Homework 6 Phys660

- Problem 1: We consider a two particle system,

$$H(x_1, p_1, x_2, p_2) = H_1(\hat{x}_1, \hat{p}_1) + H_2(\hat{x}_2, \hat{p}_2) + V(|x_1 - x_2|) \tag{1}$$

where the two Hamiltonians $H_1(x,p) = H_2(x,p)$. For this system, we have a new symmetry, exchange symmetry, i.e. $H(x_1, p_1, x_2, p_2) = H(x_2, p_2, x_1, p_1)$.
(a) Construct the quantum operator associated with the exchange symmetry and show it is conserved.
(b) What are the eigenvalues and eigenstates of the quantum operator?
(c) Let's assume that there is no interaction between two particles, i.e. $V = 0$. The complete set of eigenstates for one particle Hamiltonian H_i is given by $\{|\psi_n>, E_n\}$. Write the complete set of eigenstates for the two particle system as the eigentstates of the exchange operator.
(d) What do you expect for a system with many identical particles?

- Problem 2: (ordering in antiunitary operator)
Consider $\hat{\theta}$ is an antiunitary operator which is defined in position base, i.e. $\hat{\theta}\sum_x f(x)|x> = \sum_x f^*(x)|x>$. Let $\hat{\theta}^+$ is the adjoint operator of $\hat{\theta}$. Evaluate the following values,
(a)$< x'|\hat{\theta}^+ \sum_x f(x)|x>$
(b)$< x'|\sum_x f(x)\hat{\theta}^+|x>$
(c) $<\psi|\hat{\theta}^+ \sum_x f(x)|x>$, where $|\psi> = \sum_x g(x)|x>$.

- Problem 3:
(a)In Galilean group, a pure boost element, $(1, \vec{v}, 0, 0)$ and, a pure space translation, $(1, 0, \vec{a}, 0)$, is their product, $(1, \vec{v}, 0, 0)(1, 0, \vec{a}, 0) = (1, 0, \vec{a}, 0)(1, \vec{v}, 0, 0)$?
(b) Now consider the quantum operators associated with the pure boost and space translation,$\hat{U}(1, \vec{v}, 0, 0)$, $\hat{U}(1, 0, \vec{a}, 0)$. Do they commutate with each other? Derive the relation when you exchange these two operators.

- Problem 4:
Explicitly show that the rotation operator associated with rotating ϕ around an axis in $\vec{n}$ direction is given by $exp(-\frac{i}{\hbar}\phi\vec{n}\cdot\hat{\vec{L}})$.

- Problem 5
Consider a spin-one particle, $\hat{S}$. Let $\hat{S}(n) = \frac{\sqrt{3}}{3}(S_x + S_y + S_z)$. The eigenstates of $\hat{S}(n)$ are labeled to be $|1>, |0>, |-1>$. What are the matrix representations of S_x, S_y, S_z in the above new eigenstate base?