Homework 1 Phys660

- Problem 1:
Take position base $|x>$. On the one hand, we obtain,

$$< x|\hat{x}\hat{p}|x >= x < x|\hat{p}|x >$$

and

$$< x|\hat{p}\hat{x}|x >= x < x|\hat{p}|x >,$$

so

$$< x|\hat{x}\hat{p}|x > - < x|\hat{p}\hat{x}|x >= 0;$$

on the other hand, we have

$$< x|\hat{x}\hat{p}|x > - < x|\hat{p}\hat{x}|x >=< x|[\hat{x},\hat{p}]|x >= i\hbar < x|x > .$$

Which one is the correct answer? Why?

The following formula (Problem 2-5) are very useful in quantum mechanics:

- Problem 2: Let $f(\hat{x})$ and $g(\hat{p})$ are two functions of $\hat{x}$ and $\hat{p}$ respectively, prove,

$$[\hat{x}, g(\hat{p})] = i\hbar \frac{\partial g(\hat{p})}{\partial \hat{p}} \tag{1}$$

$$[\hat{p}, f(\hat{x})] = -i\hbar \frac{\partial f(\hat{x})}{\partial \hat{x}} \tag{2}$$

$$\tag{3}$$

- Problem 3 : Prove the following useful formula:

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2!}[\hat{A},[\hat{A},\hat{B}]] + \frac{1}{3!}[\hat{A},[\hat{A},[\hat{A},\hat{B}]]] + ... \tag{4}$$

- Problem 4: Assume $[\hat{A},\hat{C}] = i\hat{B}, [\hat{B},\hat{C}] = -i\hat{A}$, show that

$$e^{i\theta\hat{C}}\hat{B}e^{-i\theta\hat{C}} = \hat{B}cos\theta - \hat{A}sin\theta \tag{5}$$

$$e^{i\theta\hat{C}}\hat{A}e^{-i\theta\hat{C}} = \hat{A}cos\theta + \hat{B}sin\theta \tag{6}$$

- Problem 5: Let $\hat{C} = [\hat{A},\hat{B}]$, and $\hat{C}$ is commuting with both $\hat{A}$ and $\hat{B}$, show that

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}\hat{C}} = e^{\hat{B}}e^{\hat{A}}e^{\frac{1}{2}\hat{C}} \tag{7}$$

- Problem 6: In physics, there are important constants: Plank constant $\hbar$, Electron charge e, Speed of light c, electric constant ϵ_0 and Newtonian constant of Gravitation G. In quantum mechanics, we expect that the value of the physical observables become descrete. There should be a fundamental quantum unit for each physical observable. The quantum unit associated with a certain observable is called the observable quantum. For example, the quantum unit associated with angular momentum is $\hbar$, i.e. angular momentum quantum is $\hbar$. Using above constants to construct quantum unit for the following observable quantums and estimate their values (if some observable quantums can not be constructed, please state your reason):
(a) Length quantum
(b) Mass quantum
(c) Time quantum
(d) Momentum quantum
(e) Electric conductance quantum
(f) Electric resistance quantum
(g) Magnetic flux quantum
(h) Electric voltage quantum
(i) Electric current quantum
(j) Magnetic moment quantum
(k) Electric Capacitance quantum

- Problem 7: (particle number and phase operators: coherent state)
Consider an quantum operator $\hat{N}$, the complete eigenstates of the operator is given by $|n>$, with $\hat{N}|n>=n|n>, n=0,1,2,.....$ Now we construct the following state $|\phi>=\sum_n e^{in\phi}|n>$, where $0\leq\phi<2\pi$ which are the complete eigentstates of a quantum operator $\hat{\phi}$, $\hat{\phi}|\phi>=\phi|\phi>$.
(1) Prove that
$$[\hat{\phi},\hat{N}]=i$$
(2) Write down the identity operator in terms of the base: $|\phi>$

- Problem 8 (review classical physics in presence of magnetic field): Let Φ and $\vec{A}$ be the scalar and vector potential of electromagnetic field. Derive the Lagrangian and Hamiltonian for a single particle in the presence of the fields. Write down the dynamic equations for the particle.