

overview:

1. Why do we need statistical physics?

a: single particle physics; Q.M + Relativity

$$i\hbar \frac{\partial}{\partial t} \psi = \hat{H} \psi$$

b: two-body system:

Pauli exclusion principle - fermion

symmetrize wavefunction - boson

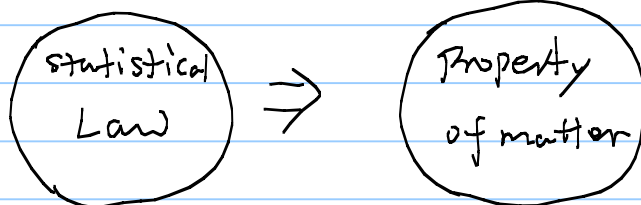
c: Many-body systems:

* individual particle law: too complicated to track

*: New principles and laws:

individual v.s. society

d: Physical systems: \rightarrow billions of particles



2, Statistical Properties of matter:

2

a: Thermal Properties:

Specific heat, entropy, free energy,
thermal conductance
heat transport

b: electric properties:

conductance, resistivity,
spin susceptibility ...

c: Phases; Classification:

Ferromagnetic
Antiferromagnetic
Ferroelectricity ...

Solid
liquid
gas

d: Low energy "single particle" physics:

{
⊗ New emergent excitation!
⊗ Collective modes
→ differs from the "original" particle!

example:

QHE:

Anyon:

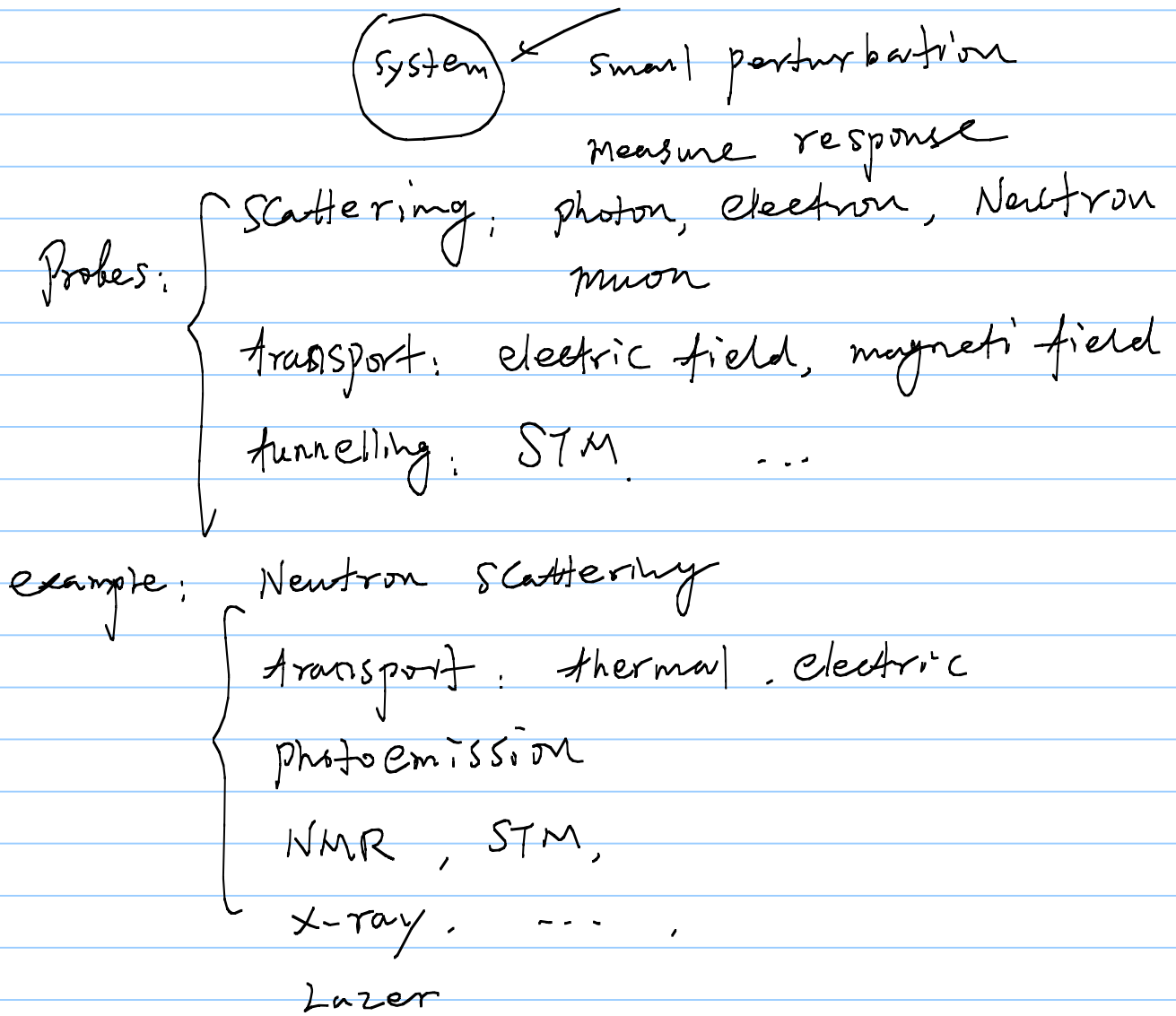
Lattice:

phonon:

Spin wave:

magnon:

3: Measurement : response function:



4: General principle:

a: phenomenology theory:

*: Landau - Ginzburg theory: (macroscopic variable)

*: Symmetry analysis

*: New principles

b: microscopic theory:

*: Q.M. or classical physics \rightarrow Hamiltonian

*: models

*: mean field solution

*: exact solution

*: Numerical simulation

c: Numerical method:

*: Band structure. DFT, LDA

*: Monte-Carlo simulation ..

5: What to expect in Phys 6705:

① models: Ising model, xy model
Heisenberg model

② techniques: * mean field solution
* Renormalization
* Landau Ginzburg approach

③: Phase transition:

* Universality: Critical exponents

* Scaling - Renormalization

* Symmetry breaking and Goldstone Theorem

* Quantum phase transition

④: ?? : disorder, nonequilibrium ----

Review of Statistical Physics;

1: Thermodynamic Variables;

inner energy
entropy
work

Example magnet;

$$dU = Tds - MdH$$

Partition function: $Z(T, H) = \sum_r e^{-\beta E_r}$

r: all microscopic configurations

Free energy;

$$F = -\frac{1}{\beta} \ln Z$$

$$\beta = \frac{1}{kT}$$

First derivative,

$$U = \frac{\partial \beta F}{\partial \beta} = F + \beta \frac{\partial F}{\partial \beta} = - \frac{\partial \ln Z}{\partial \beta}$$

$$(Z \sim e^{-\beta F} \rightarrow \ln Z = -\beta F \rightarrow F = - \frac{\partial \ln Z}{\partial \beta})$$

$$S = - \left(\frac{\partial F}{\partial T} \right)_H = \frac{1}{k_B T^2} \frac{\partial F}{\partial \beta} = \frac{1}{T} \beta \frac{\partial F}{\partial \beta} = \frac{1}{T} (U - F)$$

$$(TS + F = U)$$

(U: include free energy and entropy)

$$M = - \left(\frac{\partial F}{\partial H} \right)_T \quad \text{magnetization.}$$

Second derivative:

Specific heat:

$$C_H = T \left(\frac{\partial S}{\partial T} \right)_H$$

Spin susceptibility:

$$\chi_T = \left(\frac{\partial M}{\partial H} \right)_T$$

General:

$$\begin{cases} U = TS + \mu N + \vec{J} \cdot \vec{X} & \dots \text{fundamental relation} \\ dU = Tds + \mu \cdot dN + \vec{J} \cdot d\vec{X} & \dots \text{First-law.} \end{cases}$$

$$\Rightarrow: SdT + Ndu + \vec{X} \cdot d\vec{J} = 0 \quad \dots \text{Gibbs-Duhem relation}$$

$$\begin{cases} \text{extensive: } U, S, N, \vec{X} \\ \text{intensive: } T, \mu, \vec{J} \end{cases}$$

* Bose-Einstein statistics:

$$\langle n \rangle = \frac{1}{\exp\left(\frac{E}{kT}\right) - 1}$$

* Fermi-Dirac statistics

$$\langle n \rangle = \frac{1}{\exp\left(\frac{E}{kT}\right) + 1}$$