Quintet pairing and non-Abelian vortex string in spin-3/2 cold atomic systems

Congjun Wu,[1] Jiangping Hu,[2] and Shou-Cheng Zhang[3]

[1]*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106*
[2] *Department of Physics, Purdue University, West Lafayette, IN 47907*
[3]*Department of Physics, McCullough Building, Stanford University, Stanford CA 94305-4045*

We study the s-wave quintet Cooper pairing phase ($S_{pair}=2$) in spin-3/2 cold atomic systems and identify various novel features which do not appear in the spin-1/2 counterpart. A single quantum vortex is shown to be energetically less stable than a pair of half-quantum vortices (HQV). The HQV exhibits the global analogue of the non-Abelian Alice string and $SO(4)$ Cheshire charge in gauge theories. The non-Abelian HQV loop enables topological generation of quantum entanglement.

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Optical traps and lattices open up a whole new direction in the study of strongly correlated high spin systems by using cold atoms with hyperfine multiplets. In spin-1 bosonic systems (e.g. ^{23}Na and ^{87}Rb), spinor condensations, spin textures and nematic orders have generated a great deal of attention [1, 2]. On the other hand, high spin fermions also exhibit many exciting novel features. For instance, the multi-particle clustering instability, i.e., a multi-particle counterpart of Cooper paring, is not allowed in spin 1/2 systems due to Pauli's exclusion principle, but is possible in high spin systems [3, 4]. Furthermore, high spin fermions offer a unique playground to study high symmetries which do not appear in usual condensed matter systems. Three of us have shown that spin-3/2 fermionic systems with contact interactions, which can be realized by atoms such as ^{132}Cs, ^{9}Be, ^{135}Ba, ^{137}Ba and ^{201}Hg, enjoy a generic $SO(5)$ symmetry without any fine tuning of parameters [5]. The important effects from this high symmetry on magnetism, Cooper pairing structures and the Kondo problem in spin 3/2 systems have been extensively investigated [3, 6].

On the other hand, important progress has been made in the fault-tolerant topological quantum computation [7]. The key idea is that by using non-Abelian statistics in two dimensions, particles can be entangled in a robust way against local disturbances. The promising candidate systems to implement topological quantum computation include the non-Abelian quantum Hall states with fermions at the filling $\nu=5/2$ [8] and bosons at $\nu=1$ [9], and also the p_x+ip_y pairing state of spinless fermions [10]. In this paper, we show that due to the $SO(5)$ symmetry in spin-3/2 cold atomic systems [3, 5], the s-wave quintet Cooper pairing state ($S_{pair}=2$) in such systems provides another opportunity to topologically generate quantum entanglement between the particle and the non-Abelian half-quantum vortex (HQV) loop.

The HQV in superfluids with the spin degree of freedom is an exotic topological defect as a global analogue of the Alice string in gauge theories [11]. The HQV loop can possess spin quantum number which is an example of the Cheshire charge phenomenon. An Abelian version of the global Alice string and Cheshire charge exists in the triplet superfluid of the ^{3}He-A phase [12, 13], where the spin $SU(2)$ symmetry is broken into the $U(1)$ symmetry around the z-axis. A remarkable property is that both quasi-particles and spin wave excitations reverse the sign of their spin quantum numbers s_z when going through the HQV loop. Meanwhile the HQV loop also changes s_z to maintain spin conservation. However, due to the Abelian nature of this $U(1)$ Cheshire charge, no entanglement is generated in this process.

In this article, we investigate the non-Abelian Alice string and the topological generation of quantum entanglement through the non-Abelian Cheshire charge in spin-3/2 systems. The quintet Cooper pairing order parameters in the polar basis form a 5-vector of the $SO(5)$ symmetry group. The ground state exhibits the polar condensation where the $SO(5)$ symmetry is broken into $SO(4)$ [5]. This allows the HQV loop to possess the non-Abelian $SO(4)$ Cheshire charge, in contrast to the $U(1)$ Cheshire charge in the ^{3}He-A phase. We also explore the high symmetry effects on collective spin excitations and the structure of the HQV line as a π-disclination in the spin channel. We show that by driving the fermion quasiparticle (or spin-wave impulse) through the HQV loop, quantum entanglement between them is topologically generated. This effect has a potential application in the topological quantum computation.

Spin 3/2 systems are characterized by two independent s-wave scattering interaction parameters g_0 and g_2 in the total spin singlet ($S_T=0$) and quintet ($S_T=2$) channels, respectively. We consider the case of $g_2<0$ where the quintet channel Cooper pairing dominates, and further neglect the interaction in the singlet channel. The mean field Hamiltonian reads

$$H_{MF}=\int d^D r\Big\{\sum_{\alpha=\pm\frac{3}{2},\pm\frac{1}{2}}\psi^\dagger_\alpha(r)\big(\frac{-\hbar^2\nabla^2}{2M}-\mu\big)\psi_\alpha(r) + \sum_{a=1\sim 5}\chi^\dagger_a(r)\Delta_a(r)+h.c.-\frac{1}{g_2}\Delta^*_a(r)\Delta_a(r)\Big\}, \qquad (1)$$

with D the spatial dimension, μ the chemical potential, and M the atom mass. Δ_a is proportional to the ground state expectation value of the quintet pairing operators

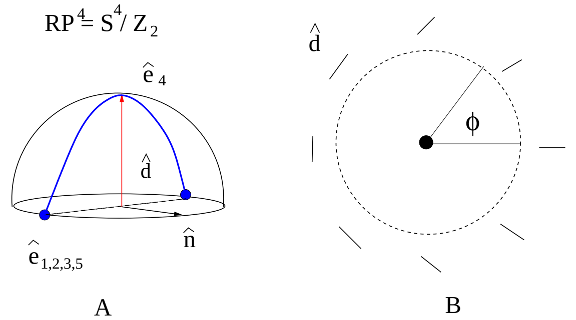

FIG. 1: A) The Goldstone manifold of $\hat{d}$ is a 5D hemisphere RP^4. It contains a class of non-contractible loops as marked by the solid curve. B) The π-disclination of $\hat{d}$ as a HQV. Assume that $\hat{d} \parallel \hat{e}_4$ at $\phi = 0$. As the azimuthal angle ϕ goes from 0 to 2π, $\hat{d}$ is rotated at the angle of $\phi/2$ around any axis $\hat{n}$ in the S^3 equator spanned by $\hat{e}_{1,2,3,5}$.

χ_a by $\Delta_a(r) = g_2\langle\chi_a^\dagger(r)\rangle$ $(a = 1 \sim 5)$. The five χ_a operators are the spin channel counterparts of the five atomic d-orbitals $(d_{xy}, d_{xz}, d_{yz}, d_{3z^2-r^2}, d_{x^2-y^2})$, and transform as a 5-vector under the $SO(5)$ group. Explicitly, they are expressed as

$$\chi_a^\dagger(r) = -\frac{i}{2}\psi_\alpha^\dagger(r)(\Gamma^a R)_{\alpha\beta}\psi_\beta^\dagger(r), \qquad (2)$$

where the five 4×4 Dirac Γ^a $(a = 1 \sim 5)$ matrices are given in Ref. [5], and $R = \Gamma^1\Gamma^3$ is the charge conjugation matrix.

This $SO(5)$ symmetry leads to new interesting results about the pairing structure in the ground state and the corresponding Goldstone (GS) modes. Within the BCS theory, Ref. [14] showed that the ground state of Eq. 1 is an $SO(3)$ polar condensate without noticing the hidden $SO(5)$ symmetry. We conclude here that the ground state is generically an $SO(5)$ polar condensate. The order parameters can be parameterized as $\Delta_a = |\Delta|e^{i\theta}d_a$, where θ is the $U(1)$ phase, $\hat{d} = d_a\hat{e}_a$ is a $5D$ unit vector, and $\hat{e}_a(a = 1 \sim 5)$ form a set of basis for the internal spin space. Rigorously speaking, $\hat{d}$ is a directionless director instead of a true vector because Δ_a's contain a Z_2 gauge symmetry of

$$\hat{d} \to -\hat{d}, \quad \theta \to \theta + \pi. \qquad (3)$$

Thus the GS manifold is $[SO(5)\otimes U(1)]/[SO(4)\otimes Z_2] = RP^4\otimes U(1)$, where RP^4 is a $5D$ hemisphere instead of an entire S^4 sphere as depicted in Fig. 1 A. In the following, we will present the GS modes and topological defects related to this GS manifold.

At zero temperature, in addition to the usual phonon mode, four branches of spin wave modes carrying the spin quantum number $S = 2$ arise because of the spontaneous symmetry breaking from $SO(5)$ to $SO(4)$. For small fluctuations, the spin wave modes decouple from the phase mode. For the purpose of describing collective excitations, Cooper pairs can be treated as composite bosons. This treatment gives a good approximation to the phonon mode in the neutral singlet BCS superfluid [15, 16]. Here we generalize this method to the quintet pairing by using a phenomenological Hamiltonian for spin-2 bosons

$$\begin{aligned} H_{eff} &= \int d^D r \Big\{ \frac{\hbar^2}{4M} \sum_{1\le a\le 5} \nabla\Psi_a^\dagger\nabla\Psi_a + \frac{1}{2\chi_\rho}(\Psi_a^\dagger\Psi_a \\ &- \rho_0)^2 + \frac{1}{2\chi_{sp}} \sum_{1\le a<b\le 5} (\Psi_c^\dagger L_{cd}^{ab}\Psi_d)^2 \Big\}, \end{aligned} \qquad (4)$$

where Ψ_a's are the boson operators in the polar basis, the equilibrium Cooper pair density ρ_0 is half of the particle density ρ_f, χ_ρ and χ_{sp} are proportional to the compressibility and $SO(5)$ spin susceptibility respectively. We define the $SO(5)$ generators in the 5×5 vector representation as

$$L_{cd}^{ab} = i(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}). \qquad (5)$$

Taking into account the Fermi liquid correction, $\chi_\rho = N_f/[4(1+F_0^s)]$ and $\chi_{sp} = N_f/[4(1+F_0^t)]$, where N_f is the fermion density of states at the Fermi energy, $F_0^{s,t}$ are the Landau parameters defined in the $SO(5)$ scalar and tensor channels [5] respectively. We introduce $\rho(r)$ as the Cooper pair density and $l_{ab}(r)$ as the $SO(5)$ spin density, and parameterize $\Psi_a = \sqrt{\rho_0}e^{i\theta}d_a$. Using the standard commutation rules between l_{ab} and $\hat{d}_a$, ρ and θ, we arrive at

$$\begin{aligned} &\partial_t l_{ab} = \frac{\hbar^2\rho_{sp}}{2M}(d_a\nabla^2 d_b - d_b\nabla^2 d_a), \quad \chi_{sp}\partial_t d_a = -l_{ab}d_b, \\ &\chi_\rho\partial_t^2\theta - \frac{\hbar^2\rho_s}{2M}\nabla^2\theta = 0, \end{aligned} \qquad (6)$$

where ρ_{sp} is the superfluid density and ρ_s is the spin superfluid density. At $T = 0K$ in a Galilean invariant system, ρ_s is just $\rho_f/2$, while ρ_{sp} receives Fermi liquid corrections as $\rho_{sp}/\rho_s = (1+F_1^v/3)/(1+F_1^s/3)$ [17] where F_1^v is the Landau parameter in the $SO(5)$ vector channel [5]. The spin wave and sound velocities are obtained as $v_{sp} = \sqrt{\rho_0/(2\chi_{sp}M)}$ and $v_s = \sqrt{\rho_0/(2\chi_\rho M)}$, respectively.

The fundamental group of the GS manifold is $\pi_1(RP^4\otimes U(1)) = Z\otimes Z_2$. The Z_2 feature gives rise to the existence of the HQV as a stable topological defect as depicted in Fig. 1 B. As we move along a loop enclosing the HQV, the π phase mismatch in the θ field is offset by a π-disclination in the d-field, thus Δ_a's are maintained single-valued. Energetically, a single quantum vortex is less favorable than a pair of HQVs. From Eq. 4, the static energy function can be written as

$$E = \int d^D r \frac{\hbar^2}{4M}\Big\{\rho_s(\nabla\theta)^2 + \rho_{sp}(\nabla\hat{d})^2\Big\}. \qquad (7)$$

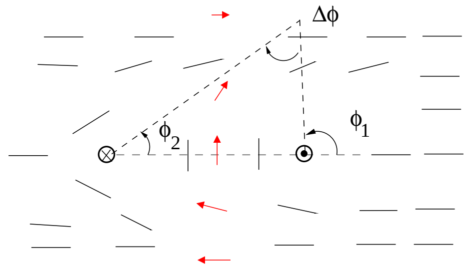

FIG. 2: The configuration of a π-disclination pair or loop described by Eq. 10. $\phi_{1,2}$ and $\Delta\phi$ are azimuthal angles and $\hat{d}(\vec{r}) \parallel \hat{e}_4$ as $\vec{r} \to \infty$. After a fermion passes the HQV loop, the components with $S_z = \pm\frac{3}{2}$ change to $S_z = \pm\frac{1}{2}$ and *vice versa* with an $SU(2)$ matrix defined in Eq. 8.

The energy density per unit length of a single quantum vortex is $E_1 = \frac{\hbar^2}{4M}\rho_s \log\frac{L}{a}$, while that of two isolated HQVs is $E_2 = \frac{\hbar^2}{8M}(\rho_s + \rho_{sp})\log\frac{L}{a}$. Although at the bare level $\rho_s = \rho_{sp}$, ρ_{sp} receives considerable Fermi liquid correction and strong reduction due to quantum fluctuations in the 5D internal space. Generally speaking, $\rho_{sp} < \rho_s$ holds in terms of their renormalized values. Then a single quantum vortex is fractionalized into a pair of HQVs. In the presence of rotation, the HQV lattice should appear instead of the usual single quantum vortex lattice. As a result of the doubling of vortex numbers, their vortex lattice constants differ by a factor of $\sqrt{2}$. The HQV was also predicted in the ^{3}He-A phase, where $\hat{d}$ is a 3D vector defined for spin-1 Cooper pairs. However, the dipole locking effect favors the d-vector aligned along the fixed direction of the l-vector, i.e., the direction of the p-wave orbital angular momentum. As a result, the two HQVs are linearly confined by a string of the mismatched d and l-vectors. In contrast, the orbital part of the quintet pairing is s-wave, no dipole locking effect exists.

The single HQV line behaves like the Alice string because a quasi-particle changes its spin quantum number after it adiabatically moves around the HQV once. For example, in the ^{3}He-A phase, a quasi-particle with spin $\uparrow$ flips its spin to $\downarrow$ up to a $U(1)$ Berry phase. The HQV in the quintet superfluid behaves as a non-Abelian generalization with the $SU(2)$ Berry phase. Without loss of generality, we assume that $\hat{d}$ is parallel to $\hat{e}_4$ at the azimuthal angle $\phi = 0$. As ϕ changes from 0 to 2π, $\hat{d}$ is rotated at the angle of $\phi/2$ in the plane spanned by $\hat{e}_4$ and $\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to $\hat{e}_4$, i.e., a vector located in the S^3 sphere spanned by $\hat{e}_{1,2,3,5}$. We define such a rotation operator as $U(\hat{n}, \phi/2)$. When U acts on an $SO(5)$ spinor, it takes the form of $U(\hat{n}, \phi/2) = \exp\{-i\frac{\phi}{2}\frac{n_b\Gamma^{b4}}{2}\}$ where $\Gamma^{b4} = i[\Gamma^b, \Gamma^4]/2$ are $SO(5)$ generators in the 4×4 spinor representation; when U acts on an $SO(5)$ vector, it behaves as $U(\hat{n}, \phi/2) = \exp\{-i\frac{\phi}{2}n_b L^{b4}\}$ where L^{ab}'s are the $SO(5)$ generators in the 5×5 vector representation explicitly defined in Eq. 5. The resulting configuration of $\hat{d}$ is $\hat{d}(\hat{n}, \phi) = U(\hat{n}, \phi/2)\hat{d}(\hat{n}, 0) = \cos\frac{\phi}{2}\hat{e}_4 - \sin\frac{\phi}{2}\hat{n}$. As fermionic quasi-particles circumscribe around the vortex line adiabatically, at $\phi = 2\pi$ fermions with $S_z = \pm\frac{3}{2}$ are rotated into $S_z = \pm\frac{1}{2}$ and *vice versa*. For convenience, we change the basis Ψ for the fermion wavefunction to $(|\frac{3}{2}\rangle, |-\frac{3}{2}\rangle, |\frac{1}{2}\rangle, |-\frac{1}{2}\rangle)^T$. After taking into account the π phase winding of θ, Ψ transforms by

$$\Psi_a \to \Psi'_a = iU(\hat{n}, \pi)_{\alpha\beta}\Psi_\beta = \begin{pmatrix} 0 & W \\ W^\dagger & 0 \end{pmatrix}_{\alpha\beta} \Psi_\beta \quad (8)$$

where W is an $SU(2)$ Berry phase depending on the direction of $\hat{n}$ on the S^3 sphere as

$$W(\hat{n}) = \begin{pmatrix} n_3 + in_2 & -n_1 - in_5 \\ n_1 - in_5 & n_3 - in_2 \end{pmatrix}. \quad (9)$$

The non-conservation of spin in this adiabatic process is not surprising because the $SO(5)$ symmetry is completely broken in the configuration depicted in Fig. 1 B.

A more interesting but related concept is the Cheshire charge, which means that a pair of the HQV loop can carry $SO(4)$ spin quantum numbers. An intersection between the HQV loop and a perpendicular plane is depicted in Fig. 2, where $\phi_{1,2}$ are respect to the vortex and anti-vortex cores respectively. Without loss of generality, we assume $\hat{d}(\vec{r}) \to \hat{e}_4$ as $r \to \infty$ where an $SO(4)$ symmetry generated by $\Gamma_{ab}(a, b = 1, 2, 3, 5)$ is preserved. In analogy to Fig. 1 B, the $\hat{d}$ vector is described by the difference between two azimuthal angles $\Delta\phi = \phi_2 - \phi_1$ as

$$\hat{d}(\hat{n}, \Delta\phi) = \cos\frac{\Delta\phi}{2}\hat{e}_4 - \sin\frac{\Delta\phi}{2}\hat{n}, \quad (10)$$

where $\hat{n}$ again is a unit vector on the S^3 equator. This classical configuration is called a phase-sharp state denoted as $|\hat{n}\rangle_{vt}$. Because the above $SO(4)$ symmetry is only broken within a small region around the HQV loop, quantum fluctuations of $\hat{n}$ dynamically restore the $SO(4)$ symmetry as described by the Hamiltonian

$$H_{rot} = \sum_{a,b=1,2,3,5} \frac{M_{ab}^2}{2I}, \; M_{ab} = i(\hat{n}_a\partial_{\hat{n}_b} - \hat{n}_b\partial_{\hat{n}_a}), \quad (11)$$

with the moment of inertial $I = \chi_{sp}\int d^D r \; \rho_0 \sin^2\frac{\Delta\phi}{2}$. Thus the zero modes $|\hat{n}\rangle_{vt}$ are quantized into the global $SO(4)$ Cheshire charge states, which are a non-Abelian generalization of the $U(1)$ case in the ^{3}He-A phase [13]. The global Cheshire charge density is localized around the HQV loop. In contrast, the Cheshire charge in gauge theories is non-localized [11].

The $SO(4)$ algebra can be grouped into two commutable sets of $SU(2)$ generators as $T_1(T'_1) = \frac{1}{4}(\pm\Gamma_{35} - \Gamma_{12}), T_2(T'_2) = \frac{1}{4}(\pm\Gamma_{31} - \Gamma_{25}), T_3(T'_3) = \frac{1}{4}(\pm\Gamma_{23} - \Gamma_{15})$. $T_{1,2,3}$ and $T'_{1,2,3}$ act in the subspaces spanned by $|\pm\frac{3}{2}\rangle$ and $|\pm\frac{1}{2}\rangle$, respectively. $SO(4)$ representations are denoted

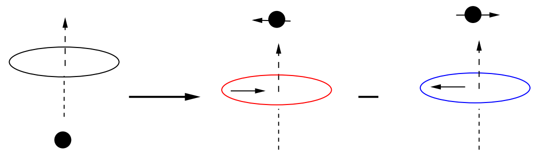

FIG. 3: The topological generation of quantum entanglement. The initial state $|i\rangle$ is a product state of a zero charged HQV loop and a quasiparticle with $S_z = 3/2$, both of which are a singlet of the $SU(2)$ group of T' acting in the subspace spanned by $|\pm\frac{1}{2}\rangle$. The final state $|f\rangle$ is an EPR state of the T' group as described by Eq. 14.

by $|T, T_3; T', T_3'\rangle$, i.e., the direct-product of representations of two $SU(2)$ groups. The HQV loop in the $SO(4)$ Cheshire charge eigenstates is defined as $|TT_3; T'T_3'\rangle_{vt} = \int_{\hat{n}\in S^3} d\hat{n}\ F_{TT_3;T'T_3'}(\hat{n})\ |\hat{n}\rangle_{vt}$, where $F_{TT_3;T'T_3'}(\hat{n})$ are the S^3 sphere harmonic functions. Thus $|TT_3; T'T_3'\rangle_{vt}$ is the non-Abelian generalization of the usual number-sharp state in $U(1)$ theories.

When a particle passes the HQV loop, $\Delta\phi$ changes from 0 to 2π. The conservation of the $SO(4)$ spin is ensured by exciting the Cheshire charges and generating quantum entanglement between the particle and the HQV loop. We demonstrate this process explicitly through a concrete example, with the initial state $|i\rangle$ made up from a zero charged HQV loop and a quasi-particle with $S_z = \frac{3}{2}$ as

$$|i\rangle \;=\; \int_{\hat{n}\in S^3} d\hat{n}\ |\hat{n}\rangle_{vt} \otimes (u\ c^\dagger_{\frac{3}{2}} + v\ c_{-\frac{3}{2}})|\Omega\rangle_{qp}, \quad (12)$$

where $|\Omega\rangle_{qp}$ is the vacuum for Bogoliubov particles. For each phase-sharp state $|\hat{n}\rangle_{vt}$, the particle changes spin according to Eq. 8 in the final state $|f\rangle$. The superposition of the non-Abelian phase gives

$$\begin{aligned}
|f\rangle \;=\; & \int_{\hat{n}\in S^3} d\hat{n}\ \Big\{u\ (W^\dagger_{11} c^\dagger_{\frac{1}{2}} + W^\dagger_{21} c^\dagger_{-\frac{1}{2}}) + v\ (W^T_{12} c_{\frac{1}{2}} \\
& +\ W^T_{22} c_{-\frac{1}{2}})\Big\} \quad |\hat{n}\rangle_{vt} \otimes |\Omega\rangle_{qp} \\
=& \int_{\hat{n}\in S^3} d\hat{n}(\hat{n}_3 - i\hat{n}_2)|\hat{n}\rangle_{vt} \otimes (u\ c^\dagger_{\frac{1}{2}} + v\ c_{-\frac{1}{2}})|\Omega\rangle_{qp} \\
-& \int_{\hat{n}\in S^3} d\hat{n}(\hat{n}_1 - i\hat{n}_5)|\hat{n}\rangle_{vt} \otimes (u\ c^\dagger_{-\frac{1}{2}} - v\ c_{\frac{1}{2}})|\Omega\rangle_{qp}, \quad (13)
\end{aligned}$$

as depicted in Fig. 3. In terms of the $SO(4)$ quantum numbers, $|i\rangle$ is a product state of $|00;00\rangle_{vt} \otimes |\frac{1}{2}\frac{1}{2};00\rangle_{qp}$, and $|f\rangle$ is

$$|\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{-1}{2}\rangle_{vt} \otimes |00;\frac{1}{2}\frac{1}{2}\rangle_{qp} - |\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}\rangle_{vt} \otimes |00;\frac{1}{2}\frac{-1}{2}\rangle_{qp}. \quad (14)$$

In the channel of (T', T_3'), the final state is exactly an entangled Einstein-Podolsky-Rosen (EPR) pair made up from the HQV loop and the quasi-particle. We note that this mechanism of generating the quantum entanglement is entirely topological, dependent only on whether the trajectory of the quasi-particle lies inside or outside of the HQV loop. In contrast, the HQV loop in ^{3}He-A system only exhibits the $U(1)$ Cheshire charge, thus the final state is still a product state without the generation of entanglement.

Similarly, the entanglement between a spin wave impulse and the HQV loop can also be generated. We consider the four local transverse bases $\hat{e}_b (b = 1, 2, 3, 5)$ at $\Delta\phi = 0$. Assume that the initial state made up from the HQV loop and the spin wave is $|i'\rangle = \int_{\hat{n}\in S^3} d\hat{n}\ |\hat{n}\rangle_{vt} \otimes (\hat{e}_1 + i\hat{e}_5)_{sw}$, where spin wave impulse carries $S_z = 2$. For each phase-sharp state $|\hat{n}\rangle_{vt}$ of the HQV, the frame bases at $\Delta\phi = 2\pi$ transform to $\hat{e}_a \to \hat{e}_a - 2\hat{n}(\hat{e}_a \cdot \hat{n})$. Thus the entanglement is generated in the final state $|f'\rangle$ as

$$\begin{aligned}
|f'\rangle \;=\; & \int_{\hat{n}\in S^3} d\hat{n}\ |\hat{n}\rangle_{vt} \otimes (\hat{e}_1 + i\hat{e}_5)_{sw} \\
& -\ 2\int_{\hat{n}\in S^3} d\hat{n}\ (n_1 + in_5) n_b |\hat{n}\rangle_{vt} \otimes \hat{e}_{b,sw}. \quad (15)
\end{aligned}$$

Recently, Bose condensation of the ^{174}Yb atom and sympathetic cooling between ^{174}Yb and the fermionic atom of ^{171}Yb [18] have been achieved. Their electron configurations are the same as the Ba atoms except an inside full-filled $4f$ shell, thus the spin-3/2 systems of ^{135}Ba and ^{137}Ba can be possibly realized in the near future. At the present time, scattering lengths of these two Ba atoms are not available. However, considering the rapid developments in this field, we are optimistic about the realization of the quintet pairing state and the associated non-Abelian topological defects.

In summary, we have studied the quintet pairing state in spin 3/2 fermionic systems with the $SO(5)$ symmetry, including its Goldstone modes and the non-Abelian topological defects. The non-Abelian Berry phase effect and the Cheshire charge behavior are analyzed in detail. The topological mechanism of generating the quantum entanglement between quasi-particles and the HQV loop could be useful for topological quantum computation.

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