Heisenberg Uncertainty Principle

Commutators

- Commutator:

$$[\hat{A}, \hat{B}] = \hat{C}$$

- If $\hat{C} \neq 0$, eigenvalues of $\hat{A}, \hat{B}$ can not be simultaneously determined.

Uncertainty Principles

- Define $$\sigma_A = < (\hat{A} - < A >)^2 >$$

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} < \hat{C} >\right)^2$$

- For $$\hat{A} = \hat{x}; \hat{B} = \hat{p}; \hat{C} = i\hbar$$

$$\sigma_x^2 \sigma_p^2 \geq \frac{\hbar^2}{4} \quad \Longrightarrow \quad \Delta X \Delta P \geq \frac{\hbar}{2}$$

Time-Energy Uncertainty Principle

- Define $$\Delta t \Delta E \geq \frac{\hbar}{2}$$

$$\frac{d < \hat{A} >}{dt} = \frac{i}{\hbar} < [\hat{H}, \hat{A}] > + < \frac{\partial \hat{A}}{\partial t} >$$

- Take $$\hat{A}; \hat{B} = \hat{H};$$

$$\sigma_A^2 \sigma_H^2 \geq \frac{\hbar^2}{4} \left(\frac{d < \hat{A} >}{dt}\right)^2$$

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

$$\Delta t = \frac{\sigma_A}{\left|\frac{d<\hat{A}>}{dt}\right|}$$

Δt: the amount of time it takes the expectation value of A to change by one standard deviation.