PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

# Physics 21900
# General Physics II

*Electricity, Magnetism and Optics*
*Lecture 12 – Chapter 17.4*
***Magnetic Forces on Currents***

Fall 2015 Semester

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# Summary of Magnetic Forces

- Lorentz force law:

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

$$\left|\vec{F}\right| = |q| v\,B \sin\theta$$

← Angle between $\vec{v}$ and $\vec{B}$.

- Force on a wire of length $L$ carrying current $I$ in a magnetic field:

$$F = I\,B\,L\,\sin\theta$$

← Angle between $\vec{I}$ and $\vec{B}$.

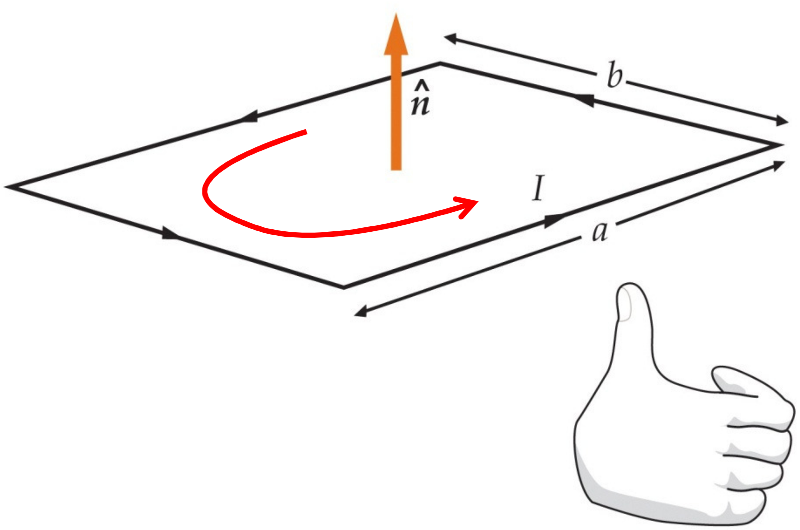
- Force on two parallel wires separated by a distance $R$:

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{R}\,L$$

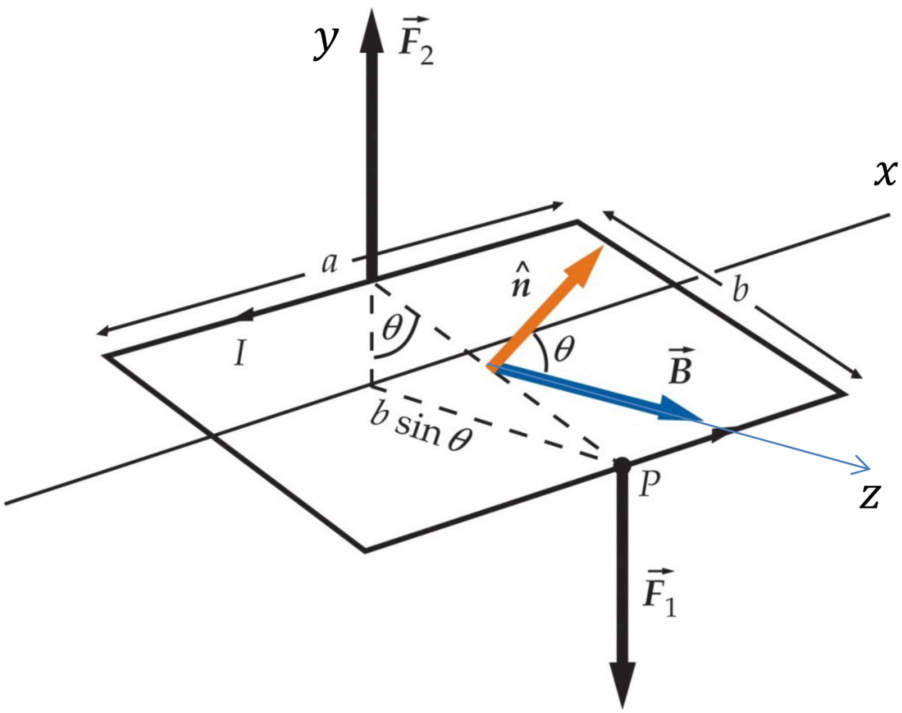
$$\mu_0 = 4\pi \times 10^{-7} N/A^2$$

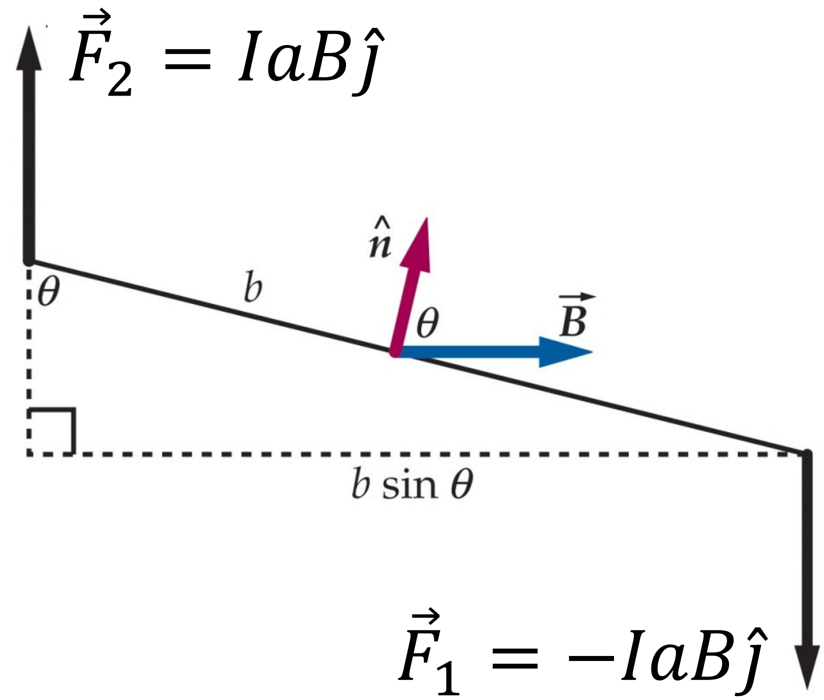
# Forces on a Loop of Current in a Magnetic Field

- Consider a rectangular loop of wire carrying current $I$ in a magnetic field.
- The orientation of the loop is given by the unit vector $\hat{n}$ perpendicular to the plane of the loop.



# Torque on a Current Loop



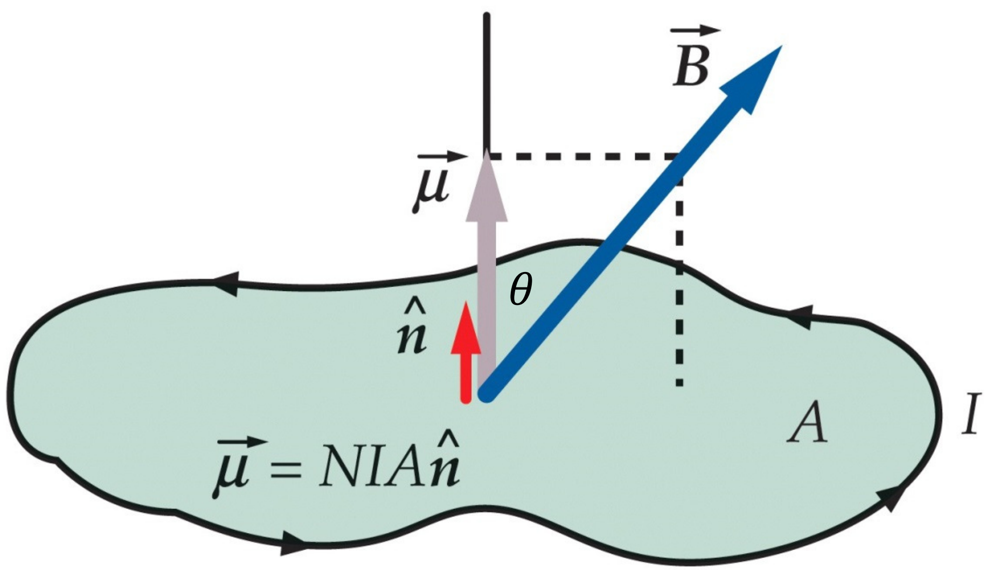

- Magnitude of torque is $\tau = IabB\sin\theta$
- Direction is perpendicular to $\vec{B}$ and $\hat{n}$:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$
$$\vec{\mu} = Iab\hat{n}$$

# Torque on a Current Loop

- In general, the torque does not depend on the shape, just the area.
- With $N$ turns of wire in the loop, multiply by $N$.

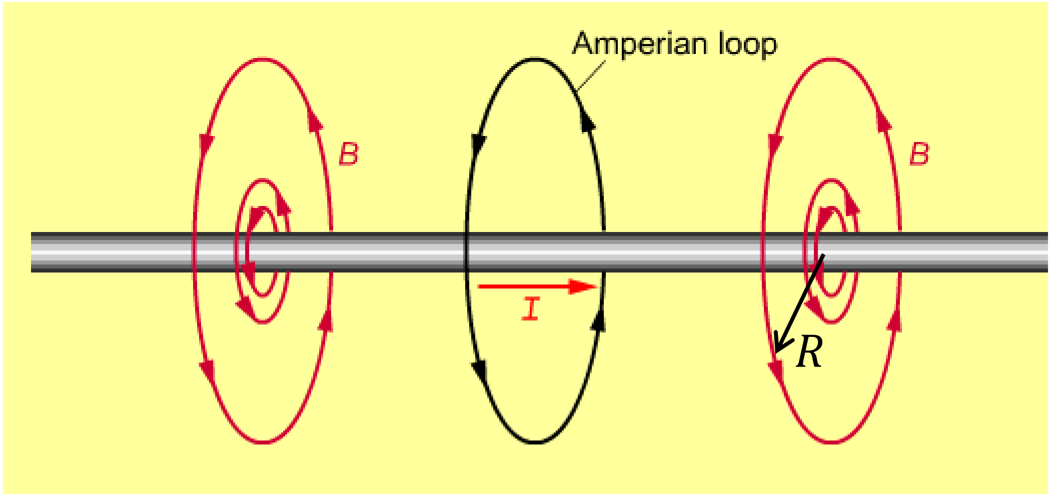
$$\vec{\mu} = NIA\,\hat{n}$$




$$\vec{\tau} = \vec{\mu} \times \vec{B}$$
$$|\vec{\tau}| = NIAB\sin\theta$$

# Magnetic Fields From Currents

- Long, straight wire:

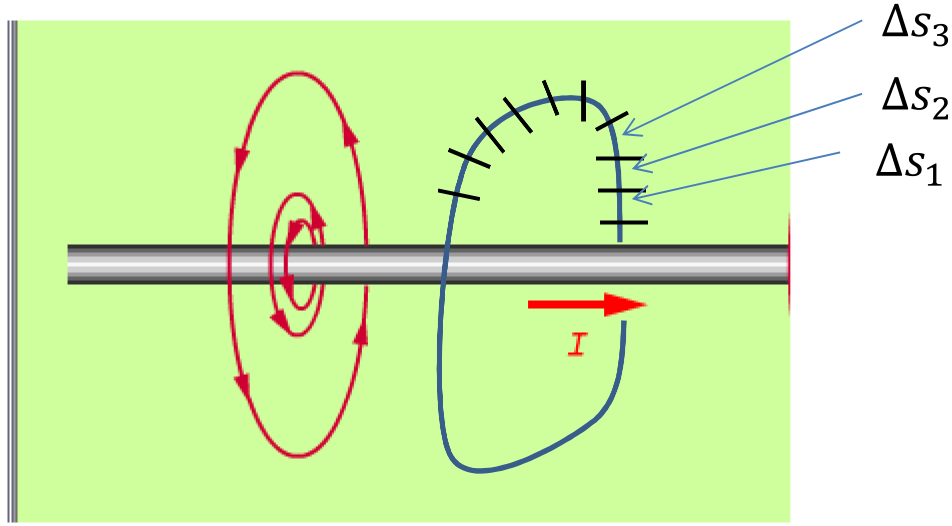



- Observation: $B \propto I/R$
- Magnetic field lines curl around the wire
  - use the right-hand rule to get the direction
- How can we work out the exact form of $B$?

# Ampere's Law

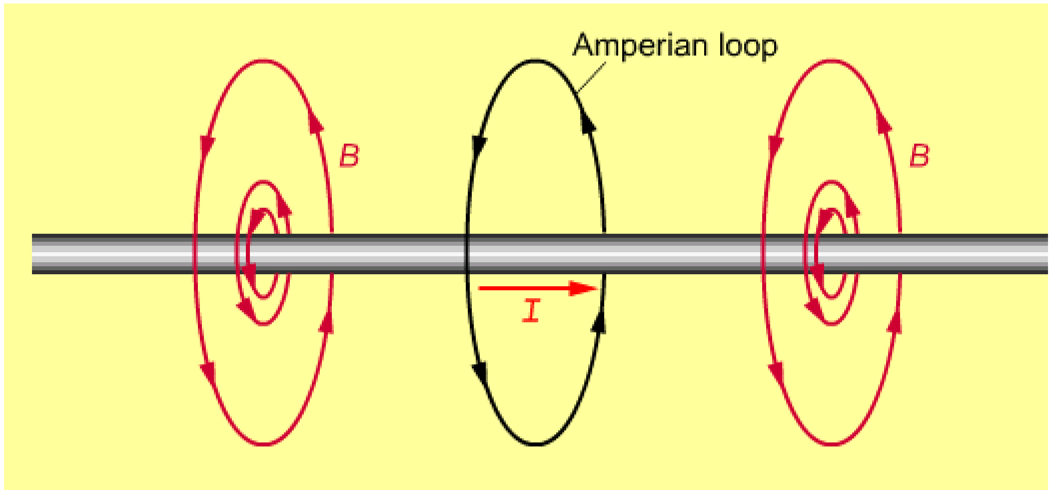
- For any closed path the encircles a current,

$$\sum B_{\parallel}\,\Delta s = \mu_0 I_{enclosed}$$




- This usually isn't useful unless we find a path that is parallel to $\vec{B}$ everywhere.
- For example, a circular path around and perpendicular to a long, straight wire.

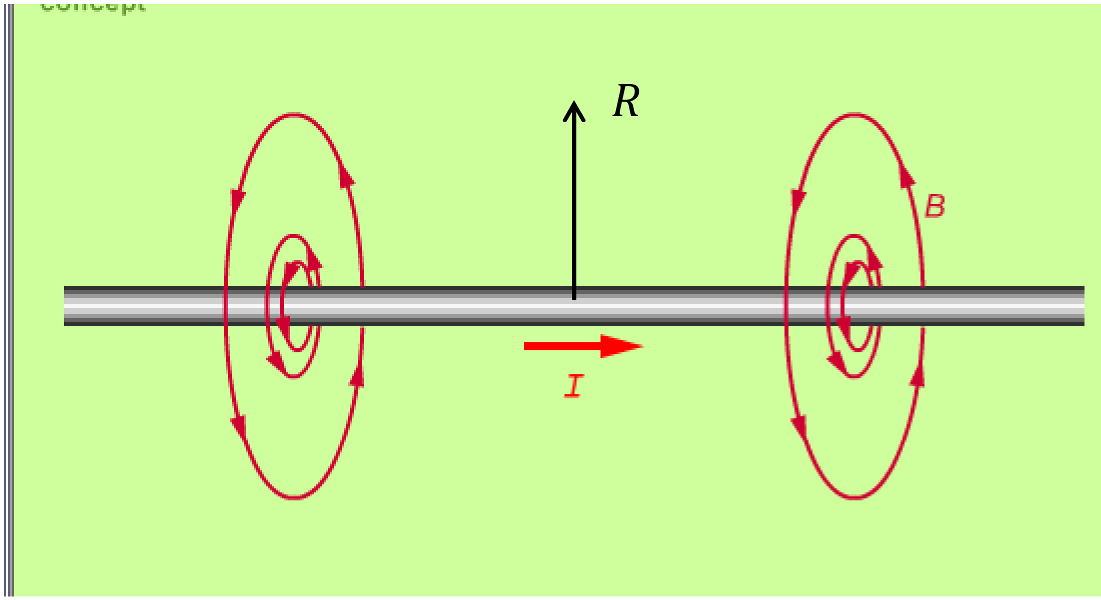
# Ampere's Law




- From the symmetry of the wire when rotated around its axis, we expect $|\vec{B}|$ to be constant along circular paths of radius $R$.

$$\sum B_{\parallel}\,\Delta s = \underbrace{2\pi\,R}_{\text{Circumference of a circle of radius } R.}\,B = \mu_0 I$$

# Ampere's Law




$$\sum B_{\parallel}\,\Delta s = 2\pi\, R\, B = \mu_0 I$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{R}$$

$$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$$

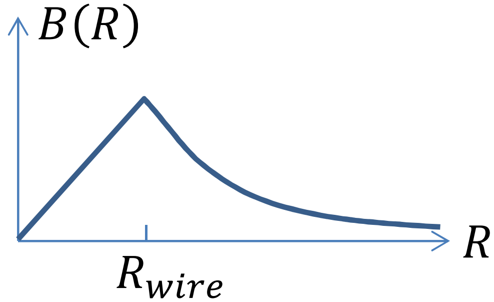
# Ampere's Law

- Example: what is the magnetic field inside the wire?
- Assume the current is uniformly distributed throughout the wire…
  - Cross-sectional area of the wire: $A_{wire} = \pi\, R_{wire}^2$
  - Area inside a loop of radius $R$: $A = \pi\, R^2$
  - Current inside a loop of radius $R < R_{wire}$:

$$I_{enclosed} = I\,\frac{R^2}{R_{wire}^2}$$

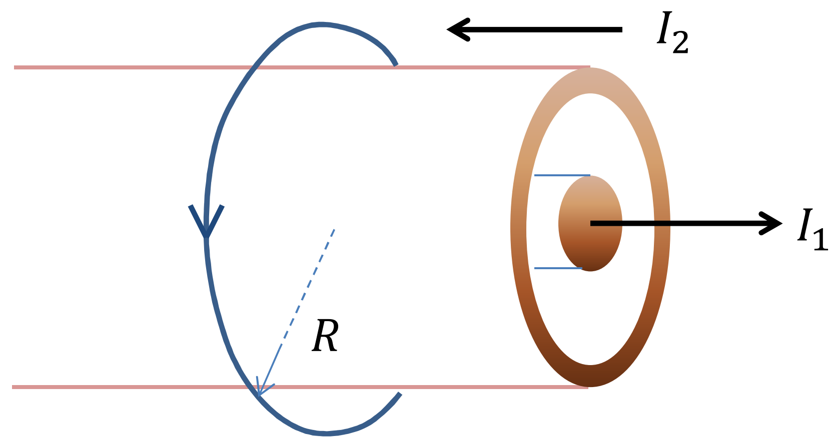
- Ampere's law:

$$2\pi\, R\, B = \mu_0 I_{enclosed} = \mu_0 I\,\frac{R^2}{R_{wire}^2}$$




$$B = \frac{\mu_0}{2\pi}\frac{IR}{R_{wire}^2}$$

# Example: Coaxial Cable

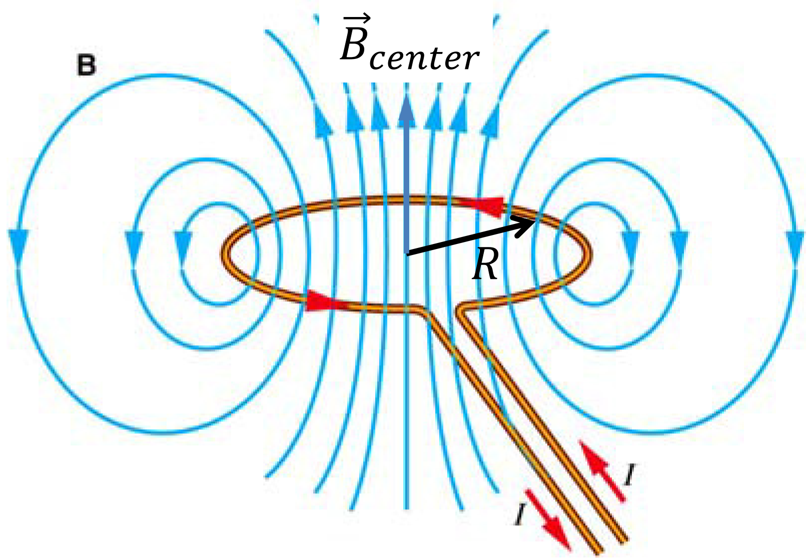



$$\sum B_{\parallel} \Delta s = 2\pi\, R\, B = \mu_0 I_{enclosed}$$

$$= \mu_0 (I_1 - I_2)$$

$$B = \frac{\mu_0}{2\pi} \frac{(I_1 - I_2)}{R}$$

# Magnetic Field at the Center of a Current Loop of Radius R



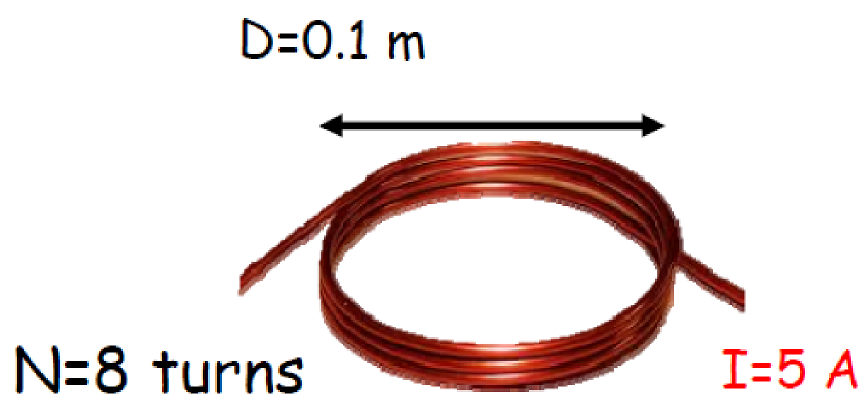

In general, the magnetic field does not have a lot of symmetry.

But we can ask about the magnitude of the magnetic field on the z-axis at the center of the loop.

The calculation is a bit more involved but the answer is…

$$B_{center} = \frac{\mu_0 I}{2R}$$

# Example




$$B_{center} = N\left(\frac{\mu_o I}{2R}\right)$$

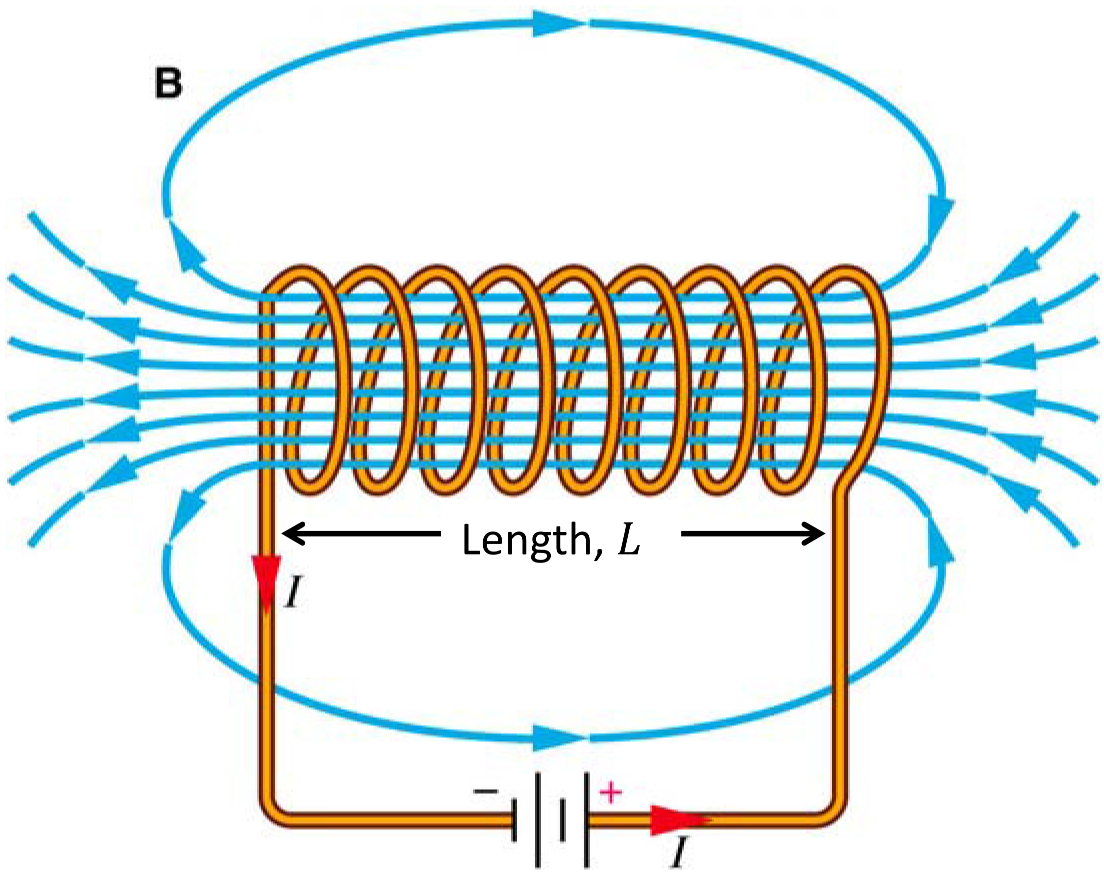
$$= 8\left(\frac{\left(4\pi \times 10^{-7}\ {}^{N}\!/\!{}_{A^2}\right)(5A)}{2(0.05m)}\right)$$

$$= 8\left(\frac{\left(20\pi \times 10^{-7}\right)}{(0.10)}\right)T$$

$$= 8\left(200\,\pi \times 10^{-7}\right)T$$

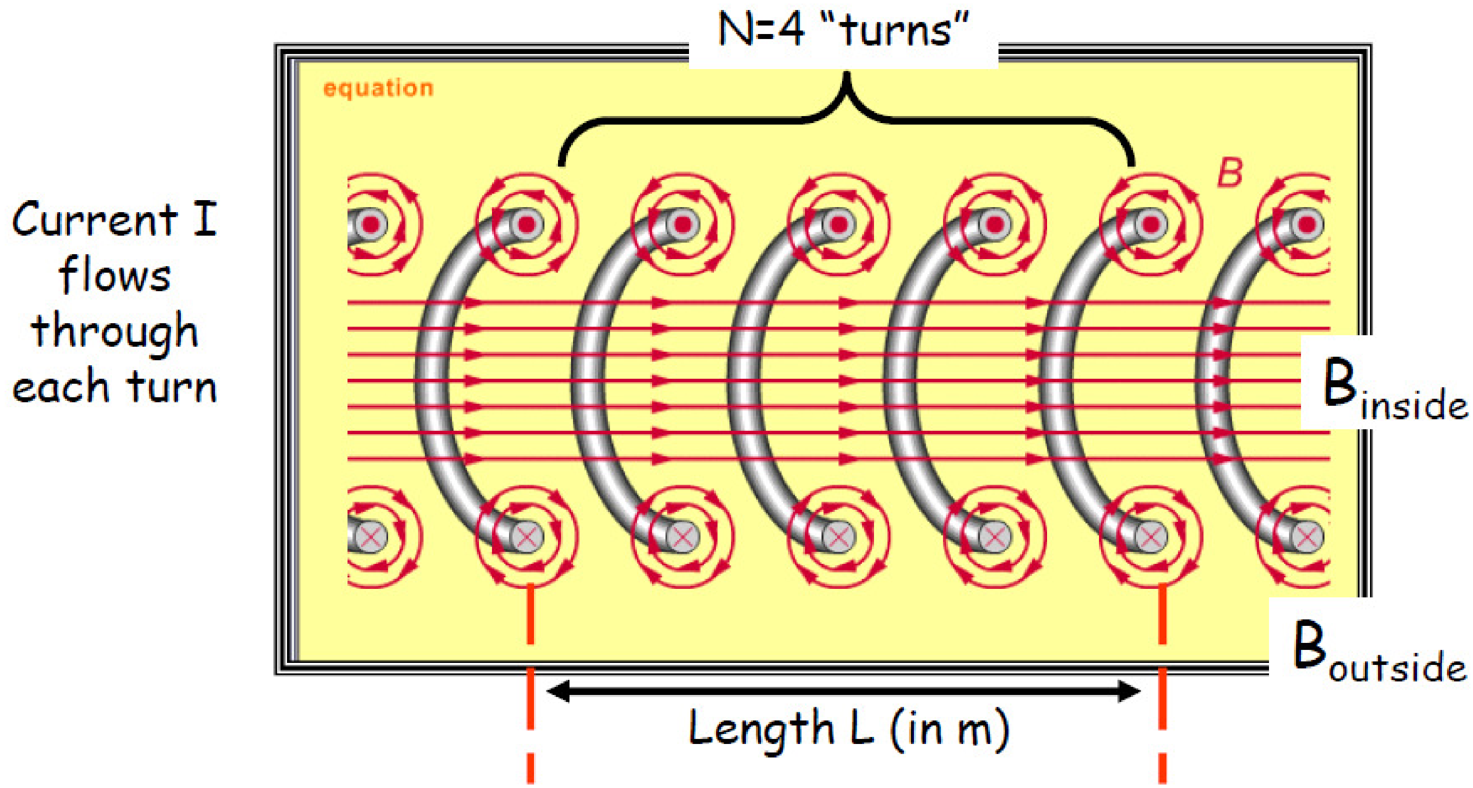
$$= 5.03 \times 10^{-4}\ T$$

# Magnetic Field Inside a Solenoid




Inside a *long* solenoid, the magnetic field is parallel to the axis.

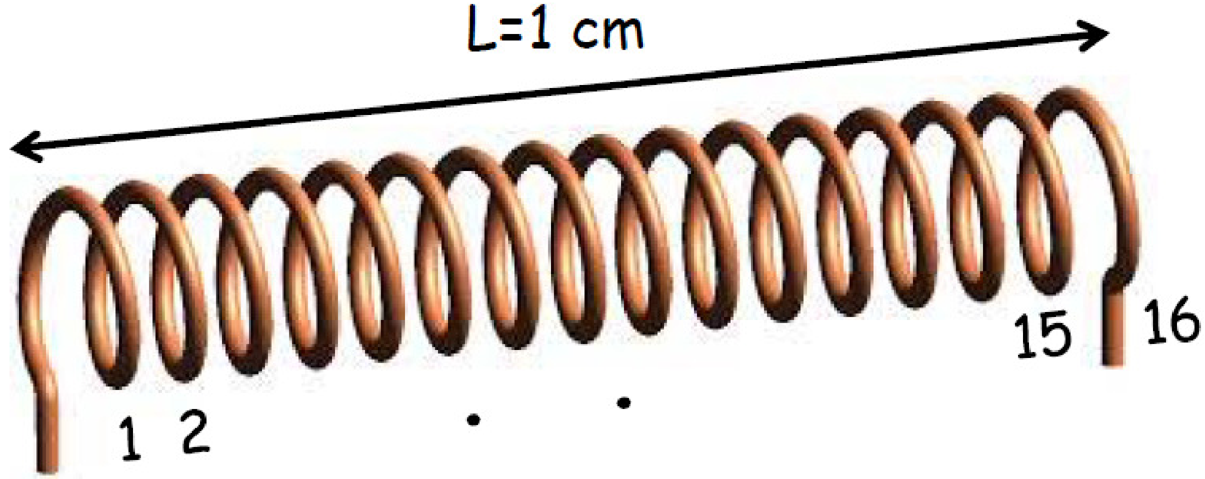
# Magnetic field produced by ***long*** solenoid




$$B_{inside} = \mu_o n I; \qquad \text{n} = \text{turns/length (m}^{-1}\text{)} = \frac{N}{L}$$

$$B_{outside} = 0$$

# Example




$$B_{inside} = \mu_o n I; \qquad n = \text{turns/length}\,(m^{-1}) = \frac{N}{L}$$

$$= \left(4\pi \times 10^{-7}\, N/A^2\right)\frac{16}{0.01\,m}(2.5A)$$

$$= 5\times 10^{-3}\,\frac{N}{A\cdot m} = 5\times 10^{-3}\,T$$