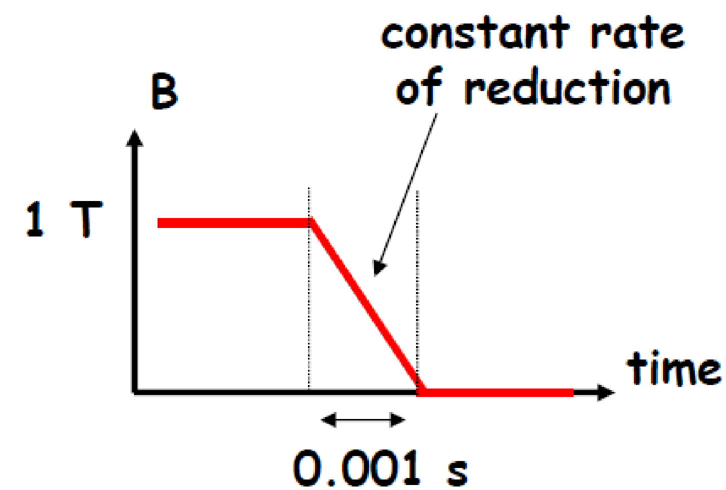
It is often easier to take the absolute value of Farady's Law to find the **magnitude** of the induced emf and then use Lenz's Law to find the **direction** of the induced current that results.

V_{ab} = 0.9 V

II. Induced Voltage: a simple example

What is V_a-V_b?



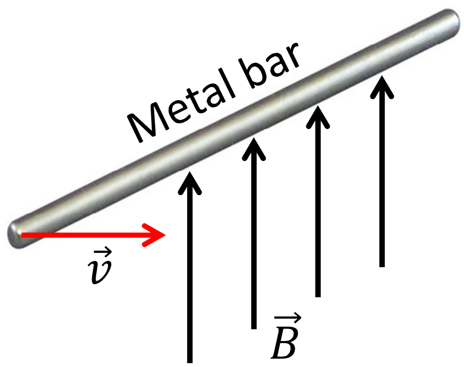


$$|\mathcal{E}| = N\left|\frac{\Delta\Phi_B}{\Delta t}\right| = NA\left|\frac{\Delta B}{\Delta t}\right| = 1\times\left(\pi(0.25)^2\right)\left|\frac{0-1\,T}{0.001\,s}\right|$$

$$= \left(0.196\,m^2\right)\left(1000\ {}^{T}\!/\!_{s}\right) = 196\,V$$

Lenz's law: $V_a > V_b$

Motional EMF

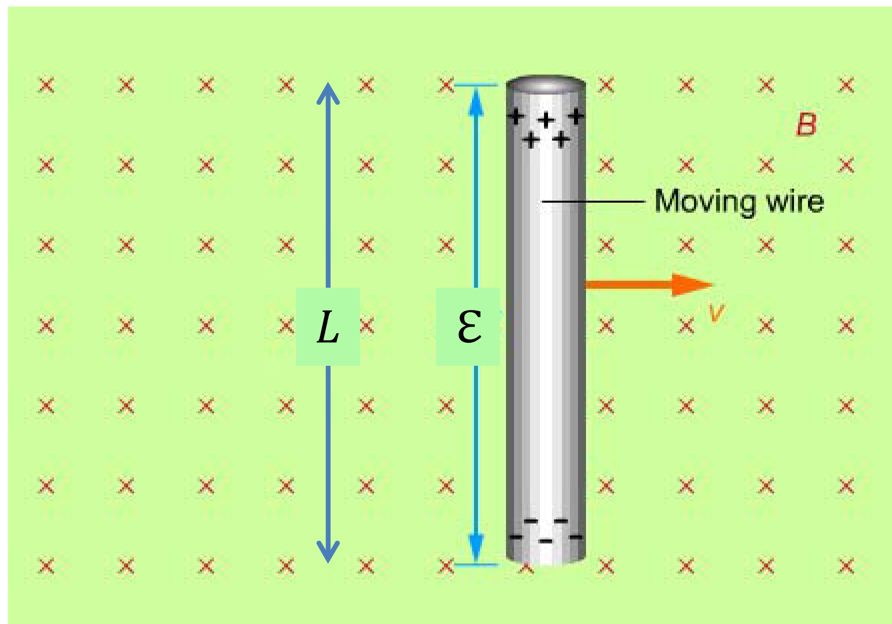


An induced EMF appears across the ends of the metal bar.

Force on a charge q is given by the Lorenz force:

$$F = qvB\sin\theta$$

This force is balanced by electrostatic forces.

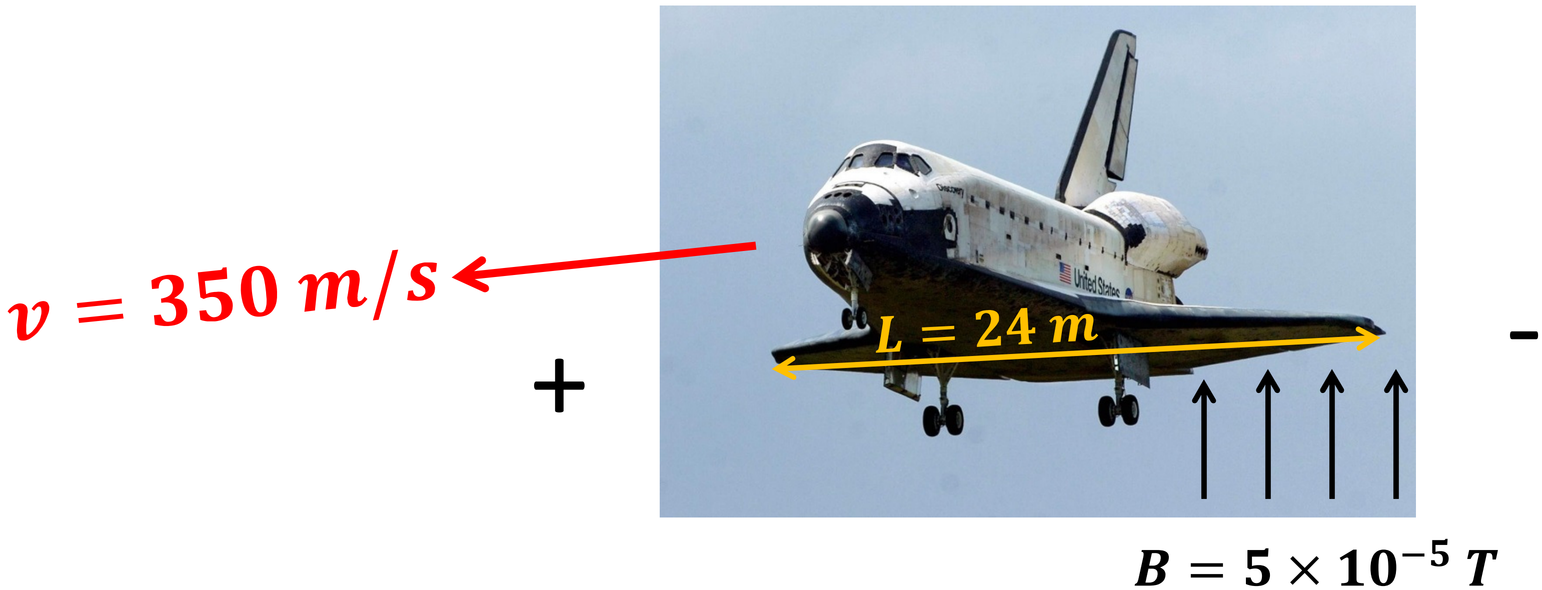


$$qE = qvB\sin\theta$$

but $E = \mathcal{E}/L$ so…

$$\mathcal{E} = vBL\sin\theta$$

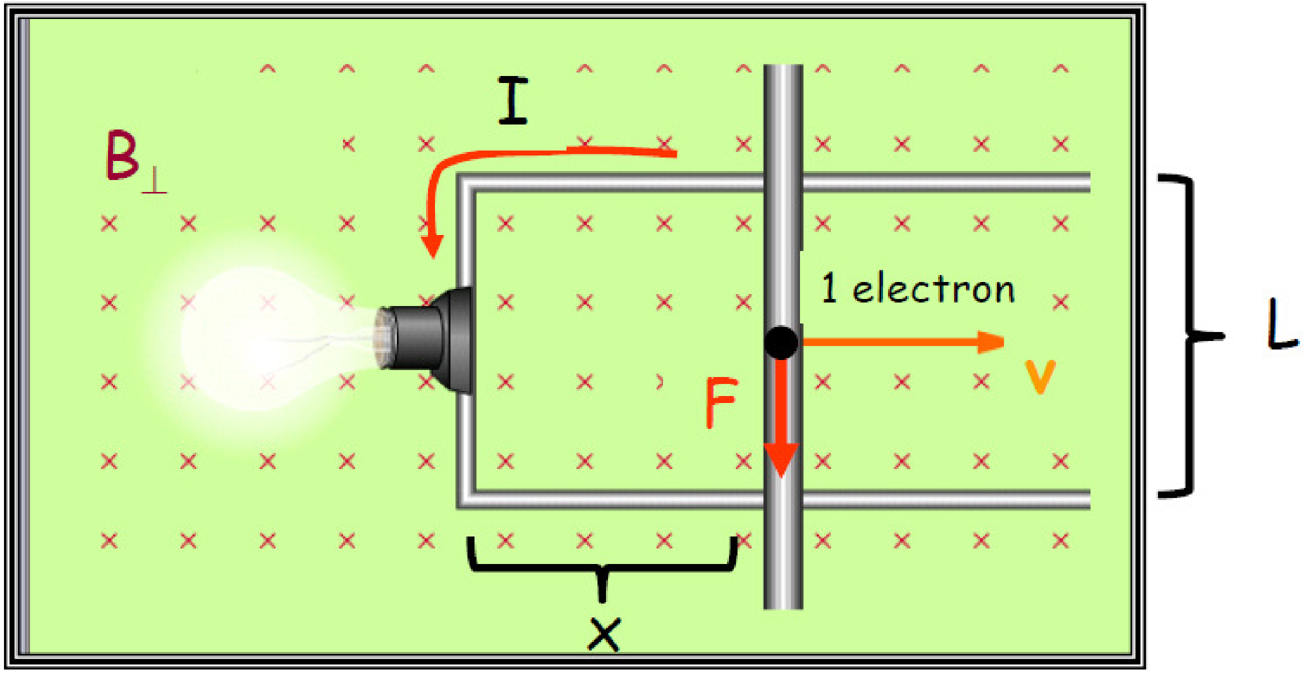
Example III: the Space Shuttle



- Assume $\vec{v}$ is perpendicular to $\vec{B}$.
- Induced EMF:
  $$\Delta V = BvL = (5 \times 10^{-5}\ T)(350\ m/s)(24\ m)$$
  $$= 0.4\ V$$

Area Changes with Time

Assume the metal rails and lightbulb have a combined resistance R



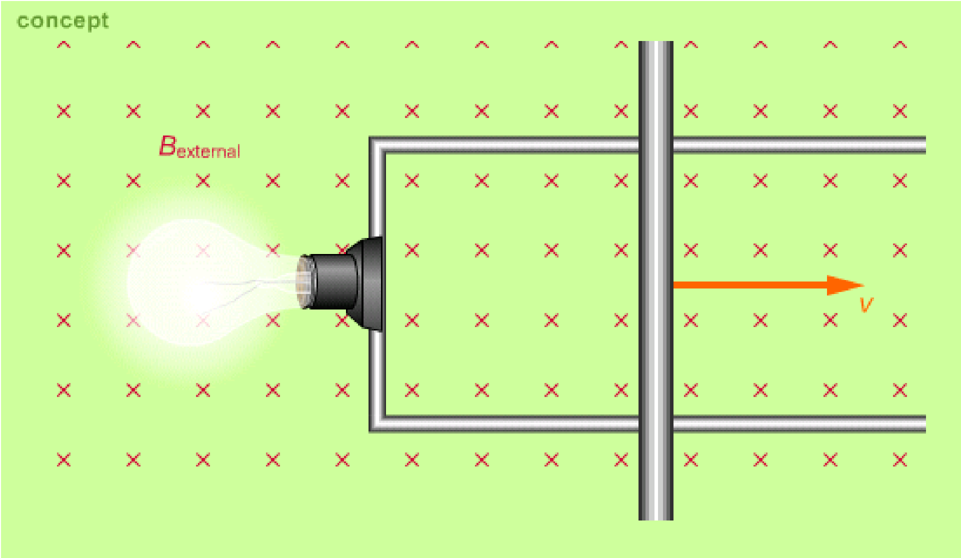
0 (because L=constant)

$$|\varepsilon| = \Delta V = \left|-\frac{\Delta\Phi}{\Delta t}\right| = B_\perp \frac{\Delta A}{\Delta t} = B_\perp \frac{\Delta(Lx)}{\Delta t} = B_\perp\left(\frac{x\,\Delta L}{\Delta t} + \frac{L\,\Delta x}{\Delta t}\right)$$

$$= B_\perp L \frac{\Delta x}{\Delta t} = B_\perp L v$$

$$\therefore I = \frac{\Delta V}{R} = \frac{B_\perp L v}{R}$$

Example



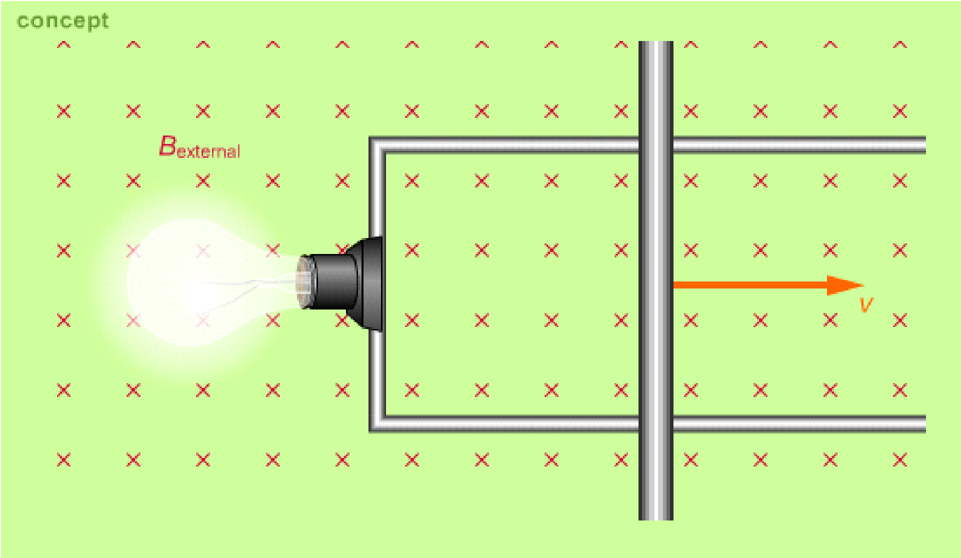
If $L = 0.5\ m$, $v = 10\ m/s$, $R = 1.5\ \Omega$ and $B = 1T$,

What is the current?

$$\mathcal{E} = \Delta V = BvL = (1\ T)(10\ m/s)(0.5\ m) = 5\ V$$

$$I = \frac{\mathcal{E}}{R} = \frac{5\ V}{1.5\ \Omega} = 3.3\ A$$

Example



$$\mathcal{E} = \Delta V = BvL = (1\ T)(10\ m/s)(0.5\ m) = 5\ V$$

$$I = \frac{\mathcal{E}}{R} = \frac{5\ V}{1.5\ \Omega} = 3.3\ A$$

Power, $P = I\,\mathcal{E} = (3.3\ A)(5\ V) = 16.5\ W$

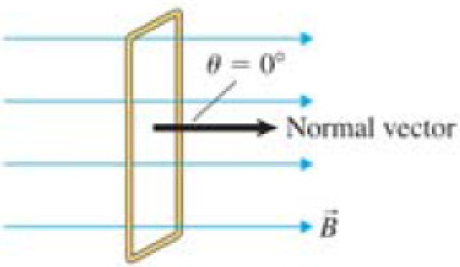
How much force is required?

$P = Fv$ so $F = P/v = (16.5\ W)/(10\ \ m/s) = 1.65\ N$

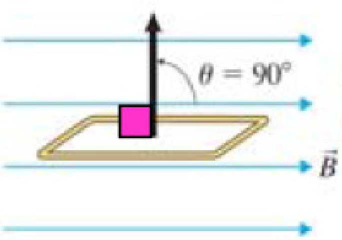
Example V. EMF produced by Rotating Loop

Stationary Loop

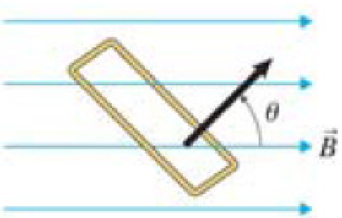
(a) Maximum flux cos 0° = 1.0



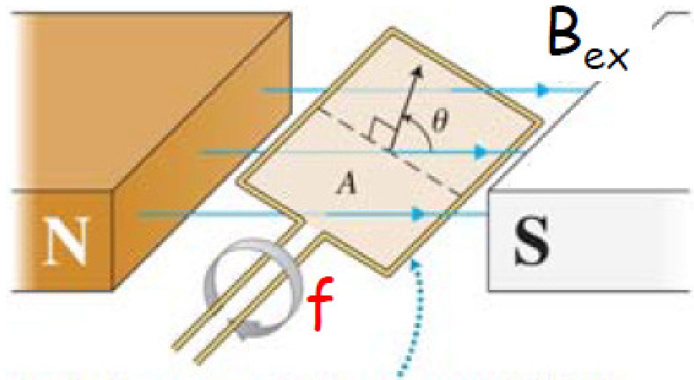
(b) Zero flux cos 90° = 0



(c) Intermediate flux 0 < cos θ < 1.0



Rotating Loop (N=1)



As the loop rotates in the magnetic field, the flux through the loop continually changes.

$$\Phi = B_{ex} A \cos(\theta)$$

If loop rotates

$$\theta = \theta_o + 2\pi f t = \theta_o + \omega t$$

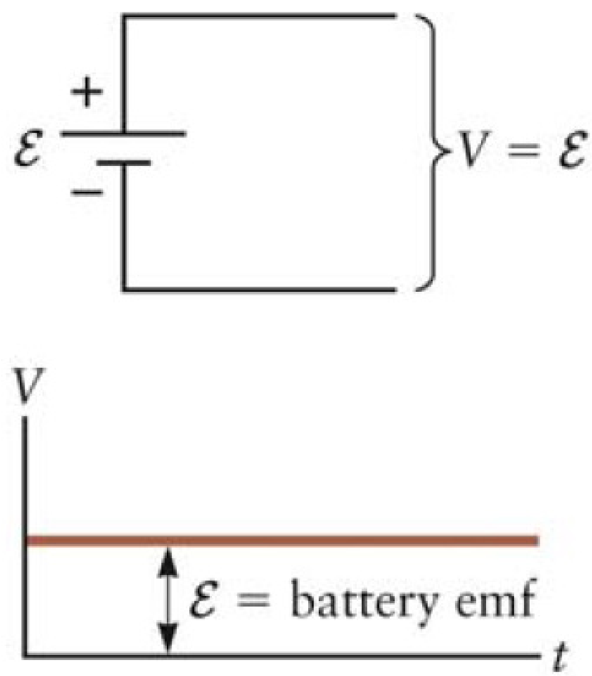
f = No. rev./s or Hz

$$|\mathcal{E}(t)| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = N B_{ex} A \omega \sin(\omega t)$$

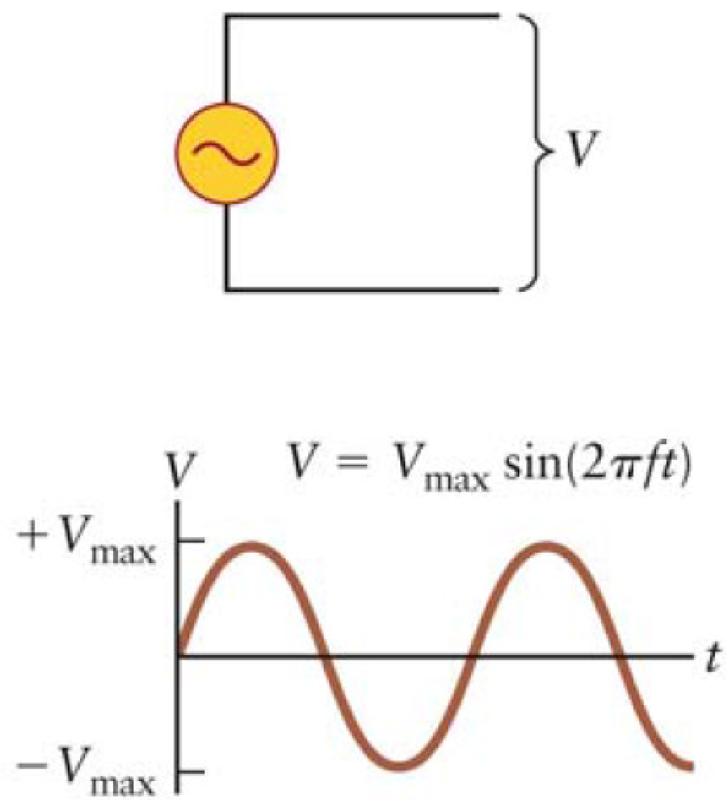
where $\omega = 2\pi f$ = angular frequency in rad/s

DC vs. AC

DC SOURCE
(Battery voltage does not vary with time)

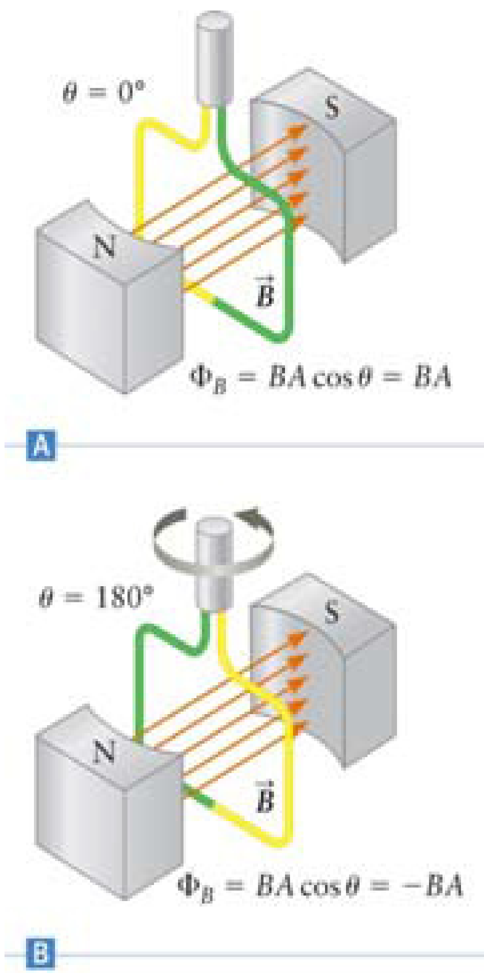


AC SOURCE

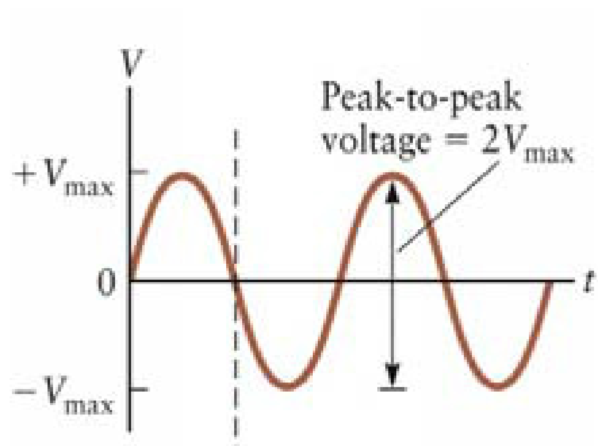


Electric Generators

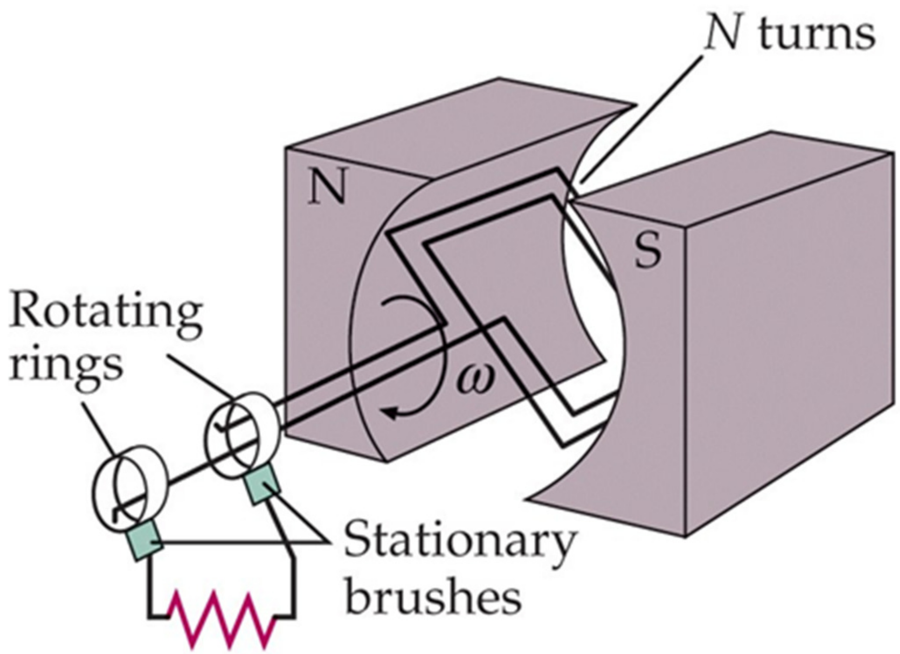
Convert mechanical energy (work=torque*angle) into electrical energy.



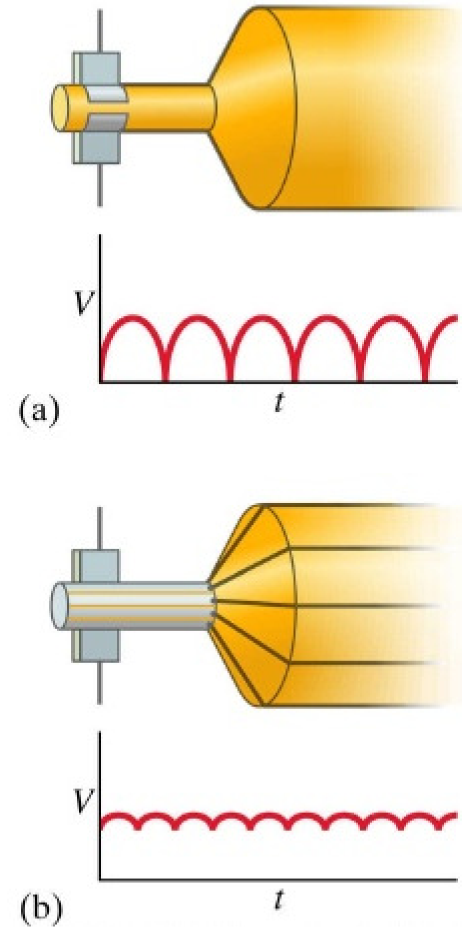
$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$
$$\Phi_B = BA\cos\theta = BA\cos(2\pi f t)$$
$$\frac{\Delta\Phi_B}{\Delta t} = -\sin(2\pi f t)$$
$$\mathcal{E} = \boldsymbol{V_{max}\,sin(2\pi f t)}$$
where $\boldsymbol{V_{max} = NBA\omega}$



Electric Generators

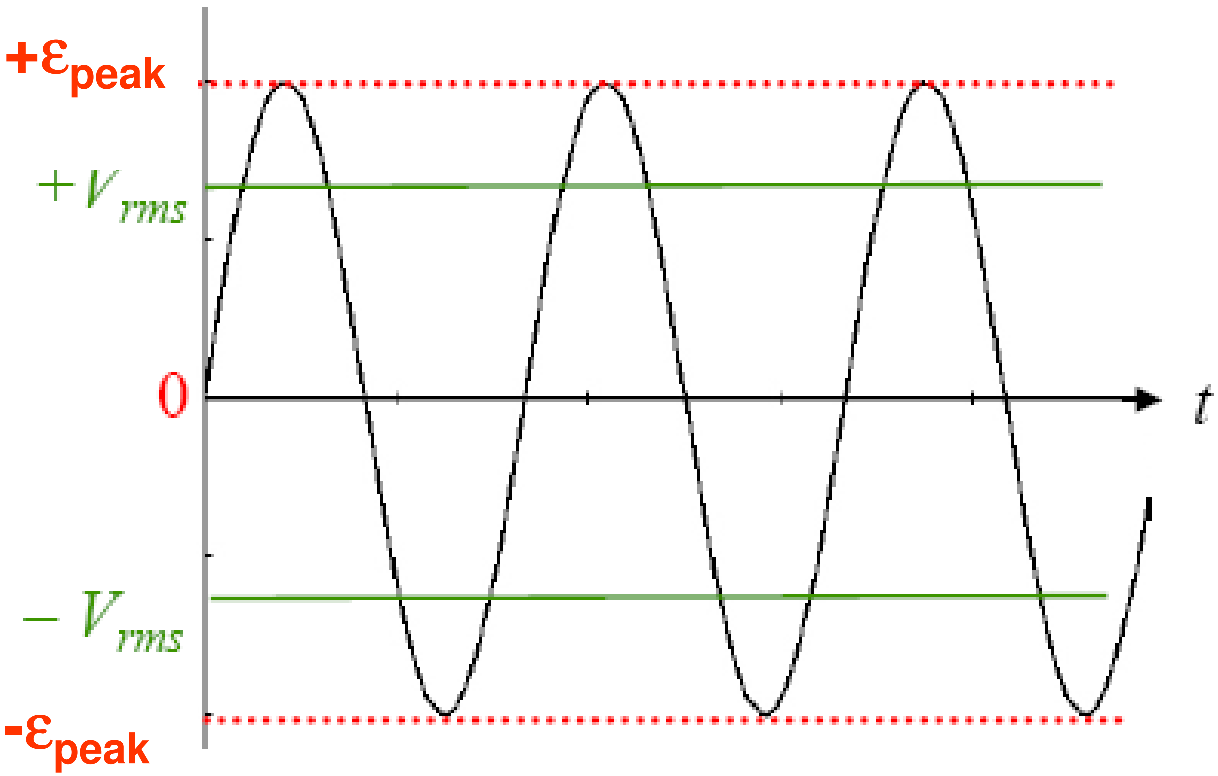


Slip contacts – AC generator



Split-ring contacts – DC generator

Standard AC Voltage in North America

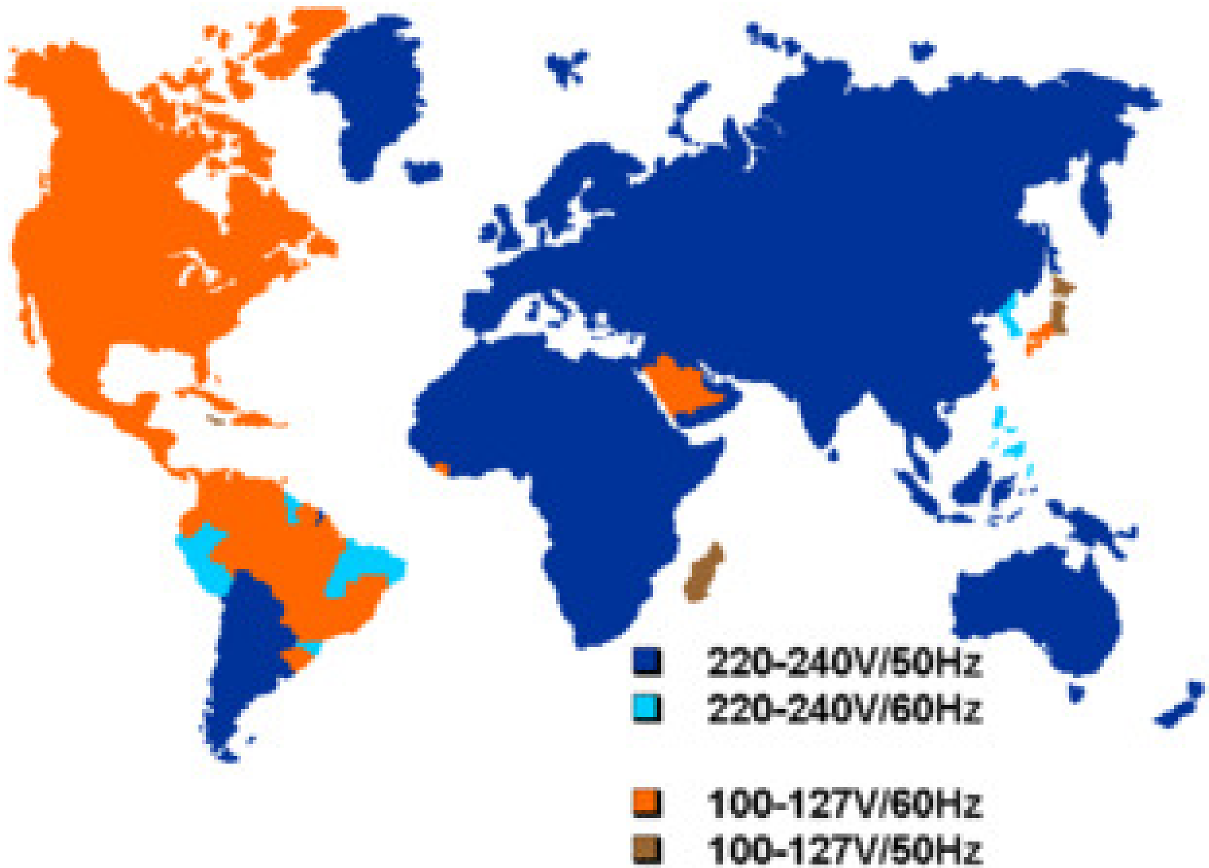


$\sim 180\ V$

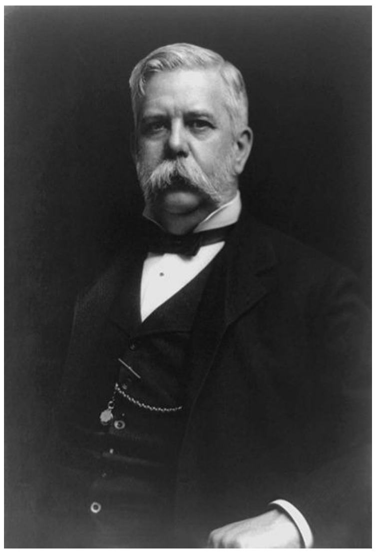
$\sim 110\ V$

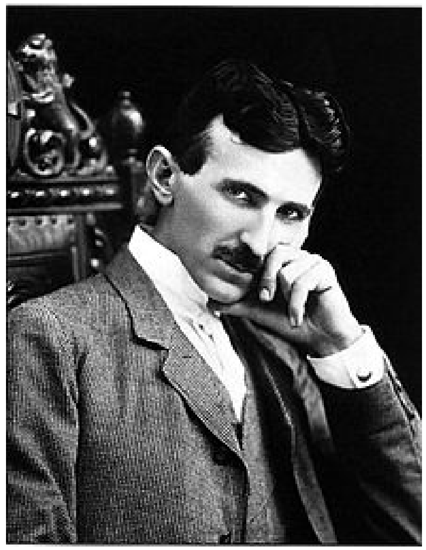
$$\frac{1}{\sqrt{2}} \approx 0.707$$

Not a world standard…

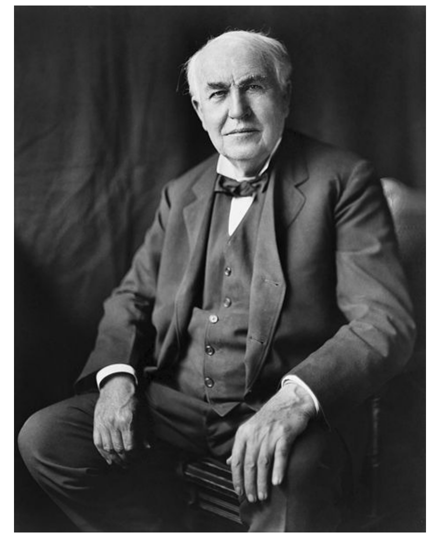


AC vs DC Power Distribution





Westinghouse and Tesla promoted AC power distribution.



Edison promoted DC power distribution.