Advanced Statistical physics

Note Title 8/5/2008

overview:

1. Why do we need statistical physics ?

a: single particle physics : Q.M + Relativity

$$i\hbar \frac{\partial}{\partial t}\psi = \hat{H}\psi$$

b: two-body system:

Pauli exclusion principle – fermion

symmetrize wavefunction – Boson

c: Many-body systems:

* Individual particle law: too complicated to track

*: New principles and laws:

Individual v.s. society

d: Physical systems: → billions of particles

statistical Law $\Rightarrow$ Property of matter

2. Statistical Properties of matter:

a: Thermal properties:

Specific heat, entropy, free energy,
thermal conductance
heat transport

b: electric properties:

Conductance, Resistivity,
spin Susceptibility ...

c: Phases: Classification: $\begin{cases} \text{solid} \\ \text{liquid} \\ \text{gas} \end{cases}$

Ferromagnetic
Antiferromagnetic
Ferroelectricity ...

d: Low energy "single particle" physics:

$\left\{ \begin{array}{l} \otimes \text{ New emergent excitation!} \\ \otimes \text{ Collective modes} \end{array} \right.$

differs from the "original" particle!

example:

QHE: Anyon:
Lattice: phonon:
spin wave: magnon:

3: Measurement : response function:

system ← small perturbation

measure response

Probes:
- Scattering: photon, electron, Neutron, muon
- transport: electric field, magneti field
- tunnelling: STM. ...

example: Neutron scattering
- transport: thermal, electric
- photoemission
- NMR, STM,
- X-ray, ... ,
- Lazer

4: General principle:

a: phenomenology theory:

*: Landau - Ginzburg theory: (macroscopic Variable)

*: Symmetry analysis

*: New principles

b: microscopic theory:

*: Q. M. or classical physics → Hamiltonian

*: models

*: mean field solution

* exact solution

* Numerical simulation

c: Numerical method:

*: Band structure. DFT, LDA

*: monte-Carlo simulation ...

5: What to expect in Phys 6705:

(1): models: Ising model, xy model

Heisenberg model

(2) techniques: * mean field solution

* Renormalization

* Landau Ginzburg approach

(3): Phase transition:

* Universality: Critical exponents.

*: Scaling - Renormalization

* Symmetry breaking and Goldstone Theorem

* Quantum phase transition

(4): ??: disorder, nonequilibrium ....

Review of statistical physics:

1: Thermodynamic Variables: $\begin{cases} \text{inner energy} \\ \text{entropy} \\ \text{work} \end{cases}$

Example magnet:

$$dU = TdS - MdH$$

Partition function: $Z(T, H) = \sum_r e^{-\beta E_r}$

r: all microscopic configuration

Free energy:

$F = -\frac{1}{\beta} \ln Z$ $\qquad \beta = \frac{1}{kT}$

First derivative.

$$U = \frac{\partial \beta F}{\partial \beta} = F + \beta \frac{\partial F}{\partial \beta} = -\frac{\partial \ln Z}{\partial \beta}$$

$$\left(Z \sim e^{-\beta F} \rightarrow \ln Z = -\beta F \rightarrow F = -\frac{\partial \ln Z}{\partial \beta} \right)$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_H = \frac{1}{k_B T^2} \frac{\partial F}{\partial \beta} = \frac{1}{T} \beta \frac{\partial F}{\partial \beta} = \frac{1}{T}(U - F)$$

$(TS + F = U)$

(U: include free energy and entropy)

$M = -\left(\frac{\partial F}{\partial H}\right)_T$ magnetization.

Second derivative:

Specific heat:

$$C_H = T\left(\frac{\partial S}{\partial T}\right)_H$$

Spin susceptibility:

$$\chi_T = \left(\frac{\partial M}{\partial H}\right)_T$$

General:

$$\begin{cases} U = TS + \mu N + \vec{J}\cdot\vec{x} & \text{... fundamental relation} \\ dU = TdS + \mu\cdot dN + \vec{J}\cdot d\vec{x} & \text{... First-law.} \end{cases}$$

$$\Rightarrow: \quad SdT + Nd\mu + \vec{x}\cdot d\vec{J} = 0 \quad \text{... Gibbs-Duhem relation}$$

$$\begin{cases} \text{extensive}: & U.\ S,\ N,\ \vec{x} \\ \text{intensive}: & T.\ \mu,\ \vec{J} \end{cases}$$

*. Bose-Einstein statistics:

$$\langle n \rangle = \frac{1}{\exp\left(\frac{E}{kT}\right) - 1}$$

*. Fermi-Dirac statistics

$$\langle n \rangle = \frac{1}{\exp\left(\frac{E}{kT}\right) + 1}$$