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A Four-Dimensional Generalization of the Quantum Hall Effect

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We construct a generalization of the quantum Hall effect, where particles move in four dimensional space under a $SU(2)$ gauge field. This system has a macroscopic number of degenerate single particle states. At appropriate integer or fractional filling fractions the system forms an incompressible quantum liquid. Gapped elementary excitation in the bulk interior and gapless elementary excitations at the boundary are investigated.

Most strongly correlated systems develop long-range order in the ground state. Familiar ordered states include superfluidity, superconductivity, antiferromagnetism, and charge density wave (J). However, there are special quantum disordered ground states with fractionalized elementary excitations. In one-dimensional (1D) systems, Bethe's Ansatz (2) gives exact ground-state wave functions of a class of Hamiltonians, and the elementary excitations are fractionalized objects called spinons and holons. In the 2D quantum Hall effect (QHE) (3, 4), Laughlin's wave function (3) describes an incompressible quantum fluid with fractionally charged elementary excitations. This incompressible liquid can also be described by a Chern-Simons-Landau-Ginzburg field theory (5), whose long-distance limit depends only on the topology but not on the metric of the un-

derlying space (6). These two special quantum disordered ground states are the focus of much theoretical and experimental studies, because they give deep insights into the interplay between quantum correlations and dimensionality and into how this interplay can give rise to fractionalized elementary excitations.

In view of their importance, it is certainly desirable to generalize these quantum wave functions to higher dimensions. However, despite repeated efforts, the Bethe's Ansatz solutions have not yet been generalized to dimensions higher than one. Laughlin's wave function uses properties that seem to be special to the 2D space. In this work, we shall report the generalization of the quantum Hall system to four space dimensions, and this system shares many compelling similarities with the 2D counterpart. In the 2D QHE, the charge current is carried in a direction perpendicular to the applied electric field (and also perpendicular to the magnetic field, which

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is applied normal to the 2D electron gas). In four space dimensions, there are three independent directions normal to the electric field, and there appears to be no unique direction for the current. A crucial ingredient of our generalization is that the particles also carry an internal $SU(2)$ spin degree of freedom. Because there are exactly three independent directions for the spin, the particle current can be uniquely carried in the direction where the spins point. At special filling factors, the quantum disordered ground state of our 4D QHE is separated from all excited states by a finite energy gap, and the lowest energy excitations are fractionally charged quasi-particles.

Although all excitations have finite energy gaps in the bulk interior, elementary excitations at the three dimensional boundary of this quantum field are gapless, in analogy with the edge states of the quantum Hall effect (7-9). These boundary excitations could be used to model the relativistic elementary particles, such as photons and gravitons. In contrast to conventional quantum field theory approach, this model has the advantage that the short-distance physics is finite and self-consistent. In fact, the magnetic length in this model provides a fundamental lower limit on all length scales. This feature shares similarity to noncommutative quantum field theory and string theory of elementary particles.

A 4D generalization of the quantum Hall problem. In the QHE problem, it is advantageous to consider compact spherical spaces that can be mapped to the flat Euclidean spaces by standard stereographical mapping (10). Eigenstates in the QHE problem are called Landau levels, and we first review the lowest Landau level (III) defined on the 2D sphere, denoted by S^2 . A point X_i on S^2 with radius R can be described by dimensionless vector coordinates $x_i = X_i/R$, with $i = 1, 2, 3$, which satisfy $x_i^2 = 1$. However, S^2 has a special property that one can also take the "square root" of the vector coordinate x_i through the introduction of the complex spinor coordinates ϕ_σ , with $\sigma = 1, 2$. These spinor coordinates are defined by

$$x_i = \bar{\phi}_\sigma(\sigma_i)_{\sigma\sigma'}\phi_{\sigma'}, \quad \bar{\phi}_\sigma\phi_\sigma = 1 \quad (1)$$

where σ_i are the three Pauli spin matrices. If there is a magnetic monopole of strength g at the center of S^2 , satisfying the Dirac quantization condition $eg = I = \text{integer or half integer}$, then the normalized eigenfunctions in the III are just the algebraic products of the spinor coordinates

$$\langle x|I,m\rangle = \sqrt{\frac{(2I)!}{(I+m)!(I-m)!}} \phi_1^{I+m} \phi_2^{I-m} \quad (2)$$

Here $m = -I, -I + 1, \dots, I - 1, I$, therefore the ground state is $2I + 1$ fold degenerate. Any states in the III can be expanded in terms of a homogeneous polynomial of ϕ_1 and ϕ_2 with degree $2I$. Notice that the conjugate coordinate $\bar{\phi}_\sigma$ does not enter the wave function in the III.

We see that the crucial algebraic structure of the QHE problem is the fractionalization of a vector coordinate into two spinor coordinates. Therefore, in seeking a higher dimensional generalization of the QHE problem, we need to find a proper generalization of Eq. 1. As the generalization of the three Pauli matrices is the five 4×4 Dirac matrices Γ_a , satisfying the Clifford algebra $\{\Gamma_a, \Gamma_b\} = 2\delta_{ab}$, we generalize Eq. 1 to

$$x_a = \bar{\Psi}_\alpha(\Gamma_a)_{\alpha\alpha'}\Psi_{\alpha'}, \quad \bar{\Psi}_\alpha\Psi_\alpha = 1 \quad (3)$$

Here, Ψ_α is a four-component complex spinor with $\alpha = 1, 2, 3, 4$, and x_a is a five-component real vector. From the normalization condition of the Ψ spinor it may be seen that $x_a^2 = 1$, therefore, $X_a = Rx_a$ describes a point of the 4D sphere S^4 with radius R . From this heuristic reasoning, one may hope to find a 4D generalization of the QHE problem, where the wave functions in the ground states are described by the products of Ψ_α spinors, in a natural generalization of Eq. 2. Equations 1 and 3 are known in the mathematical literature as the first and the second Hopf maps (11). The problem now is to find a Hamiltonian for which these are the exact ground state wave functions.

An explicit solution to Eq. 3 can be expressed as

$$\Gamma^{(1,2,3)} = \begin{pmatrix} 0 & -i\sigma_i \\ -i\sigma_i & 0 \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \sqrt{\frac{1+x_5}{2}} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad \begin{pmatrix} \Psi_3 \\ \Psi_4 \end{pmatrix} = \sqrt{\frac{1}{2(1+x_5)}} (x_4 - ix_i\sigma_i) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (5)$$

where (u_1, u_2) is an arbitrary two-component complex spinor satisfying $\bar{u}_\sigma u_\sigma = 1$. Any $SU(2)$ rotation on u_σ preserves the normalization condition and maps to the same point x_a on S^4 . From the explicit form of Ψ_α , one can compute the geometric connection (Berry's phase) $\bar{\Psi}_\alpha d\Psi_\alpha$ (11), where the differentiation operator d acts on the vector coordinates x_a , subject to the condition $x_a dx_a = 0$. One finds $\bar{\Psi}_\alpha d\Psi_\alpha = \bar{u}_\sigma (a_a dx_a)_{\sigma\sigma'} u_{\sigma'}$, $a_5 = 0$, and

$$a_\mu = \frac{-i}{1+x_5} \eta_{\mu\nu}^i x_\nu I_i, \quad \eta_{\mu\nu}^i = \epsilon_{i\mu\nu 4} + \delta_{i\mu} \delta_{4\nu} - \delta_{i\nu} \delta_{4\mu} \quad (6)$$

where $I_i = \sigma_i/2$ and $\eta_{\mu\nu}^i$ is also known as the 'tHooft symbol. a_μ is the $SU(2)$ gauge potential of a Yang monopole defined on S^4 (12). Upon a conformal transformation from S^4 to the 4D Euclidean space R^4 (13), this gauge potential is transformed to the instanton solution of the $SU(2)$ Yang-Mills theory (14). We shall call I_i a $SU(2)$ isospin matrix, and the gauge potential defined in Eq. 6 can be generalized to an arbitrary representation I of the $SU(2)$ Lie algebra $[I_i, I_j] = i\epsilon_{ijk} I_k$. The gauge field strength can be calculated from the form of the gauge potential. From the covariant derivative $D_a = \partial_a + a_a$, we define the field strength as $f_{ab} = [D_a, D_b]$. Both a_a and f_{ab} are matrix valued and can be generally expressed in terms of the isospin components $a_a = -ia_a^i I_i$ and $f_{ab} = -if_{ab}^i I_i$. In terms of these components, we find $f_{5\mu}^i = -(1+x_5)a_\mu^i$ and $f_{\mu\nu}^i = x_\nu a_\mu^i - x_\mu a_\nu^i - \eta_{\mu\nu}^i$. In addition to the dimensionless quantities a_μ and f_{ab} , we shall sometimes also use dimensional quantities defined by $A_\mu = R^{-1}a_\mu(X/R)$, and $F_{ab} = R^{-2}f_{ab}(X/R)$.

With this introduction and motivation, we are now in a position to introduce the Hamiltonian of our quantum mechanics problem. The symmetry group of S^4 is $SO(5)$, generated by the angular momentum operator $L_{ab}^{(0)} = -i(x_a \partial_b - x_b \partial_a)$. The Hamiltonian of a single particle moving on S^4

$$\text{can be expressed as } H = \frac{\hbar^2}{2MR^2} \sum_{a<b} (L_{ab}^{(0)})^2,$$

where M is the inertia mass and R is the radius of S^4 . Coupling to a gauge field a_a may be introduced by replacing ∂_a with the covariant derivative D_a . Under this replacement, $L_{ab}^{(0)}$ becomes $\Lambda_{ab} = -i(x_a D_b - x_b D_a)$. The Hamiltonian of our generalized QHE problem is therefore given by

$$H = \frac{\hbar^2}{2MR^2} \sum_{a<b} \Lambda_{ab}^2 \quad (7)$$

This Hamiltonian has an important parameter I , defined by $I_i^2 = I(I + 1)$, which specifies the dimension of the $SU(2)$ representation in the potential (Eq. 6).

Unlike $L_{ab}^{(0)}$, Λ_{ab} does not satisfy the $SO(5)$ commutation relation. However, one can define $L_{ab} = \Lambda_{ab} - if_{ab}$, which does satisfy the $SO(5)$ commutation relation. Although only a subset of $SO(5)$ irreducible representations can be generated from the $L_{ab}^{(0)}$ operators, Yang (15) showed that $L_{ab}^{(0)}$ generates all $SO(5)$ irreducible representations. In general, a $SO(5)$ irreducible representation is labeled by two integers (p, q) , with $p \geq q \geq 0$. For such a representation, the Casimir operator and the dimensionality are given by $C(p, q) = \sum_{a<b} L_{ab}^2 = \frac{p^2}{2}$

$$+ \frac{q^2}{2} + 2p + q \text{ and } d(p, q) = (1 + q)(1 + p - q) \left(1 + \frac{p}{2}\right) \left(1 + \frac{p+q}{3}\right) \text{ respectively.}$$

However, for a given I , these two integers are related by $p = 2I + q$. One can show that $\sum_{a<b} \Lambda_{ab}^2 = \sum_{a<b} L_{ab}^2 - 2I_i^2$. Therefore, for a given I , the energy eigenvalues of the Hamiltonian (Eq. 7) are given by

$$E(p = 2I + q, q) = \frac{\hbar^2}{2MR^2} [C(p = 2I + q, q) - 2I(I + 1)] \quad (8)$$

with degeneracy $d(p = 2I + q, q)$. The ground state, which is the lowest $SO(5)$ level for a given I , is obtained by setting $q = 0$, and we see that it is $\frac{1}{6}(p + 1)(p + 2)(p + 3)$ fold degenerate. Therefore, the dimension of the $SU(2)$ representation plays the role of the magnetic flux, while q plays the role of the Landau level index. States with $q > 0$ are separated from the ground state by a finite energy gap.

Besides the energy eigenvalues and the degeneracy, we need to know the explicit form of the ground-state wave function. Yang (15) did find the wave function for all the (p, q) states; however, his solution is expressed in a basis that is hard to work with for our purpose. Realizing the spinor structure we outlined above, we can express the wave functions of the lowest $SO(5)$ levels $(p, 0)$ in a very simple form. First, one can check explicitly that Ψ_α given in Eq. 5 is indeed an eigenfunction of the Hamiltonian (Eq. 7) with $I = 1/2$. This follows from the fact that it is a $SO(5)$ spinor under the generators L_{ab} :

$$L_{ab} \Psi_\alpha = -\frac{1}{2} (\Gamma_{ab})_{\alpha\beta} \Psi_\beta. \text{ From this, one can}$$

see that $\Psi_{\alpha_1 \dots \alpha_p}(x) = \Psi_{\alpha_1} \dots \Psi_{\alpha_p}$ transforms as an irreducible spinor under the $SO(5)$ group. Therefore, the complete set of normalized basis functions in the lowest $SO(5)$ level

$(p, 0)$ with orbital coordinate $x_a = \bar{\Psi} \Gamma_a \Psi$ and isospin coordinate $n_i = \bar{u} \sigma_i u$ is given by

$$\langle x_a, n_i | m_1, m_2, m_3, m_4 \rangle = \sqrt{\frac{p!}{m_1! m_2! m_3! m_4!}} \Psi_1^{m_1} \Psi_2^{m_2} \Psi_3^{m_3} \Psi_4^{m_4} \quad (9)$$

with integers $m_1 + m_2 + m_3 + m_4 = p$. These basis functions in the lowest $SO(5)$ level are the exact eigenstates of the Hamiltonian (Eq. 7) with $d(p, 0)$ fold degenerate

eigenvalue of $\frac{\hbar^2}{2MR^2} p$. They are the natural generalizations of the wave functions in the III (Eq. 2) of the QHE problem. The very simple form of the single-particle wave function (Eq.

9) introduced here greatly helps calculations of the many-body wave function.

An incompressible quantum spin liquid. We are now in the position to consider the quantum many-body problem involving N fermions. The simplest case to consider is $N = d(p, 0)$, when the lowest $SO(5)$ level is completely filled. In this case, the filling factor $\nu \equiv N/d(p, 0) = 1$, and the many-body ground-state wave function is unique.

Before presenting the explicit form of the wave function, we first need to discuss the thermodynamic limit in this problem, as it is rather nontrivial. We shall consider the limit $p = 2I \rightarrow \infty$ and $R \rightarrow \infty$ while keeping q constant. For energy eigenvalues in Eq. 8 to

be finite, we need $l_0 = \lim_{R \rightarrow \infty} \frac{R}{\sqrt{p}}$ to

approach a finite constant, which can be defined as the ‘‘magnetic length’’ in this prob-

lem. In this limit, $E(q) = \frac{\hbar^2}{2MI_0^2} (1 + q)$, and

the single-particle energy spacing is finite. At $\nu = 1$, $N \sim p^3 \sim R^6$, the naively defined particle density N/R^4 would be infinite. However, we need to keep in mind that each particle also has an infinite number of isospin degrees because $I \rightarrow \infty$. Taking this fact into account, we see that the volume of the configuration space, defined to be the product of the volume in orbital and isospin space, is $R^4 \times R^2$. Therefore, the density $n = N/R^6$ is actually finite in this limit.

Using $A = \{m_1, m_2, m_3, m_4\} = 1, \dots, d(p, 0)$ to label the single-particle states, the many-particle wave function is given by a Slater determinant

$$\Phi(x_1 \dots x_N) = \Psi_{A1}(x_1) \dots \Psi_{AN}(x_N) \epsilon_{A1 \dots AN} \quad (10)$$

The density correlation function $\rho(x, x') = \frac{1}{(N-2)!} \int dx_3 \dots dx_N |\Phi(x, x', x_3, \dots, x_N)|^2$

can be computed exactly and is given by

$$\rho(x, x') = 1 - |\bar{\Psi}_A(x) \Psi_A(x')|^2 = 1 - |\bar{\Psi}_\alpha(x) \Psi_\alpha(x')|^2 \approx 1 - e^{-\frac{1}{4l_0^2}(x_\mu^2 + N_\alpha^2)} \quad (11)$$

where the explicit form of the single-particle wave function (Eq. 9) was used. In the approximation, we placed particle x' on the north poles of both the orbital and the isospin space, i.e. $x'_a = \delta_{5a}$ and $n'_i = \delta_{3i}$, and expanded in terms of $X_\mu^2 = R^2(x_1^2 + x_2^2 + x_3^2 + x_4^2)$ and $N_\alpha^2 = R^2(n_1^2 + n_2^2)$ in the limit

$$l_0^2 = \lim_{R \rightarrow \infty} \frac{R^2}{p}. \text{ We see that just like in the}$$

QHE liquid, a particle is accompanied by a

perfect correlation hole, gaussianly localized in its vicinity. The new feature in our case is that the incompressibility applies to both the charge and isospin channel.

Having discussed the generalization to the integer QHE, let us now turn to the fractional QHE. One can see that the many-body wave function $\Phi_m = \Phi^m(x_1, \dots, x_N)$ with odd integer m is also a legitimate fermionic wave function in the lowest $SO(5)$ level. This is so because the product of the basic spinors Ψ_α is always a legitimate state in the lowest $SO(5)$ level. Φ_m is a homogeneous polynomial of $\Psi_\alpha(x_i)$ with degree $p' = mp$. Therefore, the degeneracy of the lowest $SO(5)$ level in this case is

$$d(mp, 0) = \frac{1}{6}(mp + 1)(mp + 2)(mp + 3) \rightarrow \frac{1}{6} m^3 p^3, \text{ while the particle number is still}$$

$N = d(p, 0)$. The filling factor in this case is $\nu = N/d(mp, 0) = m^{-3}$. Although Φ_m cannot be expressed in the Laughlin form of a single product, we can still use plasma analogy to understand its basic physics. $|\Phi_m|^2$ can also be interpreted as the Boltzmann weight for a classical fluid, whose effective inverse temperature is $\beta_m = m\beta_{m=1}$. As the correlation functions for the $m = 1$ case can be computed exactly, it is plausible that the $m > 1$ case has similar correlations; in particular, it is also an incompressible liquid. However, the effective parameters need to be rescaled properly in the fractional case. The effective magnetic

length is given by $l'_0 = \frac{R}{\sqrt{p'}} = \frac{R}{\sqrt{mp}}$. This

incompressible liquid supports fractionalized charge excitation with charge m^{-3} . Such a state may be described by a wave function of the form $\Phi^{m-1} \Phi_n$, where Φ_n is the wave function of the integer case, where one hole is removed from a given location in the bulk interior to the edge of the fluid. To our knowledge, this is the first time that a quantum liquid with fractional charge excitation has been identified in higher dimension $d > 2$.

Emergence of relativity at the edge.

Before we go to the discussion of our model, let us first review how 1 + 1 dimensional relativity emerges at the edge of the 2D QHE problem. We shall restrict ourselves to the integer case only. In the III, there is no kinetic energy. The only energy is supplied by the confining potential $V(r)$, which confines the particles in a circular droplet of size R . Eigenfunctions in the III take the form $\phi_n(z)$

$$= z^n \exp\left(-\frac{|z|^2}{4l_0^2}\right). \text{ From this we see that a}$$

particle is localized in the radial direction at

$r_n = nl_0$, and it carries angular momentum $L = n$. Edge excitations are particle hole excitations of the droplet. A particle hole pair with the l labels n and m near the edge has energy $E = V_n - V_m = (n - m)l_0V'(R)$, and angular momentum $L = n - m$. Therefore, a relativistic linear relation exists between the energy and the momentum of the edge excitation. Furthermore, as $n - m > 0$, the edge waves propagate only in one direction; i.e., they are chiral. Therefore, we see that relativity emerges at the edge because of a special relation between the radial and the angular part of the wave function z^n . It turns out that such a relation also exists in the present context.

In our spherical model, we can introduce a confining potential $V(X_a) = V(X_5)$, where $V(X_5)$ is a monotonic function with a minimum at the north pole $x_5 = 1$ and a maximum at $x_5 = -1$. For $N < d(p, 0)$, the quantum fluid fills the configuration space around the north pole $x_5 = 1$, up to the ‘‘Fermi latitude’’ at x_5^F . Within the lowest $SO(5)$ level, there is no kinetic energy; only the confining potential $V(x_5)$ determines the energy scale of the problem. Although the $SO(5)$ symmetry of the S^4 sphere is broken explicitly by the confining potential, the $SO(4)$ symmetry is still valid. Without loss of generality, we can fill the orbital and isospin space so that the ground state is a $SO(4)$ singlet.

The orbital $SO(4)$ symmetry is defined to be the rotation in the (x_1, x_2, x_3, x_4) subspace, generated by the angular momentum operators $L_{\mu\nu}^{(0)} = -i(x_\mu\partial_\nu - x_\nu\partial_\mu)$ where $\mu, \nu = 1, 2, 3, 4$. These angular momentum operators satisfy $SO(4)$ commutation relations, which can be decomposed into the following two sets of $SU(2)$ angular momentum operators: $K_{1i}^{(0)} = \frac{1}{2}(L_i + P_i)$ and $K_{2i}^{(0)}$

$$= \frac{1}{2}(L_i - P_i), \text{ where } L_i = \frac{1}{2}\epsilon_{ijk}L_{jk}^{(0)}, P_i =$$

$L_{4i}^{(0)}$. Because of the coupling to the Yang monopole gauge potential, these orbital $SO(4)$ generators are modified into $K_{1i} = K_{1i}^{(0)}$ and $K_{2i} = K_{2i}^{(0)} + I_i$. Therefore, all edge states can be classified by their $SO(4)$ quantum numbers (k_1, k_2) , where $K_{1i}^2 = k_1(k_1 + 1)$ and $K_{2i}^2 = k_2(k_2 + 1)$, respectively. Applying these operators to the states in the lowest $SO(5)$ level (Eq. 9), we find that the state $|m_1, m_2, m_3, m_4\rangle$ has quantum numbers $m_1 + m_2 = 2k_2, m_1 - m_2 = 2k_{2z}, m_3 + m_4 = 2k_1$ and $m_3 - m_4 = 2k_{1z}$. In particular, the elementary $SO(5)$ spinors defined in Eq. 5 transform according to the $(0, 1/2)$ and $(1/2, 0)$ representations of $SO(4)$.

In the subspace of lowest $SO(5)$ levels defined by Eq. 9, the orbital coordinate operators x_a can be represented by $x_a = \frac{1}{p}\Psi\Gamma_a\frac{\partial}{\partial\Psi}$. From this we see that the $|m_1, m_2, m_3,$

$m_4\rangle$ state is also an eigenstate of px_5 , which takes quantized values $px_5 = m_1 + m_2 - m_3 - m_4$. Because $m_1 + m_2 + m_3 + m_4 = p, \frac{px_5}{2}$ can range over $p + 1$ values: $-\frac{p}{2}, -\frac{p}{2} + 1, \dots, \frac{p}{2}$. Therefore, for a given p and at

a fixed latitude on the orbital space x_5 , the $SO(4)$ quantum numbers (k_1, k_2) are given

$$\text{by } 2k_1 = \frac{p}{2}(1 - x_5) \text{ and } 2k_2 = \frac{p}{2}(1 + x_5).$$

The role of the radial coordinate in the 2D QHE problem is played by $1 - x_5$, which measures the distance away from the origin of the droplet at $x_5 = 1$. In the 2D case, the orbital angular momentum is simply a $U(1)$ phase factor. In our case, the orbital angular momentum is a $SO(4)$ Casimir operator,

whose eigenvalue is given by $2k_1 = \frac{p}{2}(1 - x_5)$. Therefore, just as in the 2D case, the distance away from the center of the droplet directly determines the magnitude of the orbital angular momentum. Because the confining potential can be linearized near the edge of the droplet $1 - x_5^F$, this relation translates into a massless relativistic dispersion relation. Furthermore, as we shall see, the coupling to the isospin degrees of freedom gives rise to particles with nontrivial helicity.

An edge excitation is created by removing a particle (leaving behind a hole) inside the Fermi latitude x_5^F , with quantum numbers $[x_5^h, k_1^h = \frac{p}{4}(1 - x_5^h), k_{1z}^h; k_2^h = \frac{p}{4}(1 + x_5^h), k_{2z}^h]$ and creating a particle outside the Fermi latitude, with quantum numbers $[x_5^p, k_1^p = \frac{p}{4}(1 - x_5^p), k_{1z}^p; k_2^p = \frac{p}{4}(1 + x_5^p), k_{2z}^p]$. This

excitation can also be specified by the quantum numbers $(\Delta x_5 = x_5^h - x_5^p; T_1, T_{1z}; T_2, T_{2z})$, where the total angular momenta $T_{1i} = K_{1i}^h + K_{1i}^p, T_{2i} = K_{2i}^h + K_{2i}^p, T_{1i}^2 = T_1(T_1 + 1)$ and $T_{2i}^2 = T_2(T_2 + 1)$ are the sums of the $SU(2) \times SU(2)$ quantum numbers of the particle and the hole. From the usual rules of the $SU(2)$ angular momentum addition, we can determine the allowed values of the total angular momenta $T_1 = |k_1^p - k_1^h|, \dots, k_1^p + k_1^h$ and $T_2 = |k_2^p - k_2^h|, \dots, k_2^p + k_2^h$. Given x_5^h and x_5^p , we obtain $\Delta x_5 = x_5^h - x_5^p = \frac{2}{p}n$, and the energy is given by

$$E \approx \frac{\partial V}{\partial X_5} \Delta X_5 = \frac{\partial V}{\partial X_5} \frac{2R}{p} n \quad (12)$$

In the 2D QHE case, there is a unique way to combine the angular momenta of a particle and a hole; therefore, the dispersion relation has only one branch. In higher dimensions, a particle and a hole can be bound or independent,

giving rise to collective and continuum branches of the spectrum. Mathematically, this effect manifests itself in terms of the different ways of combining the $SO(4)$ angular momenta of a particle and a hole. Let us investigate the possibility of collective excitations in the spectrum. In a noninteracting Fermi system with the usual form of kinetic energy $E = \mathbf{p}^2/2M$, a particle and a hole have a well-defined relative momentum but do not have a well-defined relative position, except in one spatial dimension. Therefore, such a pair can only be ‘‘bound’’ through an attractive interaction. However, there are very special cases where the pair can be bound for kinematic reasons, without any interactions. In one dimension, the kinetic energy is approximately independent of the relative momentum; therefore, one can superpose states with different relative momenta to obtain a state with well-defined relative position. The resulting state is a bosonic collective mode. In our case, we find that the special nature of the wave function in the lowest $SO(5)$ level leads to a similar form of the kinematic binding. Basically, there is no kinetic energy in the lowest $SO(5)$ level, and a particle and a hole can be locked into a well-defined relative position without any kinetic energy cost. In our case, these collective excitations lie at the edge of the continuum states and are characterized by the total $SO(4)$ quantum numbers $(T_1 = |k_1^p - k_1^h|$

$$= \frac{n}{2}, T_2 = T_1 + |\lambda|) \text{ and } (T_1 = T_2 + |\lambda|, T_2 = |k_2^p - k_2^h| = \frac{n}{2}),$$

where $|\lambda|$ is a positive integer and the $\lambda = 0$ case is counted only once. These states are formed by a macroscopic number of contractions of the spinor wave functions (Eq. 9) of a particle and a hole, and it can be shown explicitly that the wave function in the relative orbital and isospin coordinates are gaussianly localized. In this sense, a particle and a hole form a bound state and represent collective excitations of the system.

In the flat space limit, the $SO(4)$ symmetry group of S^3 reduces to the Euclidean group E_3 of the 3D flat space. The Euclidean group has two Casimir operators, and the magnitude of the momentum operator $|\mathbf{p}|$ is determined by either T_1 or T_2 , which in our case gives $|\mathbf{p}| = n/R$. As the energy is given by Eq. 12, the collective excitations have a relativistic linear dispersion relation $E = c|\mathbf{p}|$, with the speed of light given by $c = \frac{\partial V}{\partial x_5}$

$$\frac{2R^2}{p} = 2l_0^2 \frac{\partial V}{\partial X_5}. \text{ If we take for } l_0 \text{ the Planck}$$

length $l_p = 1.6 \times 10^{-35}$ m, we can estimate

$$\text{the potential energy gradient to be } \frac{\partial V}{\partial X_5} \approx$$

$$7.7 \times 10^{62} \text{ eV m}^{-1}.$$

The second Casimir operator of the Euclidean group is the helicity, $\lambda = \mathbf{J} \cdot \mathbf{p}/|\mathbf{p}|$, where \mathbf{J} is the total angular momentum of a particle. This quantity can be obtained from the $SO(4)$ quantum numbers by $\lambda = T_1 - T_2$

(16). Therefore, the $(T_1 = \frac{n}{2}, T_2 = T_1)$ state

describes a relativistic spinless particle obeying the massless Klein-Gordon equation.

The $(T_1 = \frac{n}{2}, T_2 = T_1 + 1)$ and the $(T_1 = T_2 + 1, T_2 = \frac{n}{2})$ states describe

massless photon states with left-handed and right-handed circular polarization. The associated fields satisfy Maxwell's equation.

The $(T_1 = \frac{n}{2}, T_2 = T_1 + 2)$ and the $(T_1 = T_2$

$+ 2, T_2 = \frac{n}{2})$ states describe massless graviton states with left-handed and right-handed circular polarization.

The associated fields satisfy the linearized Einstein equation. In fact, we can proceed this way to find all massless relativistic particles with higher spins. Here the time dimension is introduced to the problem through the energy of the confining potential (Eq. 12), whereas the space dimension is introduced through the Euclidean momentum. The relativistic dispersion together with the helicity quantum numbers show that the collective excitations form nontrivial representations of the Lorentz group. The spins of these massless particles are derived from the isospin degrees of freedom in the original Hamiltonian, and the relativistic field equations have their roots in the original isospin-orbital couplings.

So far we have obtained only a noninteracting theory of relativistic particles; in particular, the equation for the graviton is only obtained to the linear order. Once we turn to interactions among the different modes, the graviton would naturally couple to the energy momentum tensor of other particles. It is known that consistency requires the graviton to couple itself exactly, according to the full nonlinear Einstein equation (17, 18). Therefore, it is likely that the interaction among the edge modes in our model also contains the nonlinear effects of quantum gravity. On the other hand, the main problem with the current model seems to be an "embarrassment of riches." In order to define a problem with large degeneracy in the single-particle spectrum, one needs to take the limit of high representation of the isospin. Therefore, each particle has a large number of internal degrees of freedom. As a result, there are not only photons and gravitons in the collective

modes spectrum, there are also other massless relativistic particles with higher spins. However, the presence of massless higher spin states may not lead to phenomenological contradictions. It is known from field theory that massless relativistic particles with spin $s > 2$ cannot have covariant couplings to photons and gravitons (19). Therefore, it is possible that they decouple in the long wavelength limit.

Hall current and noncommutative geometry. So far, we have discussed only the quantum eigenvalue problem. It is also instructive to discuss the classical Newtonian equation of motion derived from the Hamiltonian $H + V(X_a)$, where H is given by Eq. 7. The classical degrees of freedom are the isospin vector I_i , the position X_a , and the angular momentum L_{ab} ; and their equations of motion can be derived from their Poisson bracket with the Hamiltonian. As we are interested in the equations of motion in the lowest $SO(5)$ level, we can take the infinite mass limit $M \rightarrow \infty$. In this limit, we obtain the following equations of motion

$$\dot{X}_a = \frac{R^4}{I^2} \frac{\partial V}{\partial X_b} F_{ab}^i I_i, \dot{I}_i = \epsilon_{ijk} A_{\mu}^j \dot{X}_\mu I_k \quad (13)$$

where the dot denotes the time derivative. Just as in the III problem, the momentum variables can be fully eliminated. However, the price one needs to pay for this elimination is that coordinates $[X_a, X_b]$ become noncommuting. In fact, the projected Hamiltonian in the lowest $SO(5)$ level is simply $V(X_a)$. If we assume the commutation relation $[X_a, X_b] = \frac{R^4}{I^2} F_{ab}^i$, then the orbital part of Eq. 13 can be derived from the Poisson bracket of X_a with $V(X_a)$. If we expand around the north pole $X_5 = R$, we finally obtain the following commutation relation

$$[X_\mu, X_\nu] = 4i l_0^2 \eta_{\mu\nu}^i \frac{I_i}{I} \quad (14)$$

This is the central equation underlying the algebraic structure of this work. It shows that there is a fundamental limit, l_0 , for the measurability of the position of a particle.

The first equation in Eq. 13 determines the Hall current for a given spin direction J_μ^i in terms of the gradient of the potential $\eta_{\mu\nu}^i \partial V / \partial X_\nu$, giving a direct generalization of the 2D Hall effect. From the second equation in Eq. 13, we see that the spin of a particle precesses around its orbital angular momentum (which becomes linear momentum in the flat space limit) with a definite sense.

Conclusion. At the conclusion of this work, we now know three different spatial dimensions where quantum disordered liquids exist: the 1D Luttinger liquid, the 2D quantum Hall liquid, and the 4D generalization found in this work. We can ask what

makes these dimensions special. There is a special mathematical property that singles out these spatial dimensions. One, two, and four dimensional spaces have the unique mathematical property that boundaries of these spaces are isomorphic to mathematical groups, namely the groups Z_2 , $U(1)$ and $SU(2)$. No other spaces have this property. It is the deep connection between the algebra and the geometry that makes the construction of nontrivial quantum ground states possible. Other related mathematical connections are reviewed and summarized in (11). The 4D generalization of the QHE offers an ideal theoretical laboratory to study the interplay between quantum correlations and dimensionality in strongly correlated systems. It would be interesting to study our quantum wave functions on 4D manifolds with nontrivial topology and investigate whether different topologies of four manifolds correspond to degeneracies of our many-body ground states. The quantum plateau transition in the 2D QHE is still an unsolved problem; one could naturally ask if the plateau transition in four dimensions can be understood better because of the higher dimensionality. In 2D QHE, quasi-particles have both anyonic and exclusion statistics. The former cannot exist in four dimensions; the question is whether quasi-particles in our theory would obey exclusion statistics in the sense of Haldane. To address these questions, it is important to construct a field theory description of the 4D quantum Hall liquid, in analogy with the Chern-Simons-Landau-Ginzburg theory of the QHE.

In this work, we investigated the possibility of modeling relativistic elementary particles as collective boundary excitations of the 4D quantum Hall liquid. Similar connections between condensed-matter and particle physics have been explored before (20–24). There are important aspects unique to the current problem (25). The single-particle states are hugely degenerate, which enables the limit of zero inertia mass $M \rightarrow 0$ and completely removes the nonrelativistic dispersion effects. This limit is hard to take in usual condensed-matter systems. The single-particle states also have a strong gauge coupling between iso-spin and orbital degrees of freedom, which is ultimately responsible for the emergence of the relativistic helicity of the collective modes. This type of coupling is not present in usual condensed-matter systems. The vanishing of the kinetic energy is the lowest $SO(5)$ levels enables binding of a particle and a hole into a pointlike collective mode. The most remarkable mathematical structure is the noncommutative geometry (Eq. 14), which expresses a $SU(2)$ co-cycle structure of the magnetic translation. Although progress reported in this work is still very

limited, we hope that this framework can stimulate investigations on the deep connection between condensed-matter and elementary particle physics.

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REPORTS

Groundwork for a Rational Synthesis of C₆₀: Cyclodehydrogenation of a C₆₀H₃₀ Polyarene

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A C₆₀H₃₀ polycyclic aromatic hydrocarbon (PAH) that incorporates all 60 carbon atoms and 75 of the 90 carbon-carbon bonds required to form the fullerene C₆₀ has been synthesized in nine steps by conventional laboratory methods. Laser irradiation of this C₆₀H₃₀ PAH at 337 nanometers induces hydrogen loss and the formation of C₆₀, as detected by mass spectrometry. A specifically labeled [¹³C₃]C₆₀H₃₀ retains all three ¹³C atoms during the cage formation process. A structurally related C₄₈H₂₄ PAH that lacks the three peripheral benzene rings cannot be transformed into C₆₀, whereas the next higher homolog, a C₈₀H₄₀ PAH, degrades to the C₆₀H₃₀ PAH, which then loses hydrogen to give [60]fullerene. These control experiments verify that the C₆₀ is formed by a molecular transformation directly from the C₆₀H₃₀ PAH and not by fragmentation and recombination in the gas phase.

Despite more than a decade of intensive research on fullerenes (1), chemists worldwide still have no general methods or strategies available for the rational synthesis of these polyhedral carbon allotropes as discrete, preselected targets. Under carefully controlled conditions, the vaporization of graphite generates substantial amounts of C₆₀ and C₇₀; however, this complicated process remains poorly understood and is intolerant to alteration (2). Higher fullerenes can be obtained from this source only in minuscule amounts through tedious chromatographic separations (3) and likely will never be available in quantity except by rational synthesis.

Before we can hope to develop rational syntheses of individual higher fullerenes, the goal of synthesizing C₆₀ by rational methods must first be met. In this connection, the research groups of Diederich and colleagues (4), Rubin *et al.* (5), and Tobe *et al.* (6–8) have all prepared macrocyclic polyalkynes that shed multiple appendages when subjected to laser desorption/ionization (LDI), and the high-energy intermediates thus generated collapse to C₆₀ in a mass spectrometer. The considerable ambiguity about which atoms in these molecular precursors become bonded to which other atoms as the fullerene takes shape, however, precludes characterization of these processes as entirely “rational” syntheses. Prinzbach *et al.* (9) recently reported a genuinely rational synthesis of icosahedral [5]fullerene-C₂₀, in which substituents were removed from a preformed dodecahedrane cage, but extensions of this approach to syntheses of fullerenes comprising 60 or more carbon atoms are likely to be difficult. In LDI experiments, the reactive C₂₀ entities

prepared in this way can be made to oligomerize and fuse into C₆₀ (10).

Here, we report the synthesis of a stable polycyclic aromatic hydrocarbon (PAH) that incorporates all 60 of the carbon atoms and 75 of the 90 carbon-carbon bonds required to form C₆₀, and its laser-induced cyclodehydrogenation to C₆₀ (Fig. 1). Control experiments establish that the C₆₀ formed in the final step comes from a direct molecular transformation (or “zipping up”) of the synthetic PAH **6**, as suggested in Fig. 1, and not by a laser-induced degradation of the hydrocarbon to smaller fragments that recombine in a thermodynamically driven manner, as in the laser-induced vaporization of graphite (2).

Several research groups have independently conceived of a laboratory synthesis of C₆₀ from **6** or structurally related 60-carbon compounds. Wang and Shevlin in Alabama were the first to report preliminary experimental work in this direction (11). Their approach can be traced back to an earlier proposal from the same laboratory (12); however, they encountered difficulty in assembling the 60-carbon fullerene precursor. Our synthesis of **6** was designed to ensure a regular head-to-tail cyclotrimerization of a sickle-shaped 20-carbon precursor, **5**. While our work was under way, Sarobe *et al.* in the Netherlands found that attempts to synthesize **6** from a different precursor without controlling the head-to-tail regiochemistry gave the C₃-symmetric PAH **6** only as a minor component in an inseparable mixture of C₆₀H₃₀ regioisomers (13). Gomez-Lor *et al.* in Spain later prepared **6** as a single regioisomer by threefold annulation of truxene, a venerable 27-carbon, C_{3v}-symmetric PAH (14, 15). Neither Sarobe *et al.* nor Gomez-Lor *et al.*, however, were able to convert their synthetic C₆₀H₃₀ material to C₆₀.

Our synthesis of **6** (16) began with commercially available (1-bromoethyl)benzene, **1**, and 2-naphthaldehyde (17). These were joined by a Wittig reaction to give alkene **2** as a mixture of (*E*)- and (*Z*)-isomers in a combined yield of 79% after purification. Oxidative photocyclization of **2** under standard conditions

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