Topological Orbital Angular Momentum Hall Current

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We show that there is a fundamental difference between spin Hall current and orbital angular momentum Hall current in Rashba- Dresselhaus spin orbit coupling systems. The orbital angular momentum Hall current has a pure topological contribution which is originated from the existence of magnetic flux in momentum space while there is no such topological nature for the spin Hall current. Moreover, we show that the orbital Hall conductance is always larger than the spin Hall conductance in the presence of both couplings. The topological part is expected to be free from the effect of disorder due to the topological nature. Therefore, the orbital angular momentum Hall current should be the major effect in real experiments.

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The intrinsic spin Hall effect[1, 2] in spin orbit coupling systems has attracted intensive research attentions recently. The effect has been studied in a broad class of spin-orbit coupling models, such as Luttinger, Rashba and Dresselhaus Hamiltonians [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Although the spin Hall effect seems to be universal in strong spin orbit coupling systems and even most recently, two experimental groups [22, 23] have reported that their experimental results are consistent with the physics of the spin Hall effect predicted in two dimensional electron systems, it is still a fundamental question regarding whether the effect is a true "observable" effect.

There are at least two issues regarding this observability. The first is the role of the disorder. Several theoretical analyses have shown that the spin Hall effect in the Rashba or Dresselhaus systems disappears in the presence of disorder[24, 25, 26], even in the weak disorder limit. These results suggest that the spin Hall effect is only measurable in the ballistic regime. The second question is even more fundamental. In the spin orbit coupling systems, it is improper to define a pure spin current since the orbit and spin are coupled. It is the total angular momentum which is the true observable quantity. This point has been raised by Zhang and Yang[27]. They have argued that the intrinsic spin Hall current is always accompanied by an equal but opposite intrinsic orbital angular momentum Hall current. Therefore, the intrinsic spin Hall effect does not induce spin accumulation at the edge of the sample. This argument leads to a conclusion that there is no measurable effect induced by the spin Hall current.

Besides the two issues, there are also issues which are related to the proper definition of the spin current. Since spins are not conserved in spin orbit coupling systems, a conserved quantity may be needed to replace the spin operator in order to define a conserved spin current. In the Luttinger spin orbit coupling systems, such formulations have been proposed[8]. The conserved spin current has a very beautiful physical interpretation. The origin of the current is from the existence of a monopole structure in momentum space, which are directly derived from the Berry phase in spin orbit coupling systems[1, 3] when Hamiltonian is projected to double degenerated helicity bands. Due to this topological nature, the conserved spin Hall current in general can be called as topological spin Hall current. The same topological nature also accounts for the anomalous Hall effect[28, 29] in ferromagnetic metal, which has been confirmed in experimental[30] and numerical studies[31]. However, the topological spin Hall current is not universal for other spin orbit coupling systems. In the Rashba or Dresselhaus spin orbit coupling systems, the topological spin Hall current is zero[3]. The reason is that in the Luttinger case, the spin operator after being projected to each band is nontrivial while it disappears in the Rashba and Dresselhaus case.

The topological part of the current can be viewed as an intrinsic property associated with an individual band. It is protected by the topological nature and is insensitive to the effect of the weak disorder. In this letter, we study the orbital angular momentum Hall current in the presence of both Rashba and Dresselhaus spin orbit coupling. In this model, the orbital Hall current has a very different origin from the spin Hall current. The orbital Hall current has a fundamental topological nature. It is originated from the existence of the magnetic flux in momentum space while there is no such topological nature for the spin Hall current. Contradictory to the result in ref.[27], the orbital Hall current does not cancel the spin Hall current in general and the orbital Hall conductance does not change sign for any coupling parameters while it has been shown that when the Dresselhaus coupling is larger than the Rashba coupling strength, the spin Hall conductance changes the sign[18, 19]. In fact, the orbital Hall current is larger than the spin current in the presence of both couplings. Therefore, the total angular momentum current in general is dominated by the orbital Hall current and is non-vanishing. The intrinsic total angular momentum Hall effect does generate magnetization at the edge of samples. Moreover, the orbital current is free from the effect of impurity due to the topological nature. From these results, we argue that the orbital Hall current dominates the transport properties in such systems.

We consider a Hamiltonian for a two dimensional electron system with both Rashba and Dresselhaus spin orbit coupling

$$\hat{H} = \frac{p^2}{2m} + H_R + H_D, \tag{1}$$

where the Rashba term is

$$H_R = \alpha(\vec{\sigma} \times \vec{p}) \cdot \hat{z} = \alpha(p_y \sigma_x - p_x \sigma_y) \tag{2}$$

and the Dresselhaus term is

$$H_D = \beta(p_y \sigma_y - p_x \sigma_x). \tag{3}$$

The Hamiltonian can be diagonalized by a unitary matrix

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} -\imath e^{-\imath\theta} & \imath e^{-\imath\theta} \\ 1 & 1 \end{pmatrix} \tag{4}$$

with $\tan\theta = \frac{\alpha p_y - \beta p_x}{\alpha p_x - \beta p_y}$, and

$$U^\dagger H U = \frac{p^2}{2m} - E_p \sigma_z. \tag{5}$$

with

$$E_p = \sqrt{(\alpha^2 + \beta^2)p^2 - 4\alpha\beta p_x p_y}, \tag{6}$$

which is the band gap due to the spin orbit coupling.

First, we follow the standard definition of the current. Similar to the spin current, we can define the orbital angular momentum current as

$$O^i_j = \frac{1}{2}\{L_i, v_j\} \tag{7}$$

where $L_i = \epsilon_{ijk} x_j p_k$ are the orbital angular momentum operators and $v_j = \frac{\partial H}{\partial p_j}$ are velocity operators.

By a straightforward calculation following the Kubo formula in the ballistic regime which is given by

$$\sigma^{L_z}_{yx} = e \sum_{\lambda, \lambda' \neq \lambda} \int \frac{d\vec{p}}{(2\pi)^2} (f_{\lambda',p} - f_{\lambda,p}) \frac{\mathrm{Im}[\langle \lambda', p | O^z_y | \lambda, p \rangle \langle \lambda, p | v_x | \lambda', p \rangle]}{(E_{p\lambda'} - E_{p\lambda})^2}, \tag{8}$$

the orbital angular momentum Hall current is calculated to be

$$O^z_y = \sigma^{L_z}_{yx} E_x = \frac{e}{8\pi} \frac{\alpha^2 + \beta^2}{|\alpha^2 - \beta^2|} E_x \tag{9}$$

for $\alpha \neq \beta$, where we assume the applied electric field is along the x direction. For $\alpha = \beta$, $O^z_y = 0$. One can compare this result with the spin Hall current, which is given by[18, 19]

$$\frac{J^z_y}{E_x} = \begin{cases} -\frac{e}{8\pi} & \alpha > \beta \\ 0 & \alpha = \beta \\ \frac{e}{8\pi} & \alpha < \beta \end{cases} \tag{10}$$

First, we notice that the orbital conductance has the same absolute value as the spin conductance but with opposite sign when only the Rashba coupling is present, i.e. $\beta = 0$. This result is obtained in ref.[27], which can be easily understood because $L_z + S_z$ is conserved when $\beta = 0$. Secondly, the orbital conductance is the same as the spin conductance when only the Dresselhaus coupling is present. This result shows that the total angular momentum Hall current is non-vanishing in the system. The result can also be understood as follows. Since $L_z - S_z$ is conserved when $\alpha = 0$, the orbital and spin current must be equal. Thirdly, both spin and orbital conductances are discontinuous across $\alpha = \beta$. The orbital conductance remains non-negative for any spin orbit coupling parameters while the spin conductance changes the sign across $\alpha = \beta$. Fourthly, the absolute value of the spin conductance is universal in the presence of the spin orbit coupling while the orbital conductance is dependent on the coupling strength. In fact, the value of the orbital conductance is always larger than or equal to the spin conductance. Therefore, the total angular momentum current is always dominated by its orbital part. In a system with the larger value of the Rashba coupling than the Dresselhaus coupling, this result suggests that the direction of magnetization at the edge of the sample due to the flowing angular momentum Hall current should be opposite to the previous predictions based on the pure spin Hall current.

Strictly speaking, the result that one obtains from the Kubo formula is not an intrinsic property of the individual band. The spin and orbital Hall currents are not conserved quantities. It is also easy to see that in the Kubo formulism, both bands are required. In ref.[1, 8], the authors show that a conserved spin Hall current can be defined in the Luttinger spin orbit coupling systems. In this definition, the spin Hall current becomes an intrinsic property of the individual band, which can be derived from an effective Hamiltonian associated with the individual band. Moreover, the current has a fundamental topological nature which entirely comes from the Berry phase, which can be viewed as a monopole in momentum space[1]. In the Rashba-Dresselhaus spin orbit coupling systems, such a definition of the spin current does not exist. In fact, the topological contribution to the spin Hall current is zero. However, in the following, we will show that the orbital angular momentum Hall conductance can come from a fundamental topological nature due to the spin orbit coupling.

When the spin orbit coupling is very large, the independent effective Hamiltonian should exist for each individual band. Such Hamiltonian can be constructed by projecting the total Hilbert space in the original model to the space in the individual band. For the Rashba-Dresselhaus model, the effective Hamiltonians for two bands after projection are respectively given by

$$H_1 = \frac{p^2}{2m} - E_p \tag{11}$$

$$H_2 = \frac{p^2}{2m} + E_p \tag{12}$$

As shown in ref.[1], such a projection introduces a nontrivial gauge potential in momentum space. In our case, the gauge potential $\vec{A}$ is given by

$$\vec{A} = \langle L_z \rangle \vec{A}_0, \tag{13}$$

where $\langle L_z \rangle$ is the value of orbital angular momentum in the single particle state, which is given by

$$\langle L_z \rangle = -\frac{(\alpha^2 - \beta^2)p^2}{2E_p^2}. \tag{14}$$

and $\vec{A}_0 = (\frac{p_y}{p^2}, -\frac{p_x}{p^2})$. Notice that the gauge structure and the value of $\langle L_z \rangle$ are identical for both bands. Physically, this gauge describes an angular momentum flux with the value equal to $\langle L_z \rangle$ at the origin of the momentum space, i.e. $p = 0$. The coordinate operators which are the derivatives with respect to momentum are modified to the covariant derivatives, namely

$$x = i\frac{\partial}{\partial p_x} + A_x, y = i\frac{\partial}{\partial p_y} + A_y \tag{15}$$

In the Luttinger model, It has been shown that the gauge generates a noncommutative geometry[1]. In this case, the commutation relation, $[x, y]$, is zero everywhere except at the origin. However, the integral in an area which includes the origin of the momentum space is non-vanishing, namely,

$$\frac{1}{4\pi^2}\int [x, y] dp_x dp_y = i\frac{1}{4\pi^2}\oint \vec{A}\cdot d\vec{p} \neq 0 \tag{16}$$

In the presence of the electric field, the total effective Hamiltonian for the individual bands are

$$H_{eff}^{1,2} = H_{1,2} + exE_x \tag{17}$$

Thus the orbital angular momentum Hall current is given by

$$\begin{aligned}\frac{O_y^z}{E_x} &= -i\frac{e}{4\pi^2}\int dp_x dp_y \langle L_z \rangle [x, y] \\ &= \frac{e}{4\pi^2}\oint \langle L_z \rangle^2 \vec{A}_0 \cdot d\vec{p}\end{aligned} \tag{18}$$

The above integral vanishes when the Fermi surface does not contain the origin. When the Fermi energy is larger than zero, the integral is nonzero. Since the Fermi surfaces in both bands contain zero, Therefore the total topological orbital conductance in the system is doubled. Therefore,

$$\frac{O_y^z}{E_x} = \begin{cases} 0 & E_F < 0 \\ \frac{e}{4\pi}\frac{\alpha^2+\beta^2}{|\alpha^2-\beta^2|} & E_F > 0 \end{cases} \tag{19}$$

Where E_F is the Fermi energy. The topological orbital conductance is slightly different from the previous result derived from the Kubo formula for the ordinary orbital conductance, which is also the case for the topological spin current in the Luttinger case[1, 3]. This is very natural. In fact, one can easily derive different contributions to orbital Hall current based on the semiclassical approach[6].

Several remarks are in order. First, as shown above, the topological orbital angular momentum currents are the same in both bands, unlike the spin Hall current which runs opposite in two bands. Secondly, the orbital Hall current is a real topological effect. If one compares it with the topological spin current in the Luttinger case, the individual contribution to the orbital Hall current from the single particle based on our effective Hamiltonian is zero but the integral on the total Fermi surface is non-vanishing, unlike the spin Hall current in which the contribution from each single particle states is non-vanishing[1]. A simple physical origin of the orbital Hall current can be understood as follows. Since an orbital angular momentum flux exists in the momentum space, by applying a potential with an electric field, it feels the force and drifts in the perpendicular direction to the force just like the motion of the ordinary magnetic flux in real space. This motion creates the current. A simple picture is sketched in fig.1. Thirdly, the topological orbital current should be expected to be free from the disorder. The spin Hall current has been proved to vanish even in the weak disorder limit. The reason is simply that the effect of spin Hall current requires the presence of both bands which is easily seen in the Kubo formula. However, the topological nature of the orbital Hall current maintains as long as the band gap at two Fermi surfaces is larger than the energy scale of the disorder. Finally, when the Rashba and Dresselhaus coupling strength are comparable, the orbital Hall conductance can be very big. When the orbital Hall conductance is too big, it is possible that a spontaneous magnetization takes place inside the bulk.

In conclusion, a topological orbital angular momentum Hall current exists in the Rashba-Dresselhaus spin orbit coupling systems. It has an important part which is purely originated from the existence of magnetic flux in momentum space in the adiabatic limit. This orbital current is always larger than the spin Hall current when both couplings are present. Therefore, the orbital angular momentum current should be the major player in real experiments. The prediction can be tested in the pa-

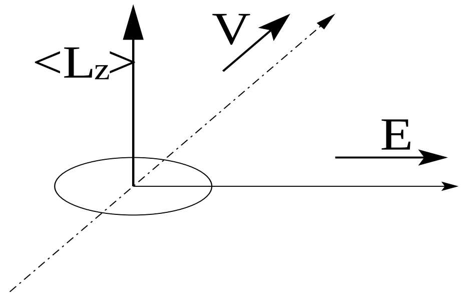

FIG. 1: A simple sketch of an orbital angular momentum flux. The applied electric field creates a force on the flux and force it to drift in the perpendicular direction, which produces the orbital current.

rameter region when the Rashba coupling is larger than the Dresselhaus coupling since the orbital and spin current runs in opposite directions. The total accumulated magnetization at edges is expected to be opposite to the prediction derived from a pure spin Hall current.

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