Schrodinger Equation

Phys 460, Fall 2009, JP Hu

Basic Idea

- Particle and wave are two different things
- Particle is described by position and momentum

Classical physics

- Particle = Wave
- Particle is described by wave-functions or states

Quantum physics

Schrodinger Equation

- Consider an non-relativistic particle with mass min a potential V(x):

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

$\Psi(x,t)$ is wavefunction.

$$\hbar = h/2\pi = 1.054(10^{-34})Js$$

Interpretation (I)

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = (-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x))\psi(x,t)$$

Connection to Energy: Kinetic Energy Potential energy

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = H\psi(x,t)$$

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

Interpretation (II)

- Statistical Interpretation

$$\int_a^b |\psi(x,t)|^2 \, dx$$

Probability to find the particle between a and b, at time t

- Normalization condition: total probability =1

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 \, dx = 1$$

Note, the wavefunction in general is a complex function.

Interpretation (III)

- Expectation values of position:

$$< x >= \int_{-\infty}^{\infty} x \,|\psi(x,t)|^2 \, dx$$

- Expectation value of velocity and momentum

$$\frac{d < x >}{dt} = -\frac{i\hbar}{m}\int_{-\infty}^{\infty} \psi^*(x,t)\frac{\partial}{\partial x}\psi(x,t)dx$$

$$< P >= -i\hbar\int_{-\infty}^{\infty} \psi^*(x,t)\frac{\partial}{\partial x}\psi(x,t)dx$$

Interpretation (IV)

- Quantum Operator

$$< x >= \int_{-\infty}^{\infty} \psi(x,t)^* x \psi(x,t) dx$$

$$< P >= \int_{-\infty}^{\infty} \psi^*(x,t)(-i\hbar \frac{\partial}{\partial x})\psi(x,t) dx$$

Position:
Momentum:

$$x \rightarrow \hat{x}$$

$$P \rightarrow \hat{P} = -i\hbar \frac{\partial}{\partial x}$$

- For any quantum operators

$$< Q(x,P) >= \int_{-\infty}^{\infty} \psi^*(x,t) Q(x, -i\hbar \frac{\partial}{\partial x}) \psi(x,t) dx$$

Example: Kinetic energy $T=p^2/2m$

Ehrenfest Theorem

- Ehrenfest Theorem:

$$\frac{d<P>}{dt} = < -\frac{\partial V}{\partial x} >$$

The expectation value satisfies the classical equation.

Commutation Relation

• Fundamental Commutation Relation:

$$\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$$

$$[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x}$$

$$[\hat{x}, \hat{p}] = i\hbar$$

Quantum Measurement: wavefunction collapse

- A wavefunction of a cat: (dead + live)
- Measurement: The cat is dead.

Is my cat dead before my measurement?